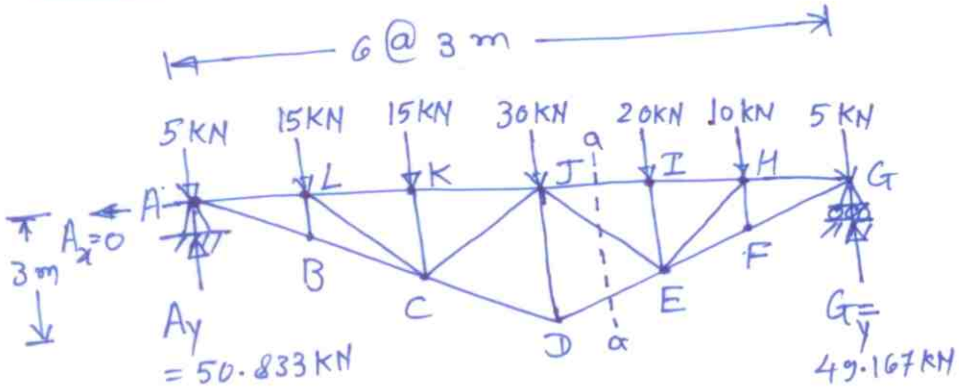
CE 222: Structural Mechanics-I

Tutorial No. 1 Solutions

1. sol^n:



$\Sigma F_x = 0$

$\therefore A_x = 0$

$\circlearrowright^{+} \Sigma M_G = 0;$

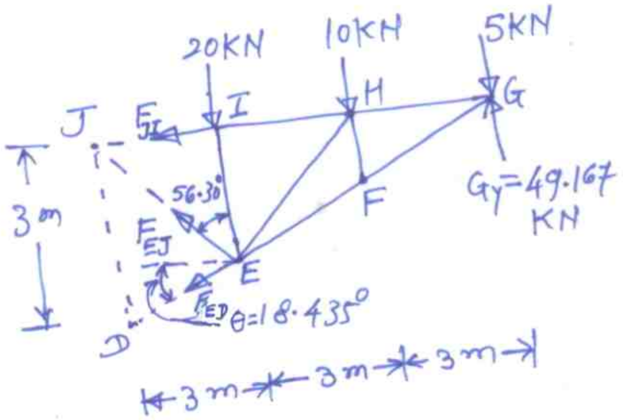
$A_y \times 18 - 5 \times 18 - 15 \times 15 - 15 \times 12 - 30 \times 9 - 20 \times 6 - 10 \times 3 = 0$

$\boxed{\therefore A_y = 50.833 \text{ KN} \uparrow}$

$\Sigma F_y = 0;$

$\Rightarrow \boxed{G_y = 49.167 \text{ KN} \uparrow}$

Consider F.B.D of portion of truss on R.H.S. of section a-a



$\circlearrowright^{+} \Sigma M_E = 0;$

$F_{JI} \times 2 - 10 \times 3 - 5 \times 6 + G_y \times 6 = 0$

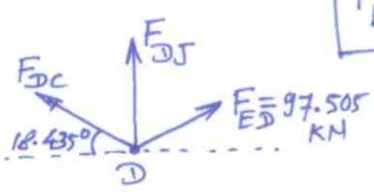
$\boxed{F_{JI} = -117.50 \text{ KN}}$ (comp)

$\circlearrowright^{+} \Sigma M_J = 0;$

$-F_{ED} \cos(18.435°) \times 2 - F_{ED} \sin(18.435°) \times 3 - 20 \times 3 - 10 \times 6 - 5 \times 9 + 49.167 \times 9 = 0$

$\boxed{F_{ED} = 97.505 \text{ KN}}$ (Tens.)

Joint D:



From F.B.D. of joint D

$\xrightarrow{+} \Sigma F_x = 0;$

$$97.505 \cos(18.435^\circ) - F_{DC} \cos(18.435^\circ) = 0$$

$$\Rightarrow F_{DC} = 97.505 \text{ KN} \quad \text{(Tens)}$$

$+\uparrow \Sigma F_y = 0;$

$$F_{DJ} + 2 \times 97.505 \times \sin(18.435^\circ) = 0$$

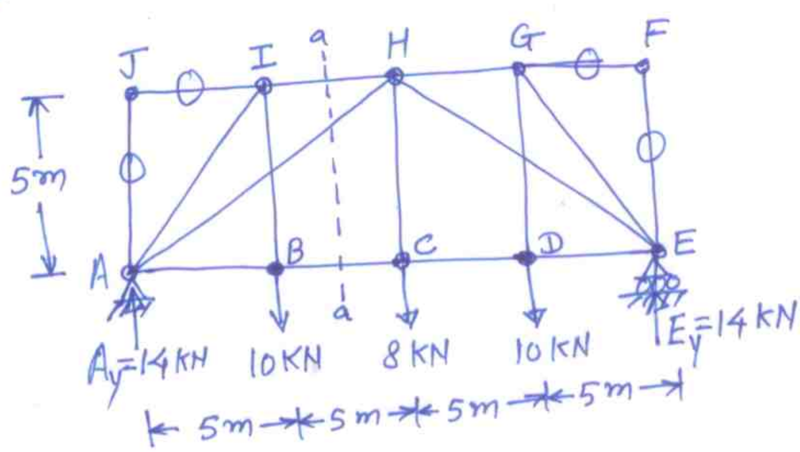
$$\Rightarrow \boxed{F_{DJ} = -61.67 \text{ KN}} \quad \text{(comp.)}$$

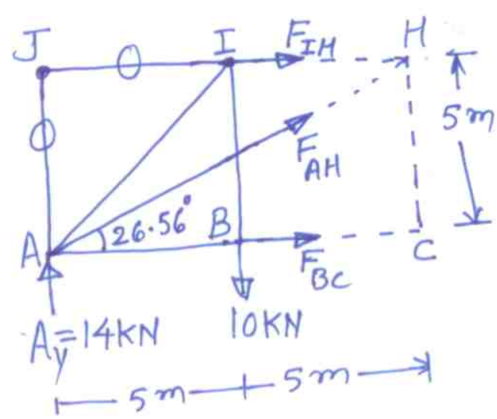
Ans:- $F_{JI} = 117.50$ KN (comp); $F_{ED} = 97.505$ KN (Tens.)

