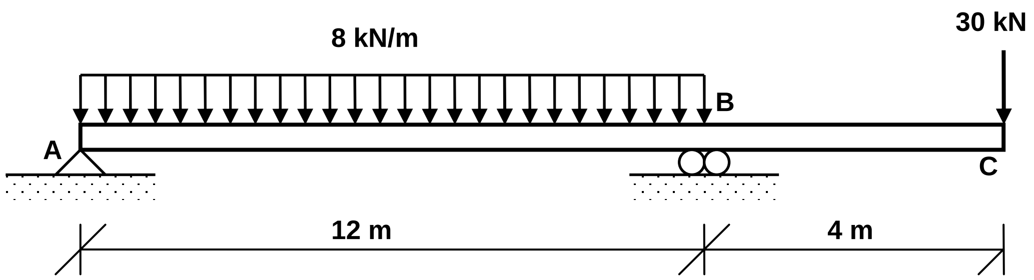
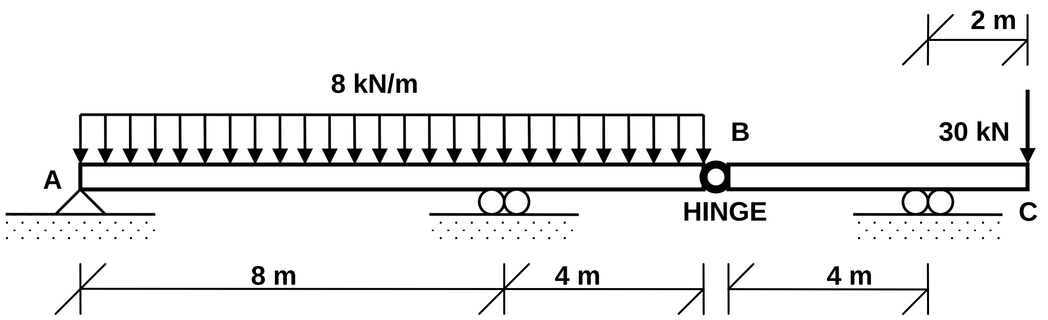
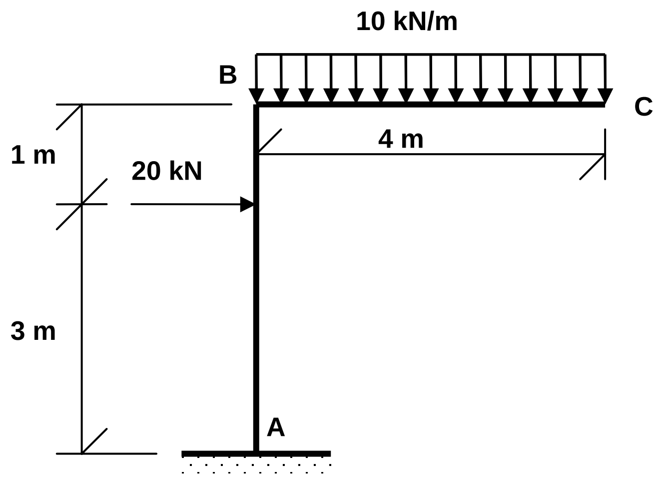
CE-222 STRUCTURAL MECHANICS I
DEPARTMENT OF CIVIL ENGINEERING
Tutorial Assignment # 6: Deflection of Beams and Frames

Problem 1

Calculate **vertical displacement of point C** for the following systems. Take EI constant and neglect axial deformations.

