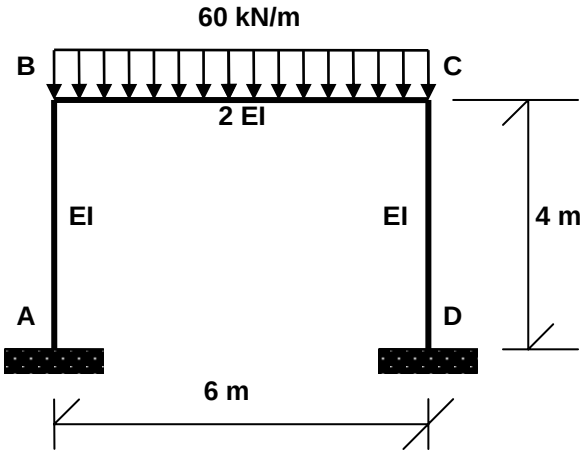
CE-222 STRUCTURAL MECHANICS I

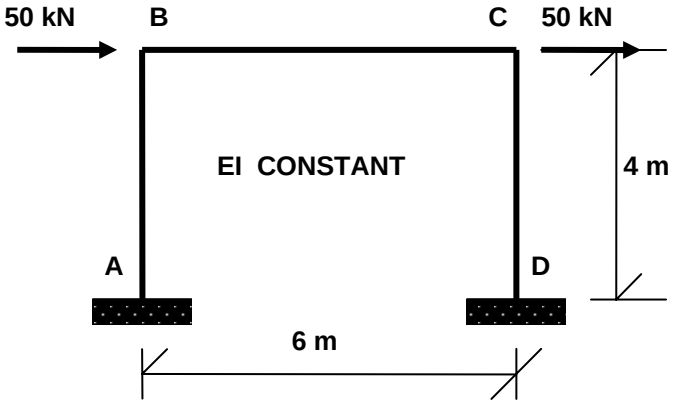
DEPARTMENT OF CIVIL ENGINEERING

Tutorial Assignment # 12: Statically Indeterminate Structures

Reduction of Degree of Indeterminacy using Symmetry

Draw the **Shear Force and Bending Moment Diagrams** for the following systems. Sketch the **Qualitative Deflected Shapes**. Use method of consistent deformations and standard notations and signs.





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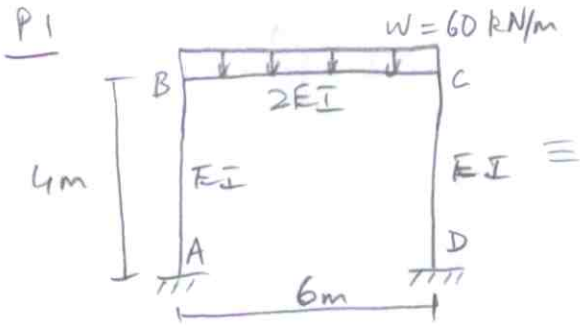

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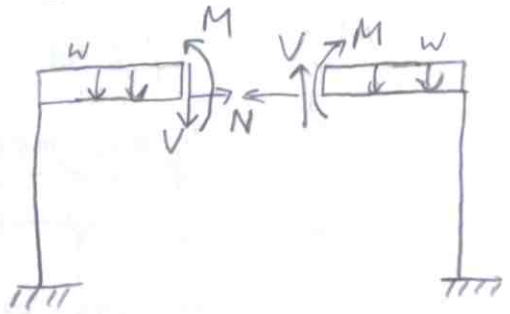
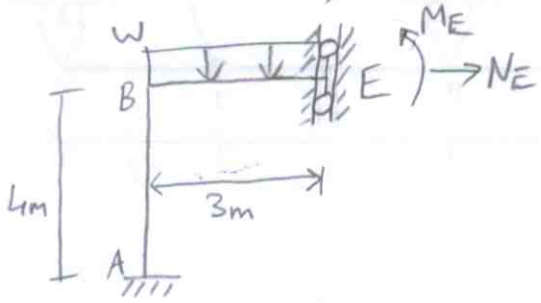


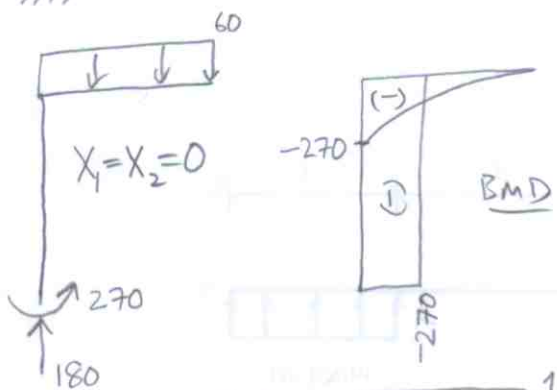
Symmetry ⇒ V = 0 (ie look from behind and all internal forces should be same magnitude & direction).