$F_{DJ} = 61.67$ KN (comp.)

xox

2. Forces in members IH, AH and BC ?





Due to symmetric loading

$$\boxed{A_y = E_y = 14 \text{ KN}}$$

Consider F.B.D. of L.H.S. Portion of truss due to section a-a

$\circlearrowleft + \Sigma M_H = 0;$ $F_{BC} \times 5 + 10 \times 5 - 14 \times 10 = 0$

$$\Rightarrow \boxed{F_{BC} = 18\ KN} \quad (\text{Tens.})$$

$\curvearrowleft + \Sigma M_A = 0$; $\quad - F_{IH} \times 5 - 10 \times 5 = 0$

$$\Rightarrow \boxed{F_{IH} = -10\ KN} \quad (\text{comp.})$$

$+\uparrow \Sigma F_y = 0$; $\quad F_{AH} \cdot \sin(26.56^\circ) - 10 + 14 = 0$

$$\Rightarrow \boxed{F_{AH} = -8.946\ KN} \quad (\text{comp})$$

$F_{BC} = 18\,KN$ (Tens); $F_{IH} = +10\ KN$ (comp); $F_{AH} = 8.946\,KN$ (comp.)

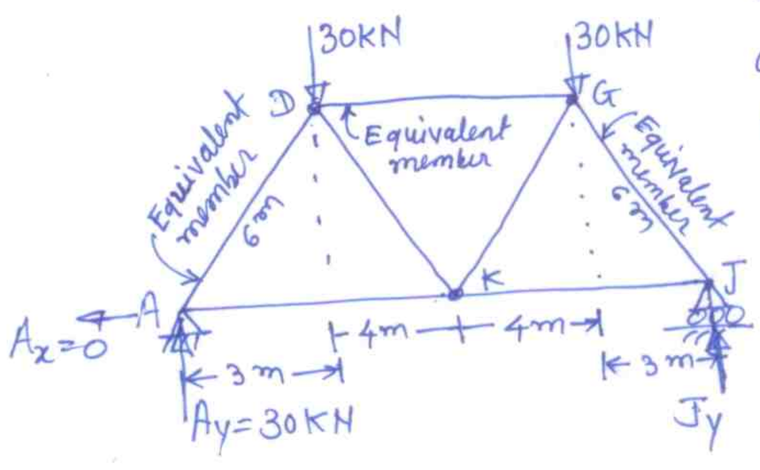
x o x

3. In Fig. 3, truss has $m = 19$, $r = 3$, $j = 11$, Therefore it satisfies the equation

$$m = 2j - r$$

and from figure, we confirm the truss as statically determinate and stable.

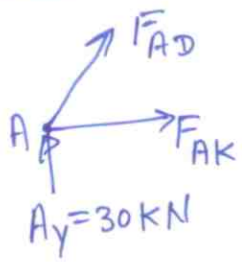
Replacing trusses ABDC, GHJI and DEGF by equivalent simple members AD, GJ and DG resp. simplifies the problem.



Here $A_y = J_y = 30\ KN$

Joint equilibrium method

Joint A :



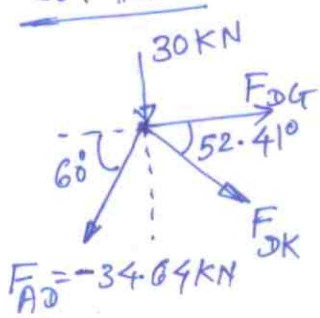
$+\uparrow \sum F_y = 0$; $F_{AD} \sin(60^\circ) + 30 = 0$

$\Rightarrow \boxed{F_{AD} = -34.64\,KN}$ (comp)

$\xrightarrow{+} \sum F_x = 0$; $F_{AK} + F_{AD} \cos(60^\circ) = 0$

$\Rightarrow \boxed{F_{AK} = 17.32\,KN}$ (Tension)

Joint D



$+\uparrow \sum F_y = 0$; $F_{DK} \cdot \sin 52.41 - 34.64 \sin 60^\circ + 30 = 0$

$\Rightarrow \boxed{F_{DK} = 0}$

$\xrightarrow{+} \sum F_x = 0$; $F_{DG} + 34.64 \cos(60) = 0$

$\Rightarrow \boxed{F_{DG} = -17.32\,KN}$ (comp)

From symmetry of loading,

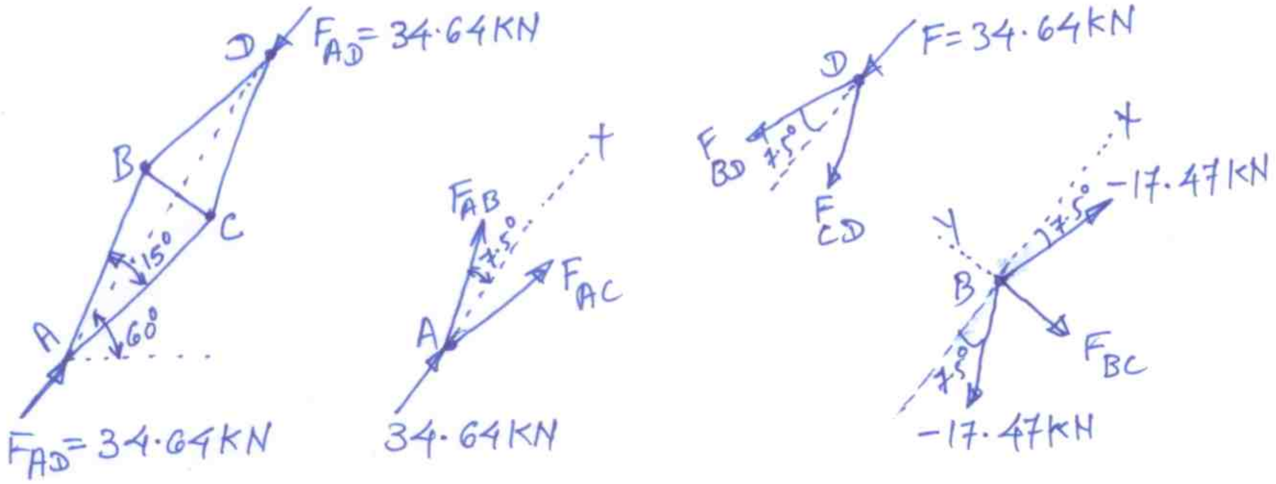
$\boxed{F_{GJ} = -34.64\,KN}$ (comp)

$\boxed{F_{KJ} = 17.32\,KN}$ (Tens.)