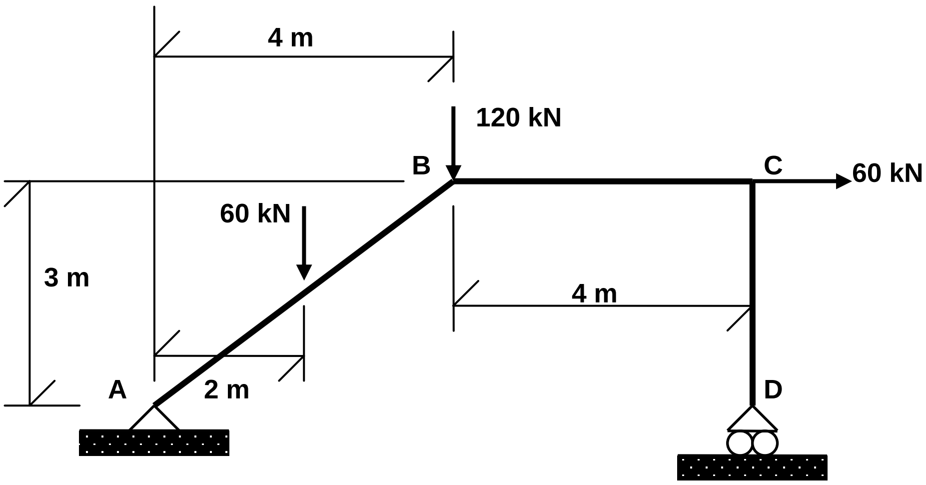




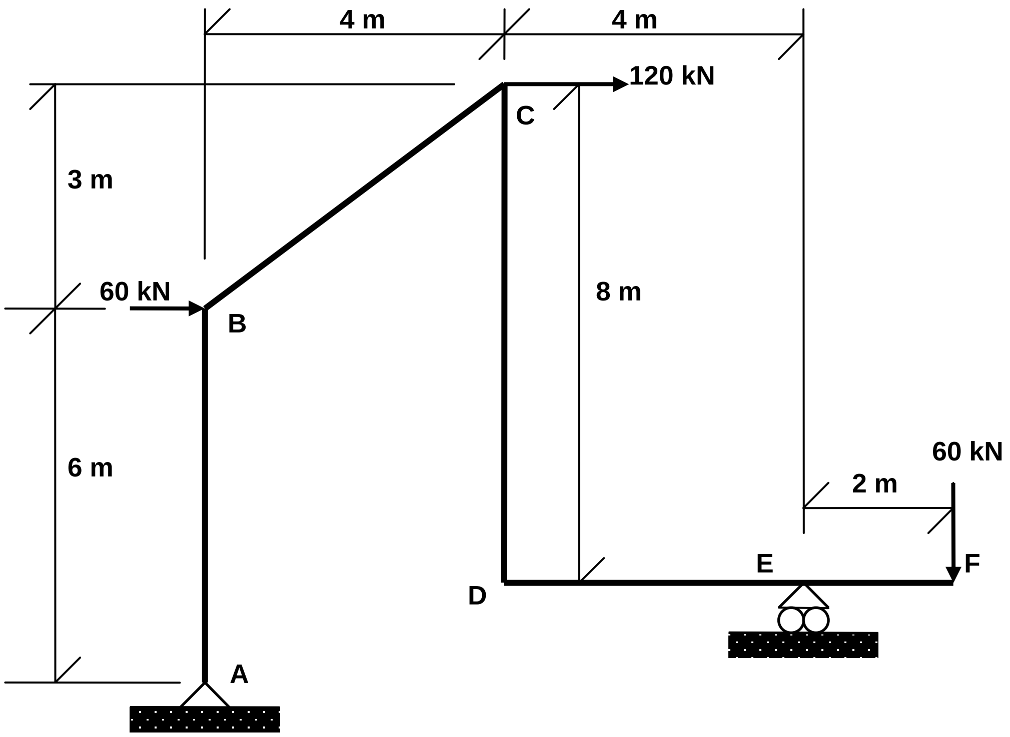


Problem 2

For the frame systems shown below, calculate the horizontal displacement, vertical displacement and rotation **for all joints** by considering **both flexural and axial deformations**. Draw the **qualitative deflected** shape of the frames clearly showing the directions of displacements and rotations **considering only flexural deformations**.

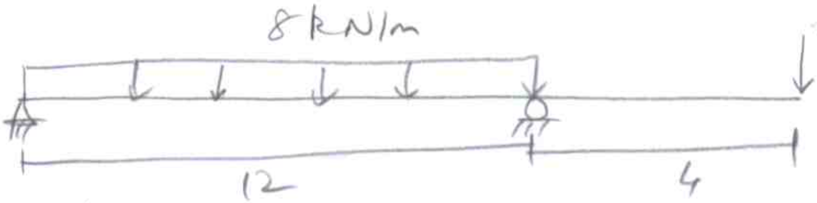


EI CONSTANT
AE CONSTANT



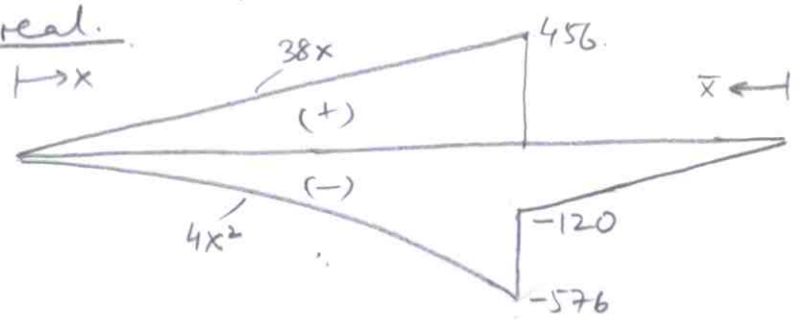
CE222 - TUTORIAL 6

1)



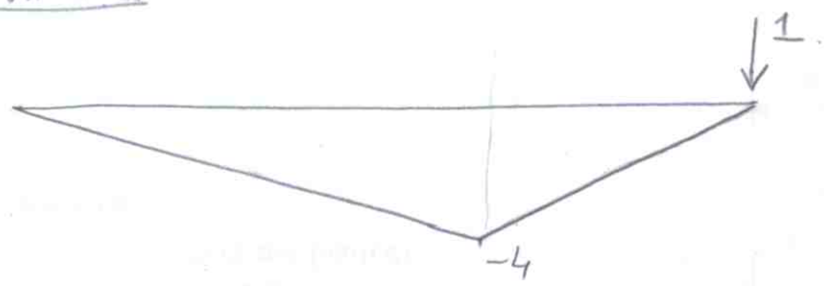
$$A_y = \frac{1}{12}\left[\frac{8(12^2)}{2} - 30(4)\right] = 38$$

BM-real.



BM diagram shown as superposition of basic components (shapes).

