DPARTMENT OF CIVIL ENGINEERING
CE-222 STRUCTURAL MECHANICS I
Tutorial Assignment # 4: Influence Line Diagrams

Problem 1: Draw the influence lines for (a) the vertical reaction at C, (b) the moment at B, and (c) the vertical reaction at D. Assume the supports at A, B, and C are rollers. Solve this problem using Muller-Breslau's principle (Fig.1).

Problem 2: Draw the influence line for (a) the force in the cable BC, (b) the vertical reaction at A, and (c) the moment at D (Fig.2).

Problem 3: Draw the influence line for the shear in panel BC of the girder. Determine the maximum negative live shear in panel BC due to uniform live load of 500 kN/m acting on the top beams. Assume the supports for these beams can exert both upward and downward forces on the beams (Fig.3).

Problem 4: Draw the influence line for the force in member IH of the bridge truss. Determine the maximum live force (tension or compression) that can be developed in this member due to a 18.5 kN truck having the wheel loads shown. Assume the truck can travel in *either direction* along the centre of the deck, so that the *half* the load shown is transferred to each of the two side trusses. Also assume the members are pin connected at the gusset plates (Fig.4).

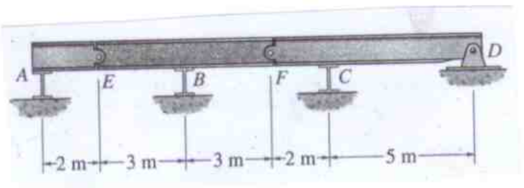


Fig. 1

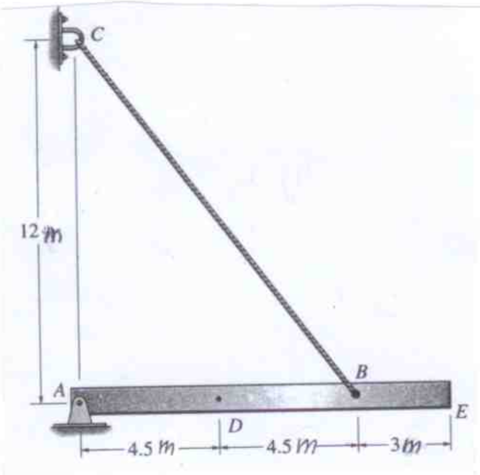


Fig. 2

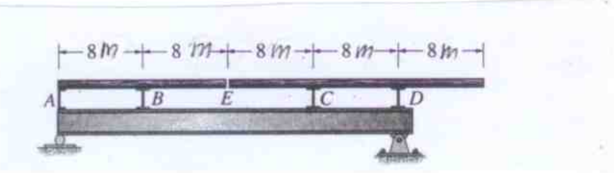


Fig. 3

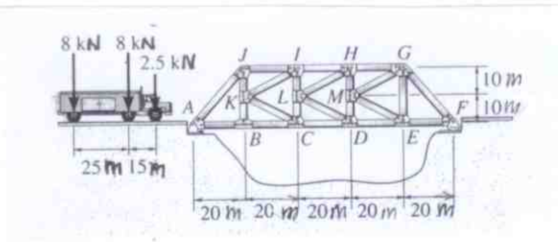


Fig. 4

Problem 5: The truck and trailer exerts the wheel reactions shown on the deck of the girder bridge. Determine the largest moment it exerts in the splice. Assume the truck travels in *either direction* along the centre of the deck, and therefore transfers half of the load shown to each of the two side girders. Assume the splice is a fixed connection and, like the girder, can support both shear and moment (Fig.5).

Problem 6: Determine the absolute maximum live moment in the girder bridge due to the loading shown. The load is applied directly to the girder (Fig.6).