$\boxed{F_{GK} = 0}$

Truss ABDC :

Here the forces at the ends A & D of secondary truss ABDC are equal to the force in equivalent member AD i.e $F_{AD} = -34.64$ KN in the direction of AD



Joint A of ABDC

$\Sigma F_y = 0$; $\quad F_{AB} = F_{AC}$

$\Sigma F_x = 0$; $\quad F_{AC}\cos(7.5^\circ) + F_{AB}\cos(7.5^\circ) + 34.64 = 0$

$$\Rightarrow 2F_{AC} = -\frac{34.64}{\cos(7.5^\circ)}$$

$$\Rightarrow \boxed{F_{AC} = F_{AB} = -17.47 \text{ KN}} \text{ (comp)}$$

Joint D :

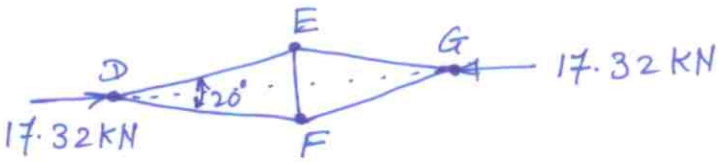
$$\boxed{F_{BD} = F_{CD} = -17.47 \text{ KN}} \text{ (comp)}$$

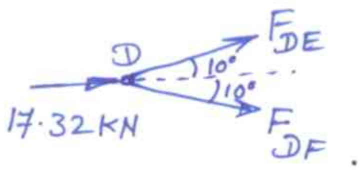
Joint B :

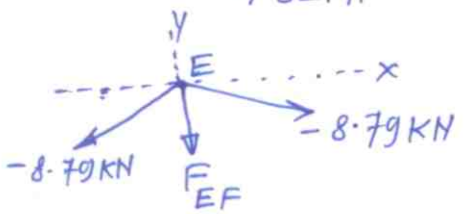
$\Sigma F_y = 0$; $\quad F_{BC} = 2 \times 17.47 \sin(7.5^\circ)$

$$\boxed{F_{BC} = 4.56 \text{ KN}} \text{ (Tens.)}$$

Truss DEGF :- Forces acting at the ends of truss DEGF are equal to $F_{DG} = -17.32\text{ KN}$ in the direction DG.







Joint D of DEGF

$+\uparrow \sum F_y = 0 ;\Rightarrow F_{DE} = F_{DF}$

$\overset{+}{\rightarrow} \sum F_x = 0 ; \quad 2F_{DE} = -\dfrac{17.32}{\cos(10^\circ)} \Rightarrow F_{DE} = -8.79\text{ KN}$

$\Rightarrow \boxed{F_{DE} = F_{DF} = -8.79\text{ KN}}$ (comp)

Joint G : From symmetry

$\boxed{F_{GE} = F_{GF} = -8.79\text{ KN}}$ (comp)

Joint E :

$+\uparrow \sum F_y = 0 ; \Rightarrow F_{EF} = 2 \times 8.79 \times \sin(10^\circ)$

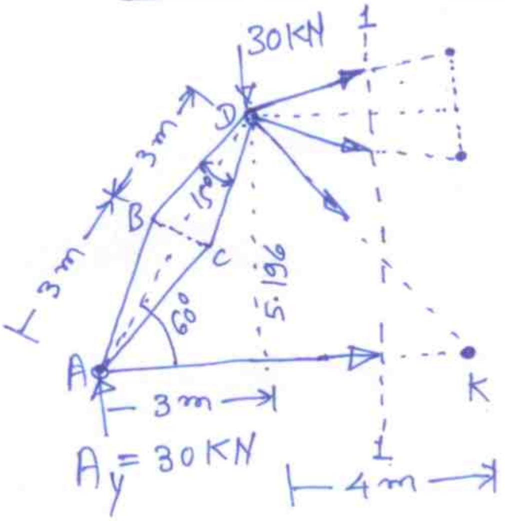
$\boxed{F_{EF} = 3.05\text{ KN}}$ (Tens.)

Truss GHJI : Due to symmetry forces in the members of secondary truss GHJI are,

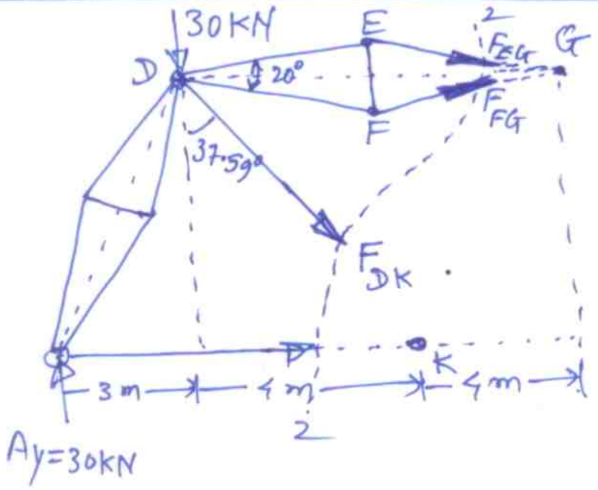
$\boxed{F_{JH} = F_{JI} = F_{HG} = F_{IG} = 17.47\text{ KN}}$ (comp)

$\boxed{F_{IH} = 4.56\text{ KN}}$ (Tens.)

3. Alternative method — Section method & joint method



Section 1-1.



