DEPARTMENT OF CIVIL ENGINEERING

CE-222 STRUCTURAL MECHANICS I

Quiz-2 27/3/10

Problem 1

Beam *ABC* is supported at *A* and *B*. Beam *CDE* is supported at *D*. Both beams are hinged together at *C* **(Fig. 1)**. Find the vertical deflection of *E* **using Conjugate beam method.**

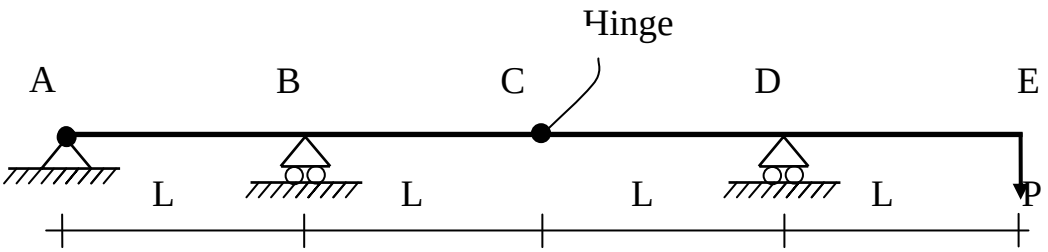

Fig. 1

Problem 2

Frame *ABCD* is supported and loaded as shown in **Fig. 2**. Find the rotation of *D* **using Castigliano's theorem.**

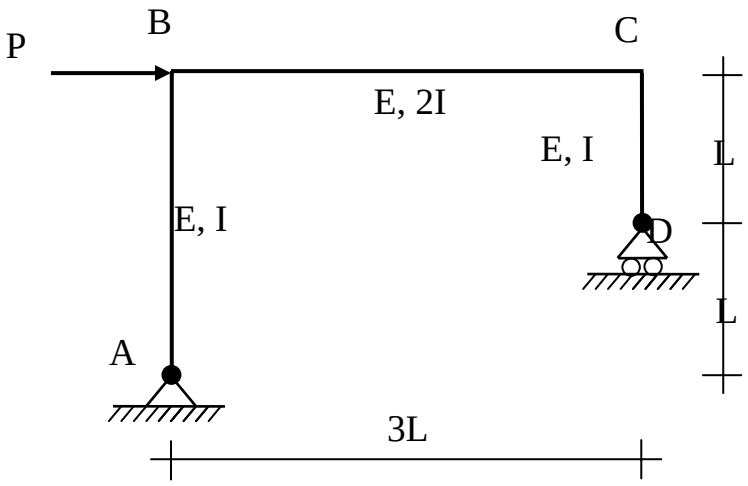

Fig. 2

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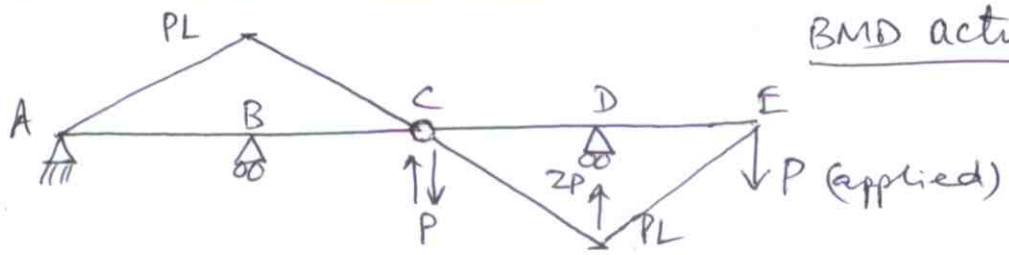
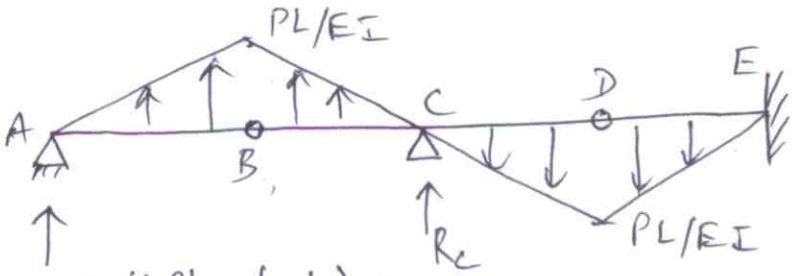
1. BMD actual beam.