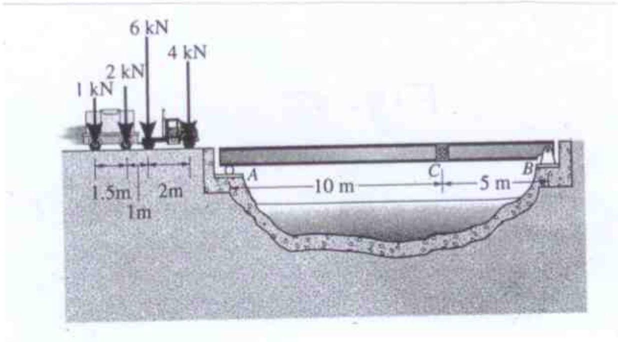


Fig. 5

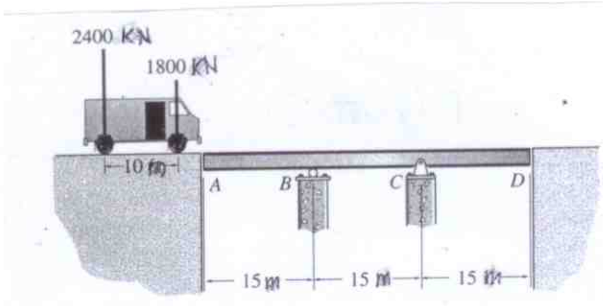
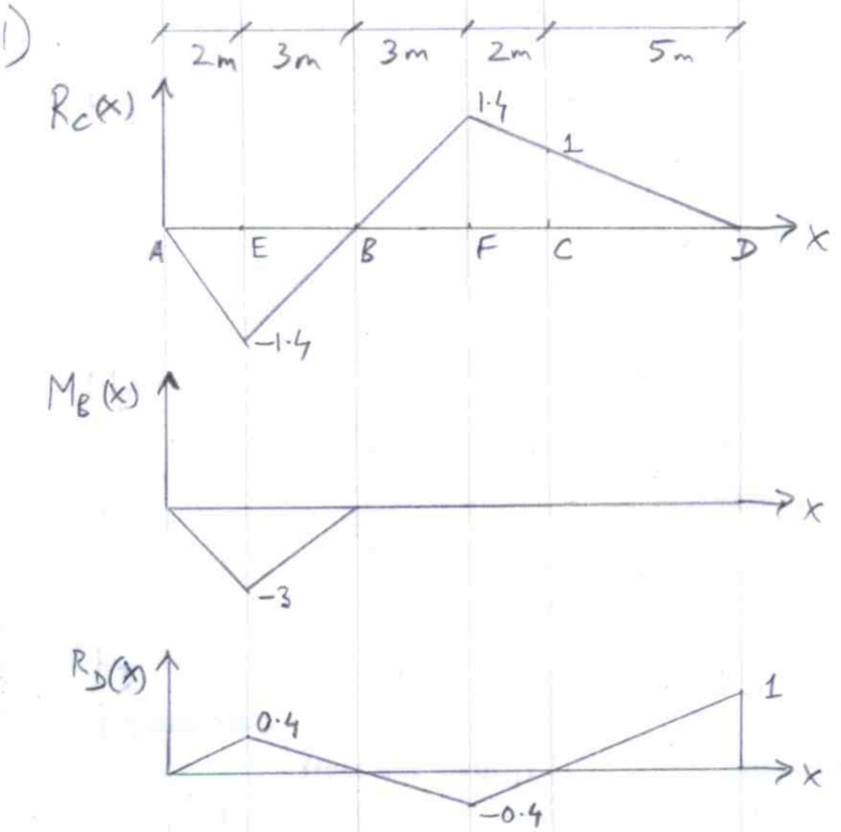
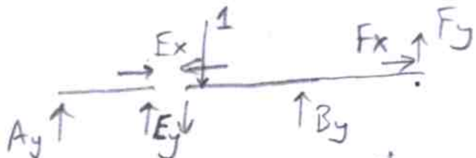


Fig. 6

CE222 TUTORIAL 4

1)