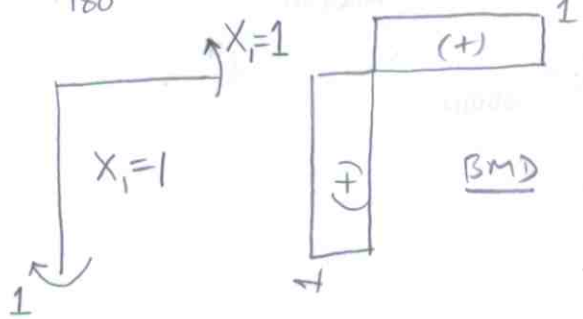
So we solve,

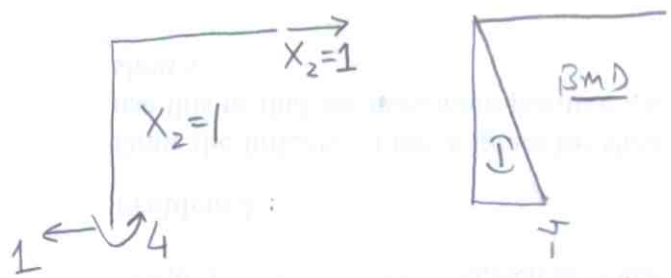


Done by VW. Can also do by Castigliano's.

$X_1 = M_E$, $X_2 = N_E$







$$\Delta_{10} = (1)\frac{(-270)(4)}{EI} + \int_0^3 -\frac{60}{2}x^2(1)\cdot\frac{1}{2EI}\,dx$$

$$= -1215/EI$$

$$f_{11} = (1)(1)(4)/EI + (1)(1)(3)/2EI = \frac{5.5}{EI}$$

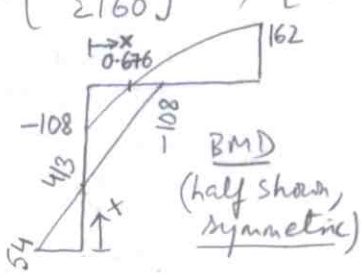
$$f_{12} = \frac{1}{2}(-4)(1)(4)/EI = -8/EI$$

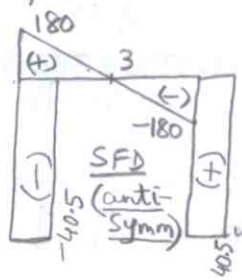
$$\Delta_{20} = \frac{1}{2}(-4)(-270)(4)/EI = 2160/EI$$

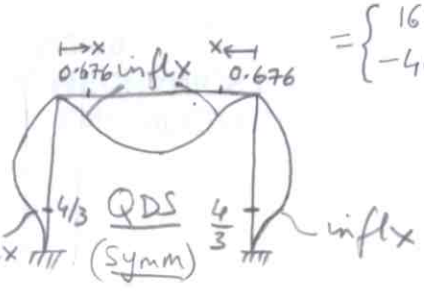
$$f_{22} = \frac{1}{3}(-4)(-4)(4)/EI = 64/3EI$$

$$\frac{1}{EI}\begin{Bmatrix}-1215\\2160\end{Bmatrix} + \frac{1}{EI}\begin{bmatrix}5.5 & -8\\ -8 & 64/3\end{bmatrix}\begin{Bmatrix}X_1\\X_2\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix}\begin{matrix}\Delta_1\\ \Delta_2\end{matrix} \Rightarrow \begin{Bmatrix}X_1\\X_2\end{Bmatrix} = \frac{3}{160}\begin{bmatrix}64/3 & 8\\ 8 & 5.5\end{bmatrix}\begin{Bmatrix}1215\\-2160\end{Bmatrix}$$