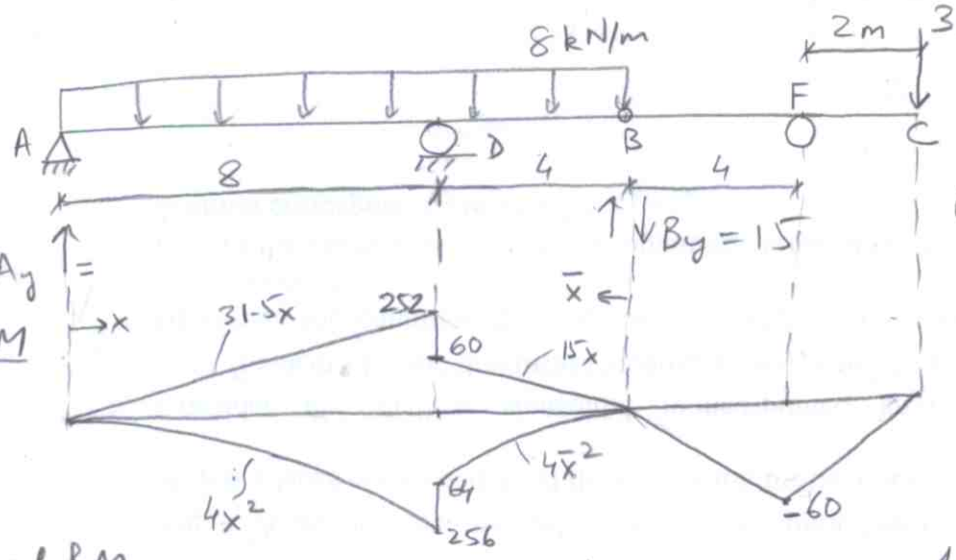
BM virtual.



$$\Delta_c = \frac{1}{EI}\left\{ \frac{1}{3}(456)(-4)(12) + \frac{1}{4}(576)(-4)(12) - (576)\frac{1}{2}(12)(-4) + \frac{1}{3}(-120)(-4)(4) \right\}$$

$$= \frac{256}{EI} \times 10^3, \quad EI \text{ in } Nm^2$$

2)



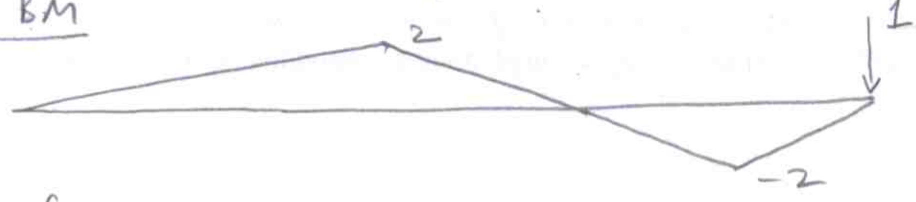
$B_y = 15$ (by observation).

$$A_y = \frac{(15)(4) + (8)(12)(2)}{8} = 31{\cdot}5.$$

→ BM diagram shown as superposition of components.

$B_y = 0{\cdot}5$
$\Rightarrow A_y = 0{\cdot}25$ } by observation

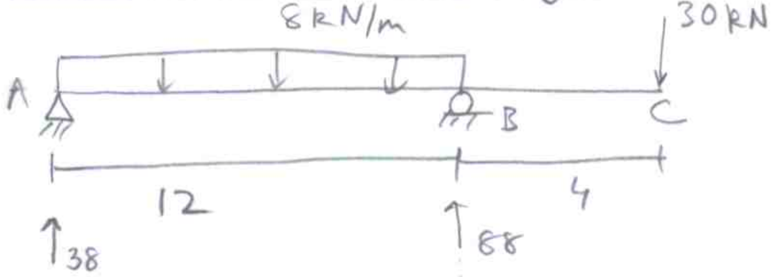
Virtual BM



$$\Delta_c = \frac{1}{EI}\left\{ \frac{1}{3}(252)(2)(8) + \frac{1}{3}(60)(2)(4) + \frac{1}{3}(-60)(-2)(4+2) + \frac{1}{4}(256)(2)(8) + \frac{1}{4}(64)(2)(4) - (256)\frac{1}{2}(8)(2) - (64)\frac{1}{2}(4)(2) \right\} = \frac{592}{EI} \times 10^3, \; EI \text{ in } N.m^2$$

Problems (1), (2) re-done by integration

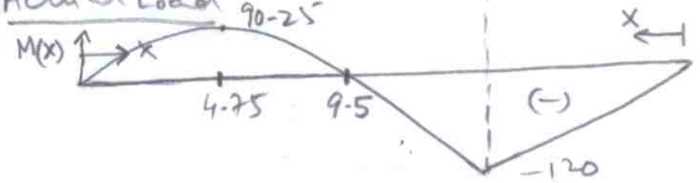
1)