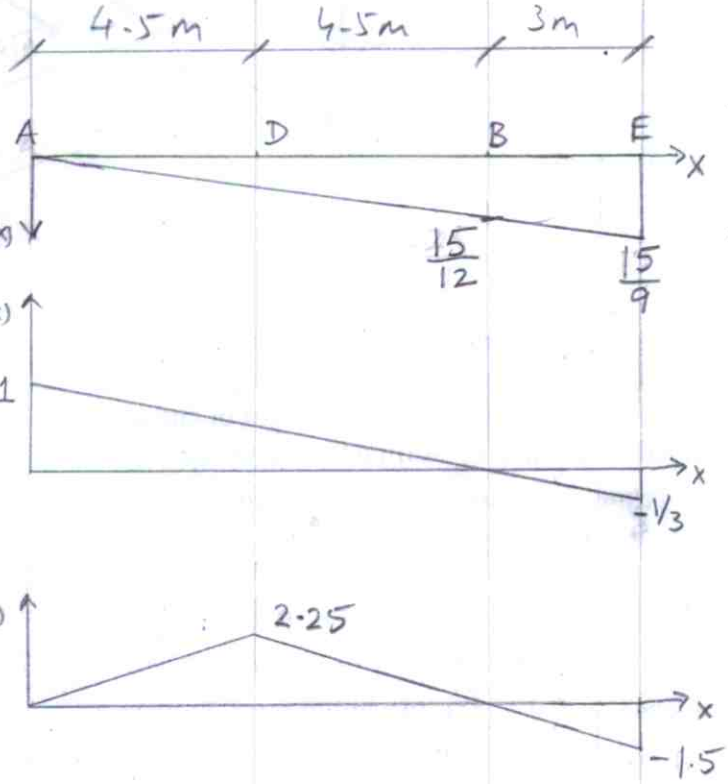


unit load at E^+

$\Sigma M_A = 0 \Rightarrow E_y = 0$

$\Rightarrow M_B = (-1)(3)$

2)



$T_{BC}\left(\frac{12}{15}\right) \underbrace{\cancel{\delta y}}_{=1} = 1\,\delta y'$

$T_{BC} = \frac{15}{12}\,\delta y'$

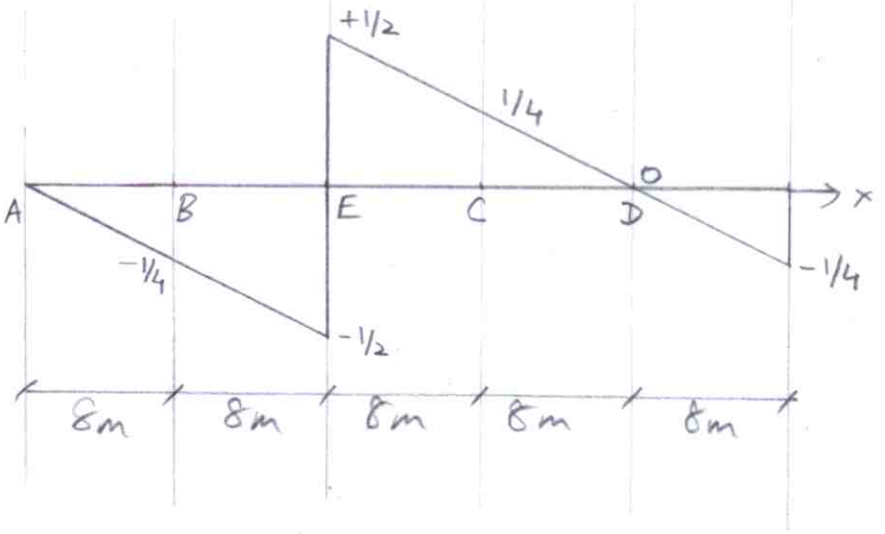
Unit load at D

$\Sigma M_B = 0 \Rightarrow A_y = 1/2$

$\Rightarrow M_D =$

(2)