Section 2-2

F.B.D. of part of truss due to section 1-1 :—

$+\circlearrowleft \sum M_D = 0$; $\quad F_{AK} \times 5.196 - 30 \times 3 = 0$

$$\boxed{F_{AK} = 17.32 \text{ KN}} \text{ (Tens.)}$$

F.B.D due to section 2-2 :—

$+\circlearrowleft \sum M_G = 0$; $\quad F_{DK} \cos(37.59) \times 8 + 17.32 \times 5.196 - 30 \times 7 + 30 \times 4 = 0$

$$\boxed{F_{DK} = 0}$$

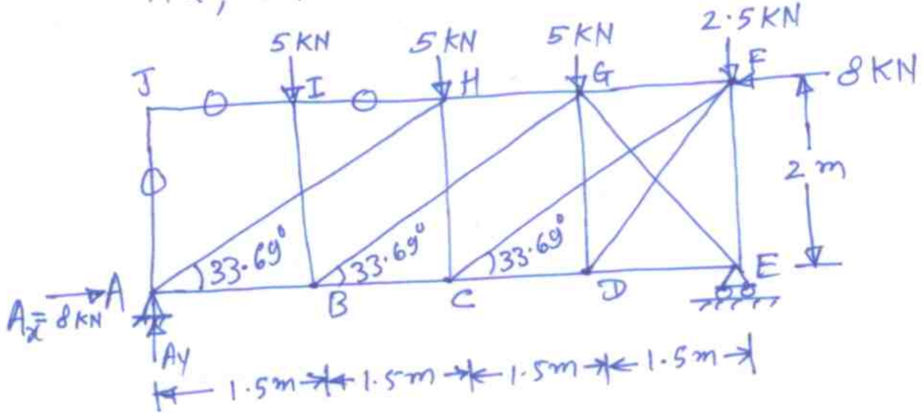
$+\uparrow \sum F_y = 0$; $\Rightarrow F_{EG} = F_{FG}$

$\xrightarrow{+} \sum F_x = 0$; $\quad F_{EG} \times \cos(10^\circ) + F_{FG} \cos(10^\circ) + 17.32 = 0$

$$\Rightarrow \boxed{F_{EG} = F_{FG} = -8.79 \text{ KN}} \text{ (comp.)}$$

We can determine the forces in the remaining members by joint equilibrium method.

Q-4 : Sol^n: Forces to be determined in members HG, BG and BC.



No. of members $= m = 18$

No. of External reaction $= r = 3$

No. of joints $= j = 10$

Check for determinacy

$$m = 2j - r$$

$$18 > 2 \times 10 - 3 = 17$$

The truss is statically indeterminate internally to degree 1. Portion GFED is over rigid.

Reactions : FBD of whole truss:

$\overset{+}{\rightarrow}\Sigma F_x = 0 \Rightarrow \boxed{A_x = 8 \text{ KN}}$

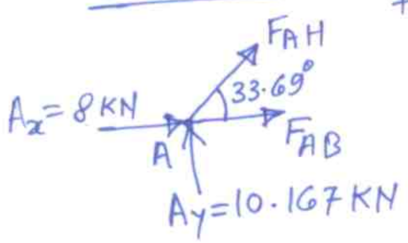
$+\circlearrowleft \Sigma M_A = 0$; $E_y \times 6 - 2.5 \times 6 + 8 \times 2 - 5 \times 4.5 - 5 \times 3 - 5 \times 1.5 = 0$

$\Rightarrow \boxed{E_y = 7.333 \text{ KN}}$

$+\uparrow \Sigma F_y = 0$; $+A_y - (+5+5+5+2.5) + 7.333 = 0$

$\Rightarrow \boxed{A_y = 10.167 \text{ KN}}$

Joint A :

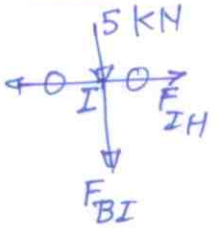


$+\uparrow \Sigma F_y = 0$; $\boxed{F_{AH} = \dfrac{-10.167}{\sin(33.69)} = -18.33 \text{ KN}}$ (comp)

$\overset{+}{\rightarrow}\Sigma F_x = 0$; $F_{AB} + 8 - 18.33 \cos(33.69°) = 0$

$\boxed{F_{AB} = 7.92 \text{ KN}}$ (Tens.)

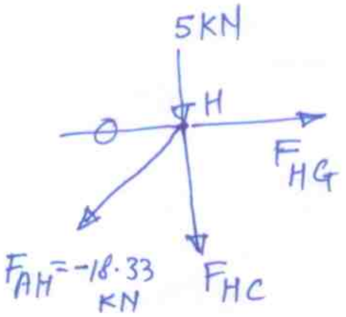
Joint I :



$\boxed{F_{BI} = -5\ KN}$ (comp)

$\boxed{F_{IH} = 0\ KN}$

Joint H :



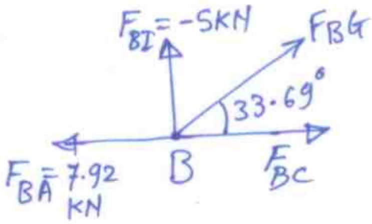
$\xrightarrow{+} \Sigma F_x = 0$; $F_{HG} - (-18.33 \times \cos 33.69) = 0$

$\Rightarrow \boxed{F_{HG} = -15.25\ KN}$ (comp.)