$$B_y = \frac{(30)(16) + (8)\left(\frac{12^2}{2}\right)}{12} = 88$$

$$A_y = 8 * 12 + 30 - 88 = 38$$

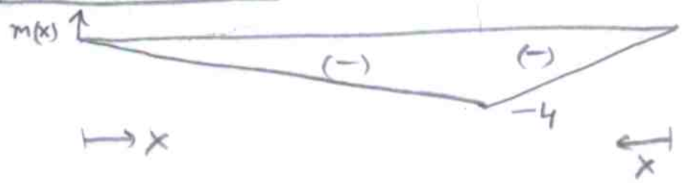
Actual Load



$M(x) = 38x - 8\frac{x^2}{2}$, in AB

$M(x) = -30x$, in CB

Unit load at C



$B_y = \frac{16}{12} = \frac{4}{3}$, $A_y = -\frac{1}{3}$

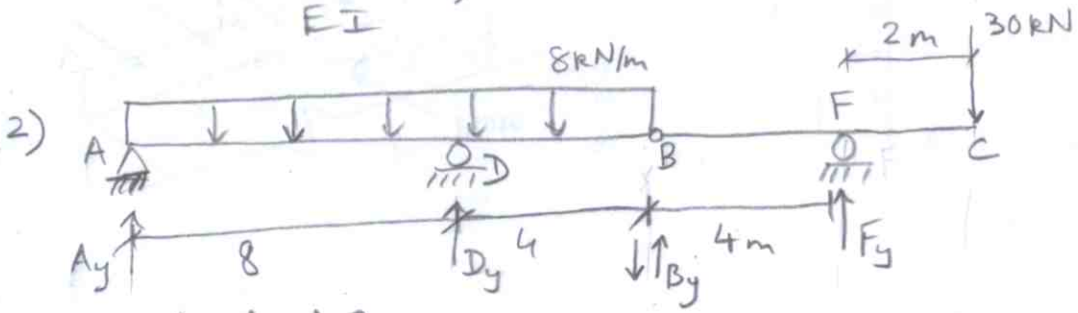
$M(x) = -x$ in CB

$= -\frac{1}{3}x$ in AB

$$EI\Delta_{CV} = \int_0^{12}(38x - 4x^2)\left(-\frac{x}{3}\right)dx + \int_0^4(-30x)(-x)\,dx = \left(\frac{4}{3}\right)\left(\frac{12^4}{4}\right) - \left(\frac{38}{3}\right)\left(\frac{12^3}{3}\right) + (30)\left(\frac{4^3}{3}\right)$$

$$\Delta_{CV} = \frac{256 * 10^3}{EI}\,m, \quad EI \text{ in } Nm^2$$

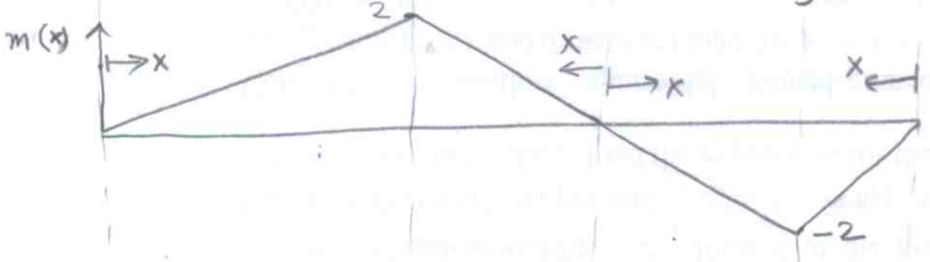
2)



Unit load at C

$\Sigma M_B = 0 \Rightarrow F_y = \frac{6}{4} = 1.5$; $\Sigma M_A = 0 \Rightarrow D_y(8) + (1.5)(16) - (1)(18) = 0 \Rightarrow D_y = -0.75$

$A_y = 1 - 1.5 + 0.75 = 0.25$.



$M(x) = -x$, in CF
$= -\frac{x}{2}$ in BF
$= \frac{x}{4}$ in AD
$= \frac{x}{2}$ in BD

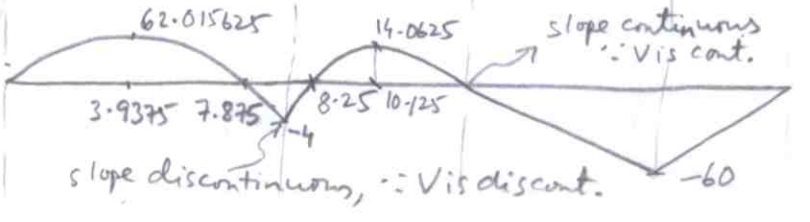
x's as shown in BMD

Actual load

$F_y = 30\left(\frac{6}{4}\right) = 45$; $D_y(8) + (45)(16) - (30)(18) - (8)\left(\frac{12^2}{2}\right) = 0 \Rightarrow D_y = 49.5$

$A_y = -49.5 - 45 + (8)(12) + 30$
$= 31.5$

$B_y = 30 - 45 = -15$



$M(x) = -30x$ in CF,
$= -15x$ in BF
$= 15x - 4x^2$, in BD
$= 31.5x - 4x^2$ in AD

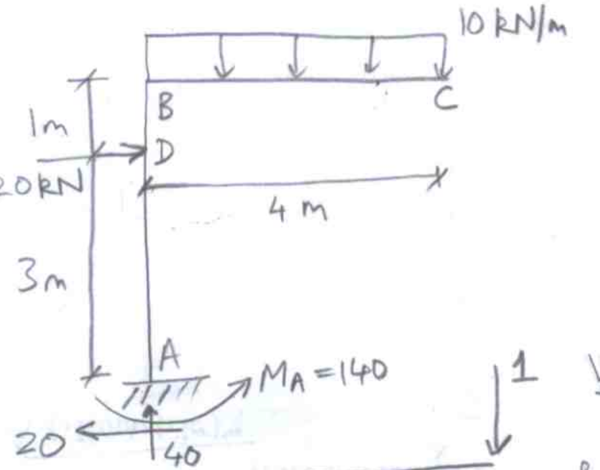
x's as in unit load BMD

$$EI\Delta_{cv} = \int_0^8 \left(\frac{x}{4}\right)(31.5x - 4x^2)\,dx + \int_0^4 \left(\frac{x}{2}\right)(15x - 4x^2)\,dx + \int_0^4 \left(-\frac{x}{2}\right)(-15x)\,dx + \int_0^2 (-x)(-30x)\,dx$$