$$= \begin{Bmatrix}162\\-40.5\end{Bmatrix}$$







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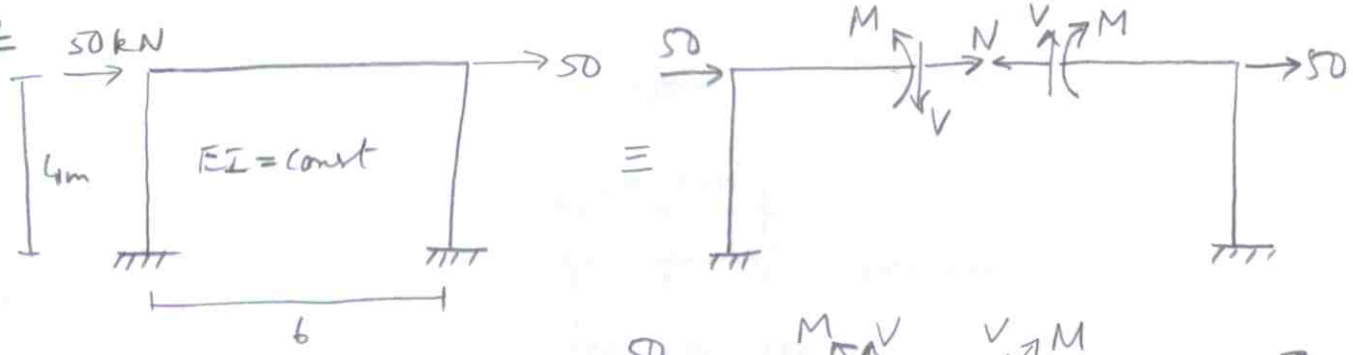
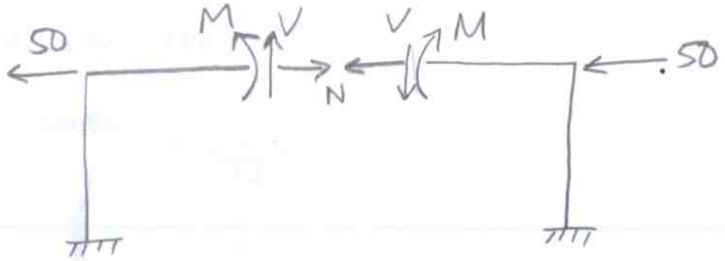
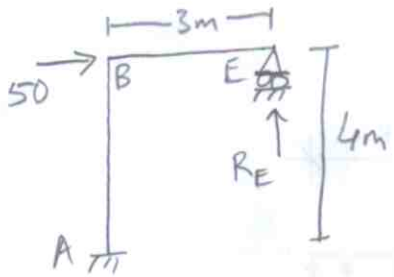


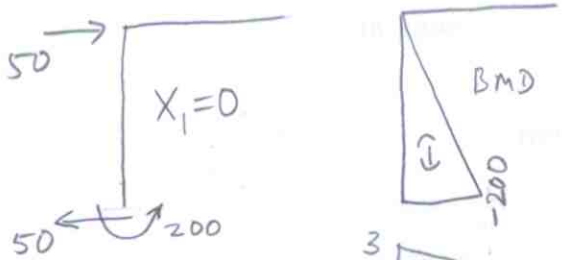


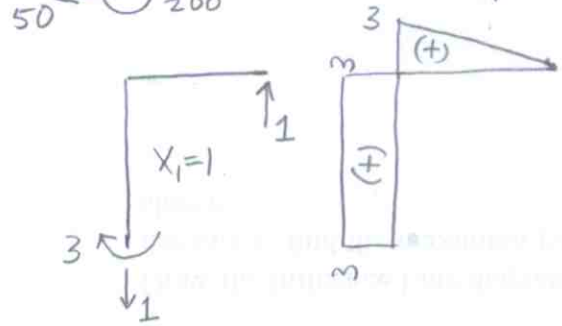
Look from backside →
ie, it looks like original with applied loads reversed
⇒ internal forces should also reverse ⇒ M = N = 0 ⇒ 1 D.O.I. (ie, V)

So we solve



$X_1 = R_E$

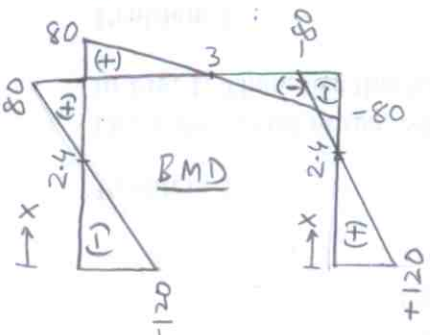




$\Delta_{10} = \frac{1}{2}(3)(-200)(4)/EI = -1200/EI$

$f_{11} = \left[(3)(3)(4) + \frac{1}{3}(3)(3)(3)\right]/EI = 45/EI$

$\Delta_{10} + f_{11} X_1 = 0 \Rightarrow X_1 = 80/3 = R_E$



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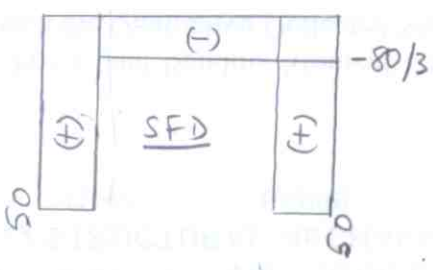
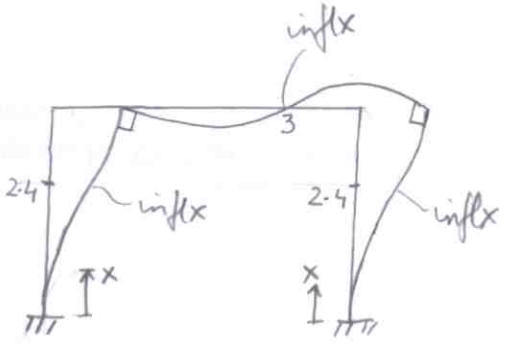


(Symmetric)



(antisymm)