$+\uparrow \Sigma F_y = 0$; $-F_{HC} - 5 - (-18.33 \sin 33.69) = 0$

$\Rightarrow \boxed{F_{HC} = 5.18\ KN}$ (Tens)

Joint B :



$+\uparrow \Sigma F_y = 0$; $F_{BG} \sin(33.69°) - 5 = 0$

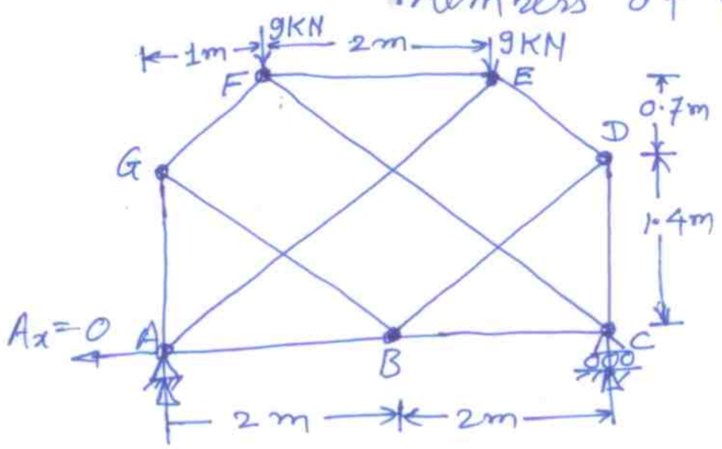
$\Rightarrow \boxed{F_{BG} = 9.014\ KN}$ (Tens.)

$\xrightarrow{+} \Sigma F_x = 0$; $F_{BC} - 7.92 + 9.014 \times \cos(33.69°) = 0$

$\Rightarrow \boxed{F_{BC} = 0.42\ KN}$ (Tens.)

———— x o x ————

Q: 5. Sol^n : determination of forces in all members of truss in Fig. 5.



No. of members, m = 11

No. of joints, j = 7

No. of reactions, r = 3

Here $m = 2j - r$

i.e $11 = 2 \times 7 - 3 = 11$

From above equation and visual inspection, it is stable and statically determinate.

Reactions :

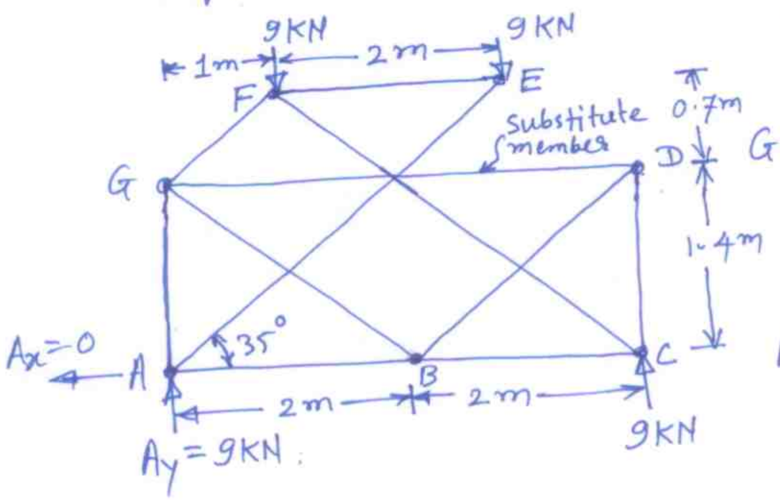
Due to symmetric loading, we can obtain reactions as

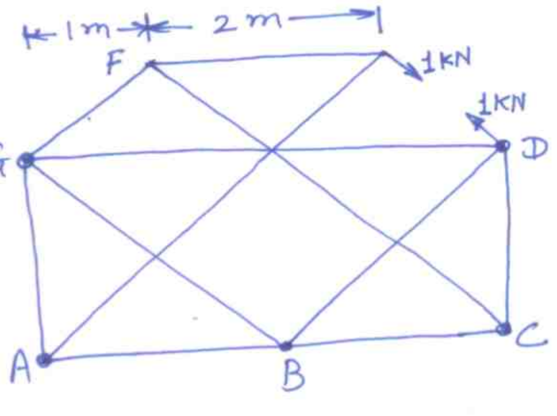
$$\boxed{\begin{array}{l} A_y = C_y = 9\text{ KN } \& \\ A_x = 0 \end{array}}$$

Forces in the members :

It is complex truss and at every joint there are three unknowns. Joint equilibrium method is possible but it involves lengthy solution of simultaneous equations.

Here analysis is done by using principle of superposition





Step 1 : Remove a member of truss such that you can start from one of the joint, however, at the same time substitute a member between other joints cleverly to keep the truss stable and determinate. In this example, the member ED is removed and substitute member GD is placed between joints G & D.

Step 2: Determine the forces in each member under external loads and reactions for this new construction Let the forces be denoted by F'

Step 3: In this step apply unit forces at joints E & D apposite to each other in the direction of ED and without external forces. Find out the forces in members due to these unit forces Let it be F''.

Step 4: Superposition:

The force in substitute member must be equal to zero. If α is the actual force in the member ED (Removed member), then,

Final force in GD = (Force in GD, calculated in step 2) + α (Force in GD due to unit load)

= 0

This eqn gives the value of α

Final force in any other member

$= F' + \alpha F''$

Members	Forces in members step 2 F' KN	Forces in members step 3 F'' KN	Factor α	Final force in members $F = F' + \alpha F''$
GD	0	0	0	0
AB	12.84 KN(T)	No need to calculate forces in members as $\alpha = 0$, in this case		12.84 KN (T)
BC	12.84 KN(T)			12.84 KN (T)
CD	0			0
DE	0			0
EF	12.84 KN(T)			12.84 KN (T)
FG	0			0
AE	−15.679 KN (Comp)			−15.679 KN (Comp)
BD	0			0
BG	0			0
CF	−15.679 KN			−15.679 KN (comp.)

Q 6:- Sol^n: Truss is resting on smooth supports at A, B, C and D, and it is symmetrically loaded Therefore reactions, $R_A = R_B = R_G = R_H = 3$ KN(↑) in vertical directions.