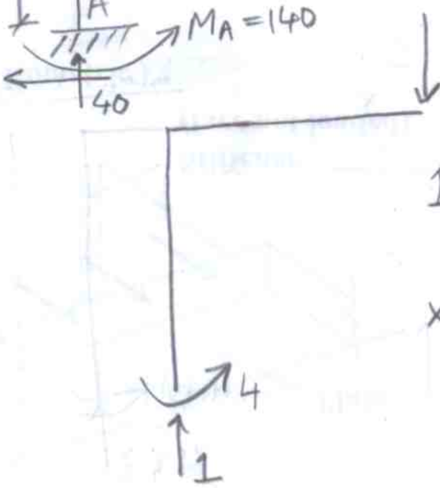
$$= \left(\frac{31.5}{4}\right)\left(\frac{8^3}{3}\right) - \left(\frac{4}{4}\right)\left(\frac{8^4}{4}\right) + \left(\frac{15}{2}\right)\left(\frac{4^3}{3}\right) - \left(\frac{4}{2}\right)\left(\frac{4^4}{4}\right) + \left(\frac{15}{2}\right)\left(\frac{4^3}{3}\right) + (30)\left(\frac{2^3}{3}\right) = 592$$

$$\Delta_{cv} = \frac{592}{EI} * 10^3 \text{ m}, \quad EI \text{ in Nm}^2.$$

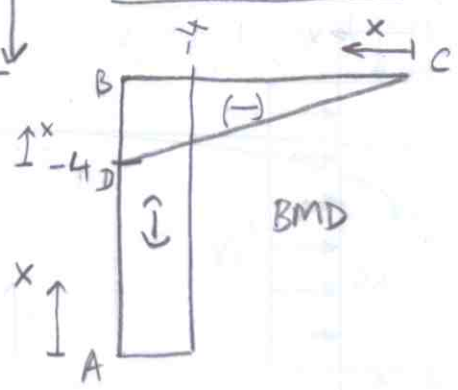
3)



$$M_A = (20)(3) + (10)\left(\frac{4^2}{2}\right) = 140$$

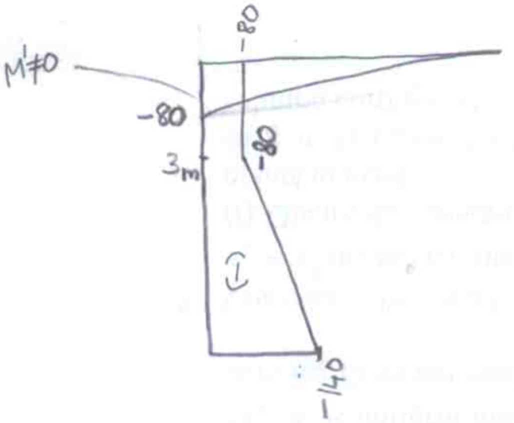


Virtual Load



$M(x) = -4$, in AB (ie AD and DB)
$= -x$, in CB

'x' as shown in BMD



$M(x) = -10\frac{x^2}{2} = -5x^2$, in CB
$= -140 + 20x$, in AD
$= -80$, in DB

'x' as shown in BMD for virtual load

$$EI\Delta_{cv} = \frac{1}{2}(-4)(-140-80)(3) + (-80)(-4)(1) + \int_0^4 (-5x^2)(-x)\,dx = 1640 + 5\left(\frac{4^4}{4}\right)$$
$$= 1960$$

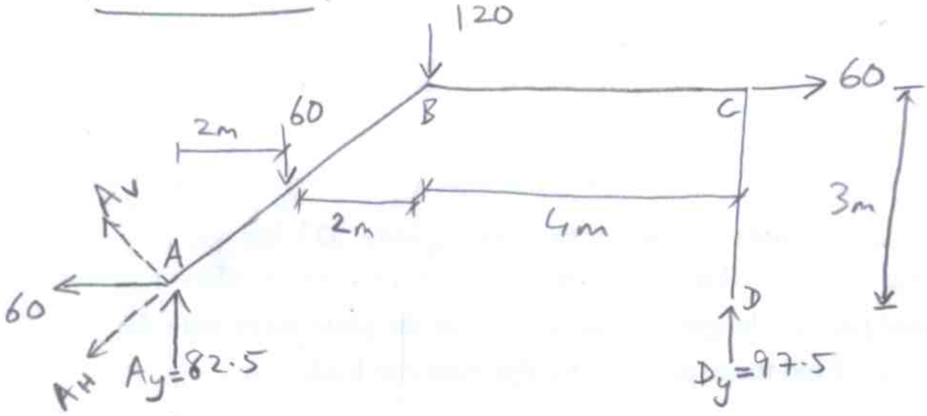
$$\Delta_{cv} = \frac{1960}{EI} * 10^3 \text{ m}, \quad EI \text{ in N.m}^2.$$

alternatively, by multiplying areas, $\int_0^4 (m+80)m'\,dx - 80\int_0^4 m'\,dx$

$$= \frac{1}{4}(80)(-4)(4) - (80)\frac{(-4)(4)}{2} = 320$$

so you get same result.