Loading on conjugate beam



$\sum M_B = 0 \Rightarrow R_A = -\frac{1}{L}\left(\frac{1}{2}PL \cdot L \cdot \frac{L}{3}\right)\cdot\frac{1}{EI}$

$= -\frac{PL^2}{6}\frac{1}{EI}$

$\sum M_D = 0 \Rightarrow R_c = -\frac{1}{L}\left[\frac{1}{2}PL\cdot L\left(\frac{5}{3}L + \frac{7}{3}L - \frac{1}{3}L\right) + \left(-\frac{PL^2}{6}\right)(3L)\right]\cdot\frac{1}{EI}$

$R_c = \frac{PL^2}{EI}\left[-\frac{11}{6}+\frac{3}{6}\right] = -\frac{4}{3}\frac{PL^2}{EI}$

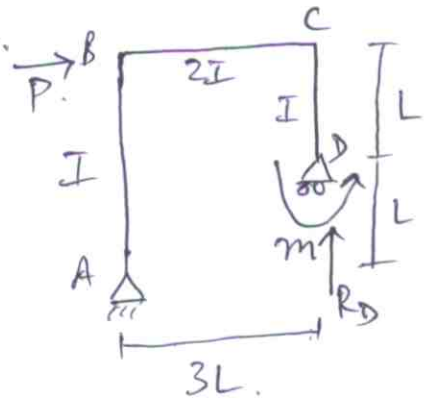
$(\Delta_{VE})_{actual} = (M_E)_{conj} = -\frac{PL^2}{6EI}(4L) - \frac{4}{3}\frac{PL^2}{EI}(2L) + \frac{1}{2}(2L)\left(\frac{PL}{EI}\right)(3L-L) = \frac{PL^3}{EI}\left(-\frac{4}{6}-\frac{8}{3}+12\right)$

$\Delta_{VE} = -PL^3\cdot\frac{4}{3}$ ◀ (ie ⇓).

Check by Castigliano's: $\Delta_{VE} = \frac{\partial U}{\partial P} = \frac{1}{2}\frac{\partial}{\partial P}\int \frac{M^2}{EI}dx = 4\cdot\frac{1}{2}\cdot\frac{\partial}{\partial P}\int_0^L \frac{P^2x^2}{EI}dx$

$= 2\cdot\frac{2P}{EI}\frac{L^3}{3} = \frac{4}{3}\frac{PL^3}{EI}$ ie ⇓ (in dirn of P) ◀

2.



Apply ↻ m at D.

$R_D = (2PL - m)/3L$

	$\frac{\partial M}{\partial m}$
$M = m$, DC	$= 1$, DC
$= m + \frac{(2PL-m)}{3L}x$, CB	$= 1 - \frac{x}{3L}$, CB
$= Px$, AB	$= 0$, AB

$\theta_D = \frac{1}{EI}\left[\int_0^L (m)(1)\,dx + \frac{1}{2}\int_0^{3L}\left[m + \left(\frac{2PL-m}{3L}\right)x\right]\left[1-\frac{x}{3L}\right]dx\right]\Big|_{m=0}$

$= \frac{1}{2EI}\int_0^{3L}\frac{2}{3}Px\left(1-\frac{x}{3L}\right)dx = \frac{1}{2EI}\frac{2}{3}P\left[\frac{(3L)^2}{2} - \frac{(3L)^3}{3\cdot 3L}\right] = \frac{PL^2}{3EI}\left[\frac{9}{2}-\frac{27}{9}\right]$

$= \frac{PL^2}{2EI}$ ↰ ◀