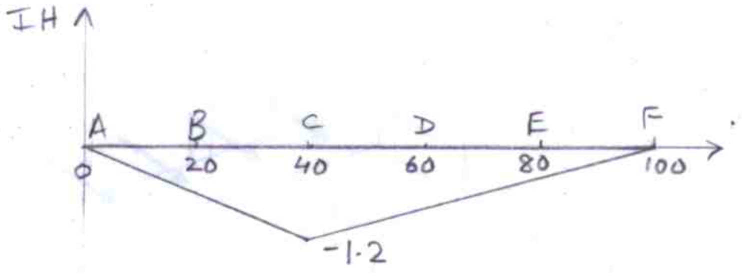
3)



x	V_{BC}
0	0
8	$-D_y = -1/4$
16^-	$-D_y = -\frac{1}{2}$
16^+	$A_y = \frac{1}{2}$
24	$A_y = 1/4$
32	0
40	$A_y = -1/4$

$$(V_{BC})_{max,-ve} = (500)\,\tfrac{1}{2}\,(16)(-\tfrac{1}{2}) = -2000\text{ kN}$$

4) MoS, section thru IH, IL, KC, BC, $\Sigma M_C = 0$.



	x	IH
	0	0
	20	$\frac{-A_y(40) + (1)(20)}{20} = -0.6$ (A_y: 4/5)
	40	$-\frac{A_y(40)}{20} = -(\frac{3}{5})(2) = -1.2$
not essential	60	$-A_y(2) = -(\frac{2}{5})(2) = -0.8$
not essential	80	$-A_y(2) = -(\frac{1}{5})(2) = -0.4$
	100	0

Not required – see note below

⇒ Forward travel:
2.5 kN at C → 1; 1st 8 kN at C → 2;
2nd 8 kN at C → 3

$$\Delta IH_{1-2} = \left((2.5)\left(\frac{1.2}{60}\right) + (2)(8)\left(-\frac{1.2}{40}\right)\right) * 15 = -6.45$$

$$\Delta IH_{2-3} = \left((2.5+8)\left(\frac{1.2}{60}\right) + (8)\left(-\frac{1.2}{40}\right)\right) * 25 = -0.75$$ ✓ will give local max

⇐ Backward travel: use same notation.

$$\Delta IH_{1-2} = \left((2.5)\left(\frac{1.2}{40}\right) + (2)(8)\left(-\frac{1.2}{60}\right)\right) * 15 = -3.675$$ ✓ will give local max

$$\Delta IH_{2-3} \quad \left((2.5+8)\left(\frac{1.2}{40}\right) + (8)\left(-\frac{1.2}{60}\right)\right) * 25 = 3.875$$

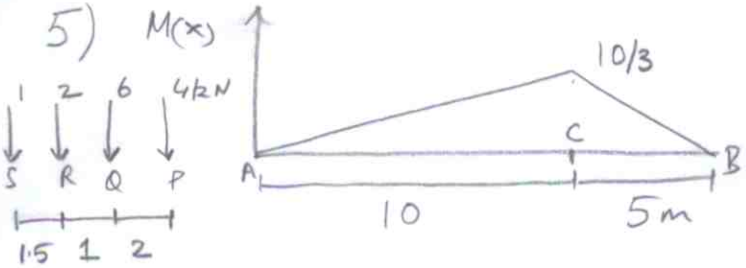
$$IH_3 \text{ (forward)} = \tfrac{1}{2}\left[(2.5)(-1.2) + (8)(-1.2)\left(\tfrac{25}{40}\right) - 6.45 - 0.75\right] = 8.1$$

MAX

$$IH_2 \text{ (backward)} = \tfrac{1}{2}\left[(2.5)(-1.2) + (8)(-1.2)\left(\tfrac{45}{60}\right) + (8)(-1.2)\left(\tfrac{20}{60}\right) - 3.675\right] = \boxed{8.5375}$$

→ when 1st 8 kN at C. → Can directly see that this gives global max ∵ CD slope < AC slope & 8 kN at C.