4) Real Load



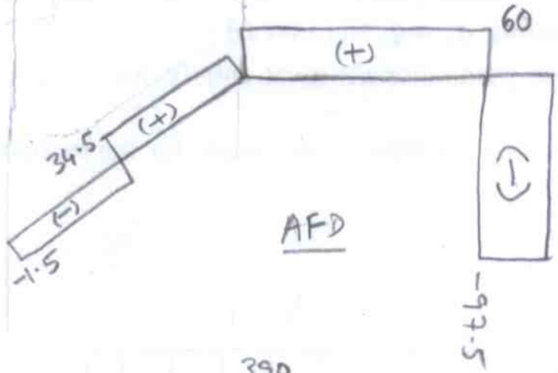
$$D_y = \frac{(60)(2)+(120)(4)+(60)(3)}{8}$$

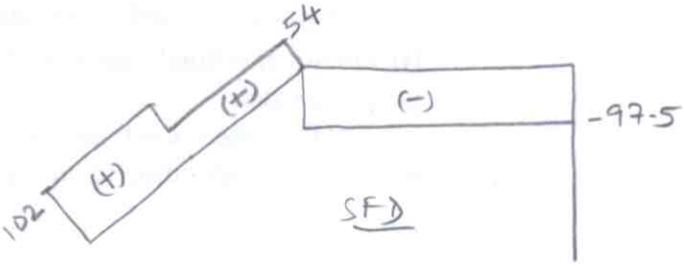
$$= 97.5$$

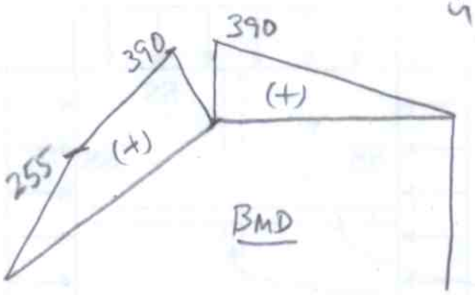
$$A_y = 60+120-97.5 = 82.5$$

$$A_V = (60)\left(\tfrac{3}{5}\right)+(82.5)\left(\tfrac{4}{5}\right) = 102$$

$$A_H = (60)\left(\tfrac{4}{5}\right)-(82.5)\left(\tfrac{3}{5}\right) = -1.5$$

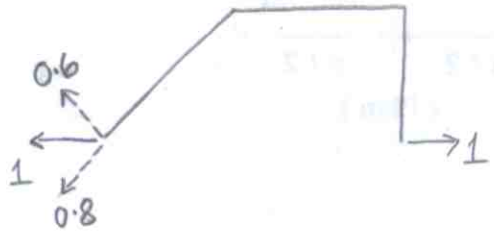


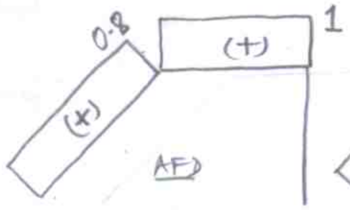


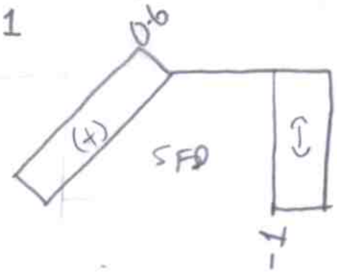


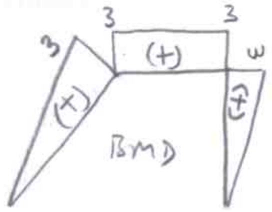
No need to write equations for AFD, BMD, SFD; can use Tables.