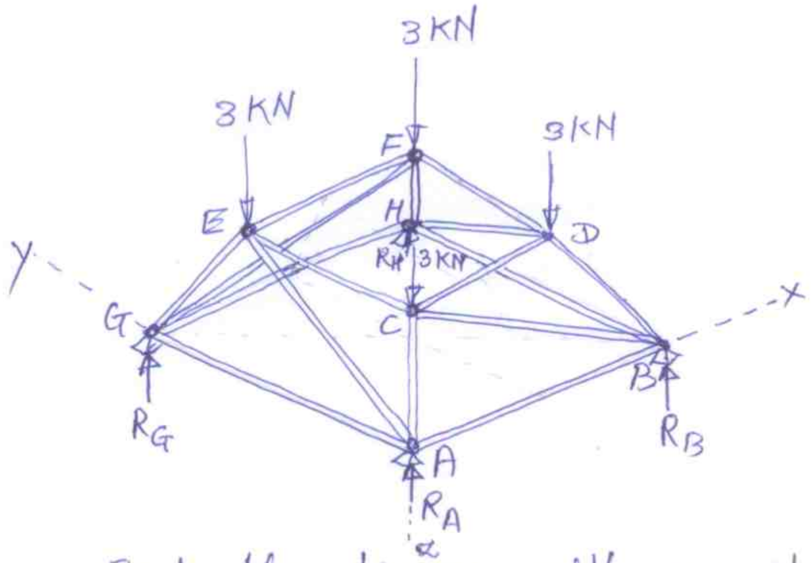
Due to symmetry of truss and loading on it, forces developed in the following members are of same value,

CB = GF = AE = HD ;

CD = DF = FE = EC ;

AB = AG = GH = HB ;

AC = BD = HF = GE ;



Cut the truss with yz plane (i.e with x as normal) exposing the forces in the members AB, CB, CD, EF, GH and GF. Note that

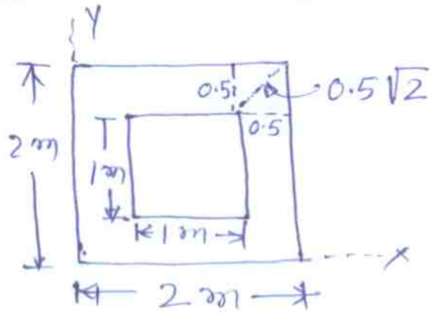
CB = GF.

Sum the forces in y-direction. Note CD, AB, GH & EF have components only in x-direction, and $(CB)_y$ & $(GF)_y$ add up to $2(CB)_y$ where $(CB)_y$

is y component of CB. Thus $\Sigma F_y = 0$, gives

$$2(CB)_y = 0 \Rightarrow CB = 0$$

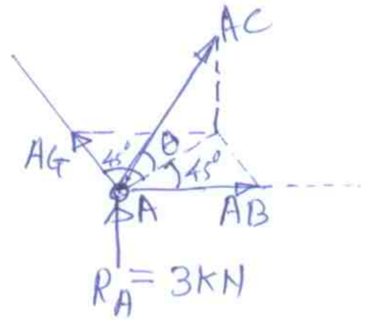
consider joint A :



The angle made by AC member with xy plane

$$\theta = \tan^{-1}\left(\frac{1.5}{0.5\sqrt{2}}\right) = 64.761^\circ$$

where height of truss = 1.5 m



By using joint equilibrium method we have,

$+\uparrow \Sigma F_z = 0$;

$$AC \sin(64.761) + 3 = 0$$

$$\Rightarrow \boxed{AC = -3.3166 \text{ KN}} \text{ (comp)}$$

component of AC in xy plane

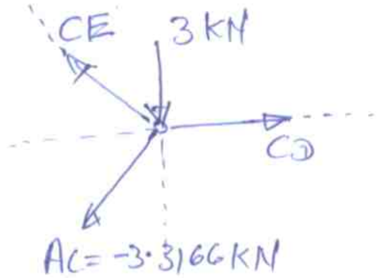
$$AC \times \cos(64.761^\circ) = -3.3166 \times \cos(64.761^\circ)$$
$$= -1.41418 \text{ KN}$$

$\Sigma F_x = 0$; $\quad AB + (-1.41418 \times \cos(45^\circ)) = 0$

$$\Rightarrow \boxed{AB = 0.99999 = 1 \text{ KN}} \text{ (Tens.)}$$

$$\Rightarrow \boxed{AG = 1 \text{ KN}} \text{ (Tens.)}$$

Joint C :-



component of AC in x-y plane

$$AC \cos(64.761) = -1.41418 \text{ KN}$$

$\Sigma F_x = 0$; $\quad CD - (-1.41418 \times \cos 45^\circ) = 0$

$$\Rightarrow \boxed{CD = -1 \text{ KN}} \text{ comp.}$$

$$\Rightarrow \boxed{CE = -1 \text{ KN}} \text{ (comp.)}$$

AC = BD = 3.166 KN (comp), AB = 1 KN (Tens.)
CD = 1 KN (comp)

Q 7

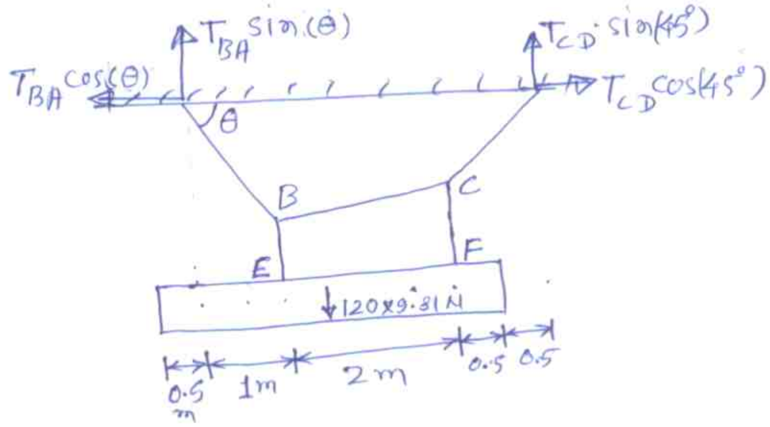


Fig- 7(b)

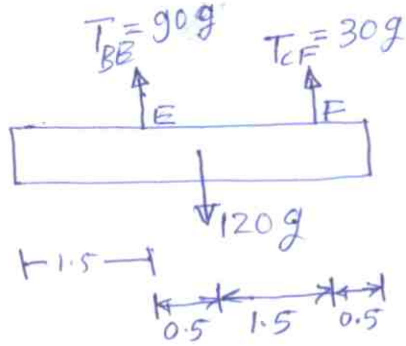


Fig. 7(a)