5)



$1 \rightarrow P$ at C
$2 \rightarrow Q$ " C
$3 \rightarrow R$ " C
$4 \rightarrow S$ " C

Forward travel.

$$\Delta M_{1-2} = (2)\left[\left(-\frac{10}{15}\right)(4) + \left(\frac{10}{30}\right)(6+2+1)\right] = 0.667 \Rightarrow$$ Case 2 forward travel gives local M_{max}

$$\Delta M_{2-3} = (1)\left[\left(-\frac{10}{15}\right)(4+6) + \left(\frac{10}{30}\right)(2+1)\right] = -5.667$$

$$\Delta M_{3-4} = (1.5)\left[\left(-\frac{10}{15}\right)(4+6+2) + \left(\frac{10}{30}\right)(1)\right] = -11.5$$

Reverse travel

$$\Delta M_{1-2} = (2)\left[\left(-\frac{10}{30}\right)(4) + \left(\frac{10}{15}\right)(6+2+1)\right] = 9.333 \Rightarrow$$ Case 2 reverse travel gives local M_{max}

$$= (1)\left[\left(-\frac{10}{30}\right)(4+6) + \left(\frac{10}{15}\right)(2+1)\right] = -1.333$$

$$= (1.5)\left[\left(-\frac{10}{30}\right)(4+6+2) + \left(\frac{10}{15}\right)(1)\right] = -5$$

Reverse $M_{max} = \frac{1}{2}\left[(4)\left(\frac{10}{3}\right) + (6)\left(\frac{10}{3} - (2)\left(\frac{10}{15}\right)\right) + (2)\left(\frac{10}{3} - (3)\left(\frac{10}{15}\right)\right) + (1)\left(\frac{10}{3} - (4.5)\left(\frac{10}{15}\right)\right) + 9.333\right]$

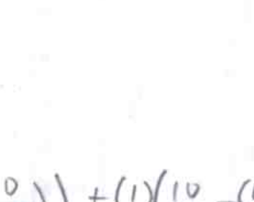
$= \boxed{18.8333} \longleftarrow$ Global M_{max}

Forward $M_{max} = \frac{1}{2}\left[4\left(\frac{10}{3}\right) + (6)\left(\frac{10}{3} - (2)\left(\frac{10}{30}\right)\right) + (2)\left(\frac{10}{3} - (3)\left(\frac{10}{30}\right)\right) + (1)\left(\frac{10}{3} - (4.5)\left(\frac{10}{30}\right)\right) + 0.667\right]$

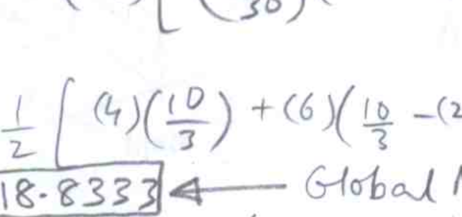
$= 18.25$

6) Either M_{max} occurs under one of the loads when load train over midspan, or M_{max} occurs at support B or C when load train enters span AB or DC

For M_{max} at B or C, $M_{max} = (2400)(15) + (1800)(5) = \boxed{45000 = M_{max}}$

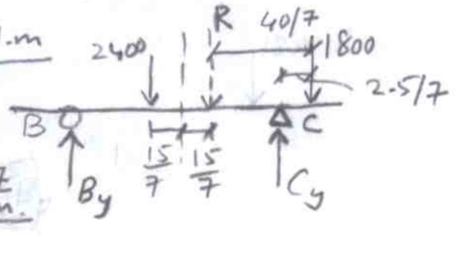


For M_{max} under 1800 kN load, $x = \frac{1}{2}\left(\frac{40}{7}\right) = \frac{20}{7}$



$C_y = (4200)\left(7.5 - \frac{20}{7}\right)/15 = 1300$

$M_{max} = (1300)\left(7.5 - \frac{20}{7}\right) = \underline{6035.7 \text{ kN.m}}$



For M_{max} under 2400 kN load, $x = \frac{15}{7}$

$B_y = (4200)(7.5 - 15/7)/15 = 1500$, $M_{max} = (1500)\left(7.5 - \frac{15}{7}\right) = \underline{\underline{8035.7 \text{ kN.m.}}}$