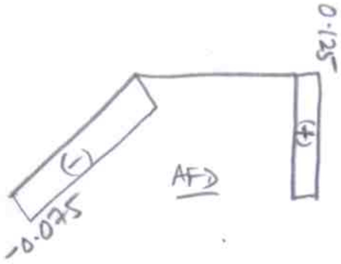
Virtual Load

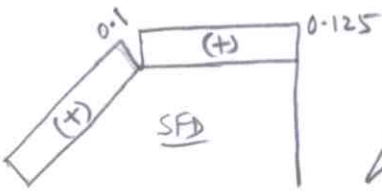


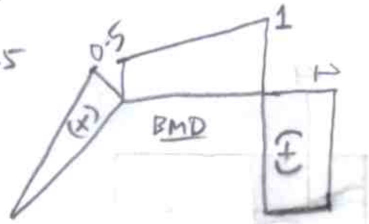


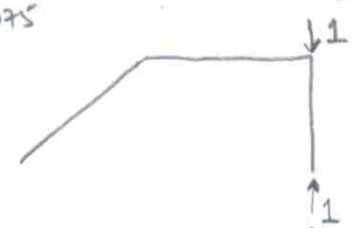




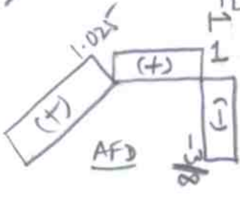


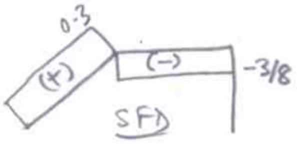


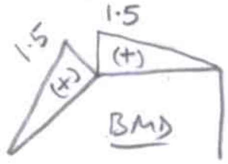


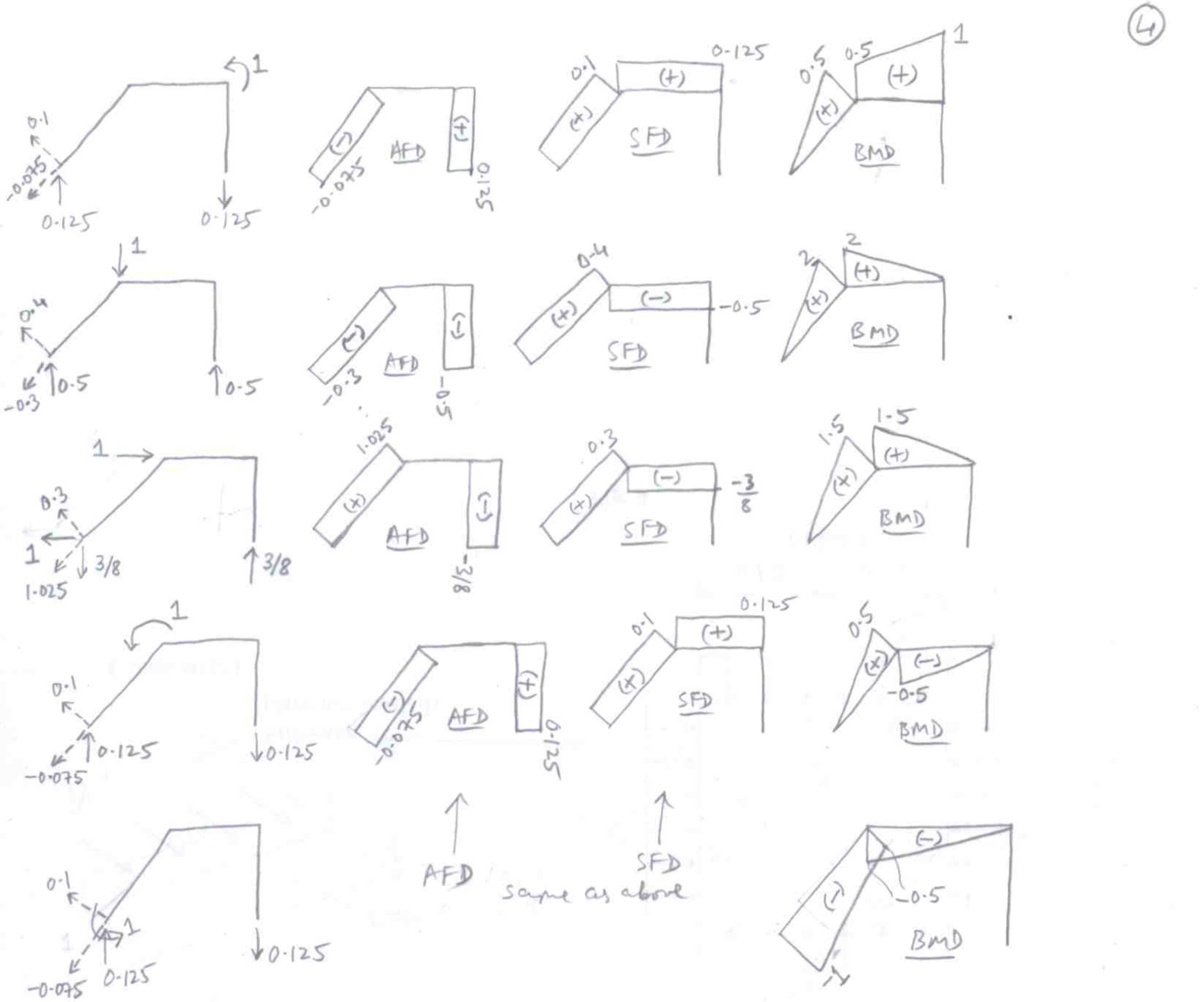


No SF, BM.









Using tables,

$$\Delta_{DH} = \frac{1}{EI}\left\{\frac{1}{3}(255)(1.5)(2.5) + \frac{1}{6}\left[(255)(2*1.5+3) + (390)(1.5+2*3)\right]*2.5 + \frac{1}{2}(390)(3)(4)\right\}$$
$$+ \frac{1}{EA}\left\{(0.8)(-1.5+34.5)(2.5) + (60)(1)(4)\right\} = \left(\frac{4515}{EI} + \frac{306}{EA}\right)*10^3\,m, \quad EI \text{ in } Nm^2, \; EA \text{ in } N.$$

$$\theta_D = \frac{1}{EI}\left\{\frac{1}{3}(255)(0.25)(2.5) + \frac{1}{6}\left[(255)(2*0.25+0.5) + (390)(0.25+2*0.5)\right]*2.5 + \frac{1}{6}(390)(2*0.5+1)\right\}$$
$$+ \frac{1}{EA}\left\{(-0.075)(-1.5+34.5)(2.5) + (0.125)(-97.5)(3)\right\} = \left(\frac{492.5}{EI} - \frac{42.75}{EA}\right)*10^3\,rad$$