FBD of beam:

$\circlearrowleft \sum M_F = 0$; $T_{BE} = \dfrac{120(g)\times 1.5}{2} = 90g \text{ N} = 882.9 \text{ N}$

$+\uparrow \sum F_y = 0$; $T_{CF} = (120-90)g = 30g \text{ N} = 294.3 \text{ N}$

FBD of whole structure:

$\circlearrowleft \sum M_A = 0$; $T_{CD}\sin(45°)\times 4 - (120\times 1.5)g = 0$

$$\boxed{T_{CD} = 63.6396\,g = 624.30 \text{ N}}$$

$\xrightarrow{+} \sum F_x = 0$; $T_{AB}\cos\theta = T_{CD}\cos(45°) = 45g$

$+\uparrow \sum F_y = 0$; $T_{AB}\sin\theta = (120-45)g$

$\Rightarrow \tan\theta = \dfrac{120-45}{45} = 1.667 \Rightarrow \boxed{\theta = 59.036°}$

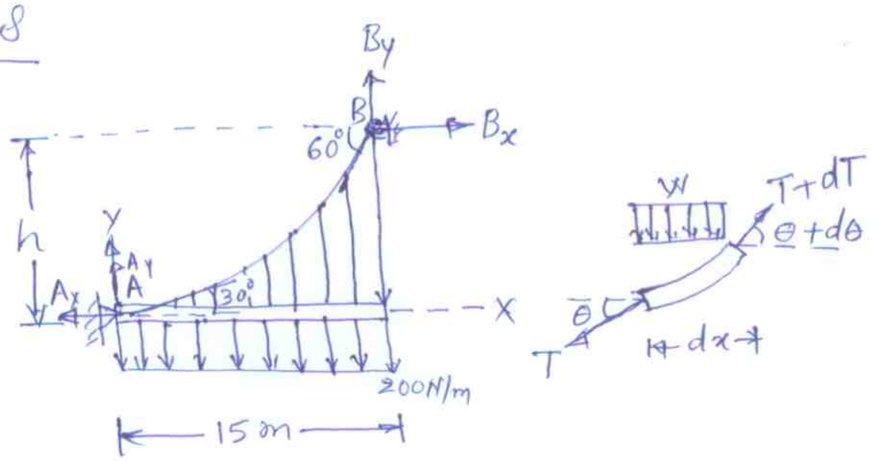
$\Rightarrow \boxed{T_{AB} = 858.02 \text{ N}}$

$\tan\theta = \dfrac{Y_B}{1} \Rightarrow \boxed{Y_B = 1.6667 \text{ m}}$

Inclination of AB with horizontal is maximum.

$\therefore \boxed{T_{max} = T_{AB} = 858.02 \text{ N} \;\&\; Y_B = 1.6667 \text{ m}}$

Q 8



$\Sigma F_x = 0$: Horizontal component of tension T is constant throughout the length of cable. Let it be F_H

$$T\cos\theta = F_H$$

$\uparrow \Sigma F_y = 0$ $\quad (T+dT)\sin(\theta+d\theta) - T\sin\theta - W\cdot dx = 0$

$$\Rightarrow T\cdot\sin\theta = Wx + C_1$$

$$\therefore \frac{dy}{dx} = \tan\theta = \frac{Wx + C_1}{F_H} = \frac{Wx}{F_H} + C_1^*$$

$$Y = \frac{Wx^2}{2F_H} + C_1^* x + C_2$$

At $x = 0$, $Y = 0 \Longrightarrow C_2 = 0$

& $\frac{dy}{dx} = \tan 30° = \frac{Wx}{F_H} + C_1^*$

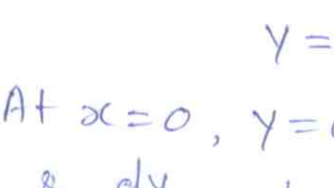
$$\Rightarrow 0.5774 = \frac{200\times 0}{F_H} + C_1^* \Rightarrow \boxed{C_1^* = 0.5774}$$

At $x = 15$ m, $\frac{dy}{dx} = \tan(60°)$

$$\Rightarrow \tan(60°) = 1.73205 = \frac{200\times 15}{F_H} + 0.5774$$

$$\Rightarrow \boxed{F_H = 2598.19\ N}$$

Maximum Tension in cable: Occures at B



$$T_B\cos(60°) = F_H = 2598.19\ N$$

$$\boxed{T_B = T_{max} = 5196.38\ N}$$

Shape of cable: $Y = \frac{Wx^2}{2F_H} + C_1^* x$

$$Y = \left(\frac{200\, x^2}{2 \times 5196.38}\right) + (0.5774)\,x$$

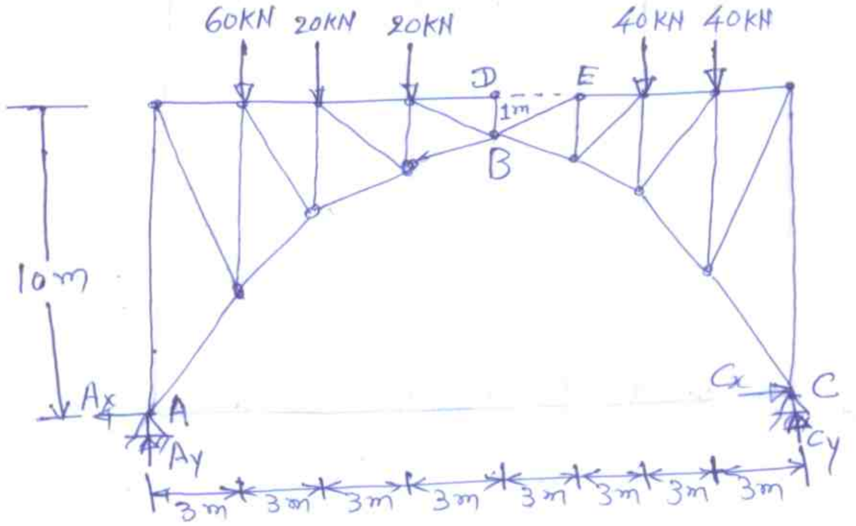
$$\boxed{Y = (0.03849)\,x^2 + (0.5774)\,x}$$

$$\Rightarrow \boxed{h = 17.3208 \text{ m}}$$

———— x o x ————

Q-9 : Refer Fig. 9 of Tuto.-1.

consider F.B.D. of whole truss.



$+\circlearrowleft \underline{\Sigma M_C = 0}$; $-A_y \times 24 + 60 \times 21 + 20 \times 18 + 20 \times 15 + 40 \times 6 + 40 \times 3 = 0$

$$\Rightarrow \boxed{A_y = 95 \text{ KN}}$$

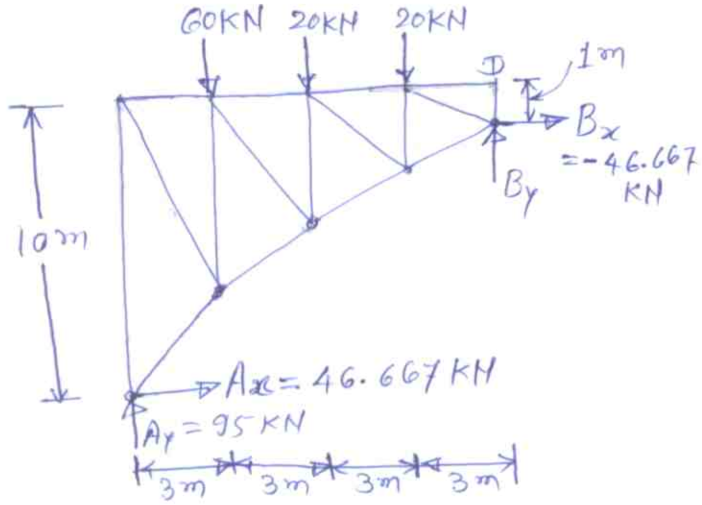
$+\uparrow \underline{\Sigma F_y = 0}$; $C_y + 95 - 60 - 20 - 20 - 40 - 40 = 0$

$$\boxed{C_y = 85 \text{KN}}$$

$\xrightarrow{+} \underline{\Sigma Fx = 0}$; $A_x = -C_x$

F.B.D. of L.H.S part from B-Hinge:

No force in member DE.
The truss can be separated from hinge B as show in fig.



$\circlearrowleft + \underline{\Sigma M_B = 0}; \quad -95\times 12 + A_x \times 9 + 60\times 9 + 20\times 6 + 20\times 3 = 0$

$\boxed{A_x = 46.667 \text{ KN}} \rightarrow$

$\Rightarrow \boxed{B_x = 46.667 \text{ KN} \leftarrow \text{ on L.H.S.}}$

$+\uparrow \underline{\Sigma F_y = 0}; \quad \boxed{B_y = -95 + 60 + 20 + 20 = 5 \text{ KN}} \uparrow$ on L.H.S.

——— x-x ———

Q 10 :-

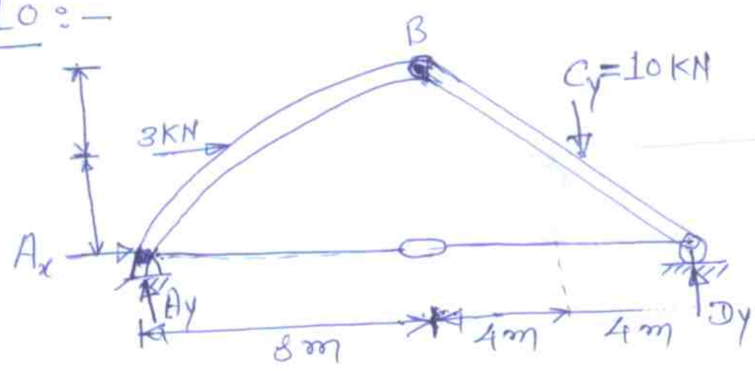


Fig - 10(a)

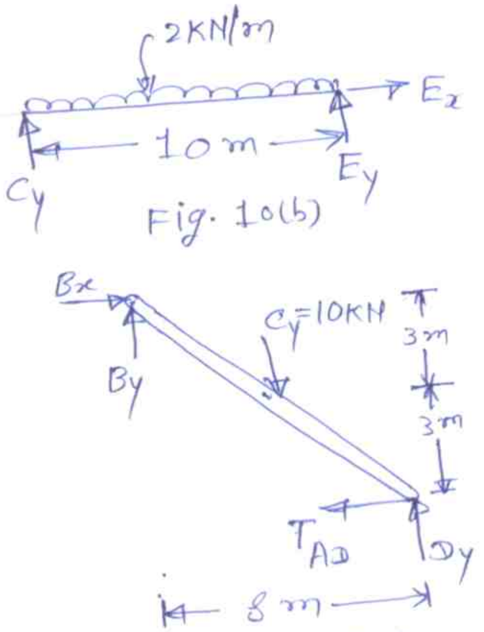


Fig 10 (c)

FBD of structure (Fig 10(b))

$\boxed{E_y = C_y = 10 \text{ KN}}$

FBD of structure in Fig 10(a) :-

$\circlearrowleft + \underline{\Sigma M_A = 0}; \quad D_y = \dfrac{10\times 12 + 3\times 3}{16} = 8.0625 \text{ KN}$

$+\uparrow \underline{\Sigma F_y = 0}; \quad A_y = 10 - 8.0625 = 1.9375 \text{ KN}$

$\xrightarrow{+} \underline{\Sigma F_x = 0}; \quad A_x = -3 \text{ KN}$

FBD of structure - Fig 10(c) :-

$\circlearrowleft \Sigma M_B = 0; \quad T = \dfrac{-10\times 4 + 8.0625\times 8}{6} = 4.0833 \text{KN}$

Ans: $A_x = 3 \text{ KN} \leftarrow, \; A_y = 1.9375 \text{ KN} \uparrow, \; D_y = 8.0625 \text{ KN} \uparrow$

Tension in Rod AD $= 4.0833$ KN