$$\Delta_{CV} = \frac{1}{EA}(-97.5)(-1)(3) = \frac{292.5}{EA}*10^3\,m$$

$$\Delta_{CH} = \frac{1}{EI}\left\{\frac{1}{3}(255)(0.75)(2.5) + \frac{1}{6}\left[(255)(2*0.75+1.5) + (390)(0.75+2*1.5)\right]*2.5 + \frac{1}{3}(1.5)(390)(4)\right\}$$
$$+ \frac{1}{EA}\left\{(1.025)(-1.5+34.5)(2.5) + (60)(1)(4) + (-97.5)\left(-\frac{3}{8}\right)(3)\right\} = \left(\frac{1867.5}{EI} + \frac{434.25}{EA}\right)*10^3\,m$$

$\theta_C = \theta_D \rightarrow$ (checks out (expected) since leg CD carries no BM, SF)

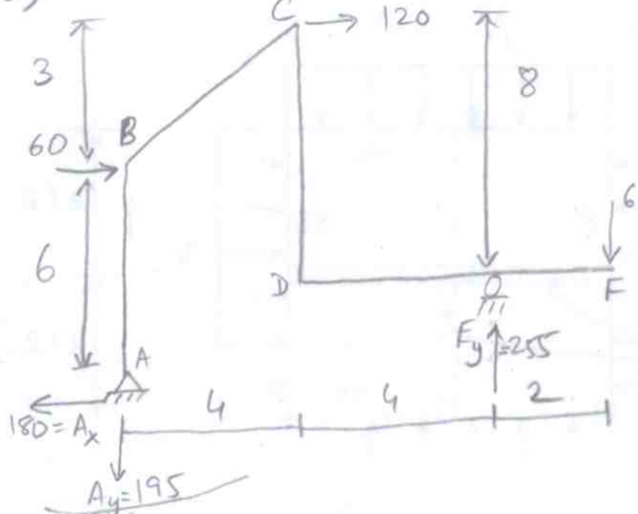
$$\Delta_{BV} = \frac{1}{EI}\left\{\frac{1}{3}(255)(1)(2.5) + \frac{1}{6}\Big[(255)(2*1+2)+(390)(1+2*2)\Big]*2.5 + \frac{1}{3}(390)(2)(4)\right\}$$
$$+\frac{1}{EA}\left\{(-0.3)(-1.5+34.5)(2.5) + (-0.5)(-97.5)(3)\right\} = \left(\frac{2490}{EI} + \frac{121.5}{EA}\right)*10^3\ \text{m.}$$

$$\Delta_{BH} = \frac{1867.5}{EI} + \frac{434.25-240}{EA} = \left(\frac{1867.5}{EI} + \frac{194.25}{EA}\right)*10^3\ \text{m}$$

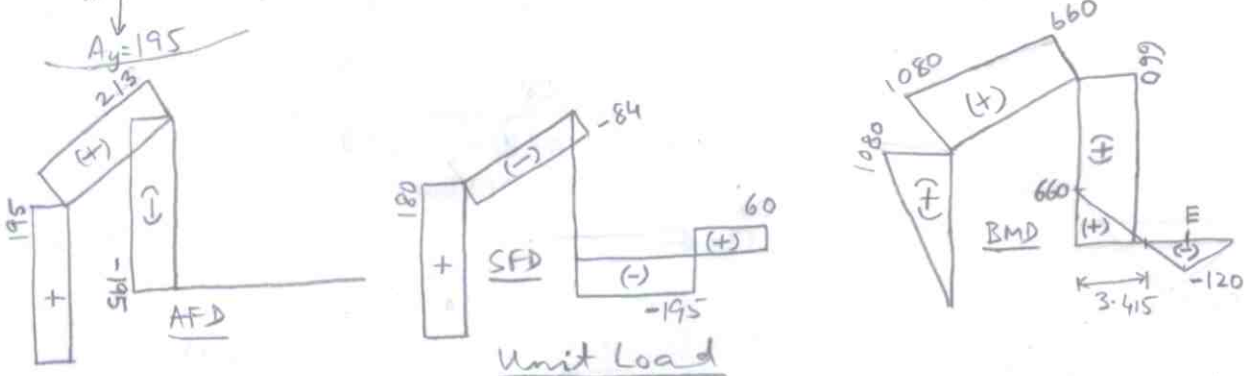
$$\theta_B = \frac{1}{EI}\left\{492.5 - \frac{1}{6}(390)(2*0.5+1) + \frac{1}{3}(-0.5)(390)(4)\right\} - \frac{42.75}{EA} = \left(\frac{102.5}{EI} - \frac{42.75}{EA}\right)*10^3\ \text{rad}$$

$$\theta_A = \frac{1}{EI}\left\{\frac{1}{6}(255)(-1+2*(-0.75))(2.5) + \frac{1}{6}\Big[(255)(-2*0.75-0.5)+(390)(-0.75-2*0.5)\Big](2.5) + \frac{1}{3}(-0.5)(390)(4)\right\}$$
$$-42.75/EA$$
$$= \left(-\frac{1022.5}{EI} - \frac{42.75}{EA}\right)*10^3\ \text{rad}$$

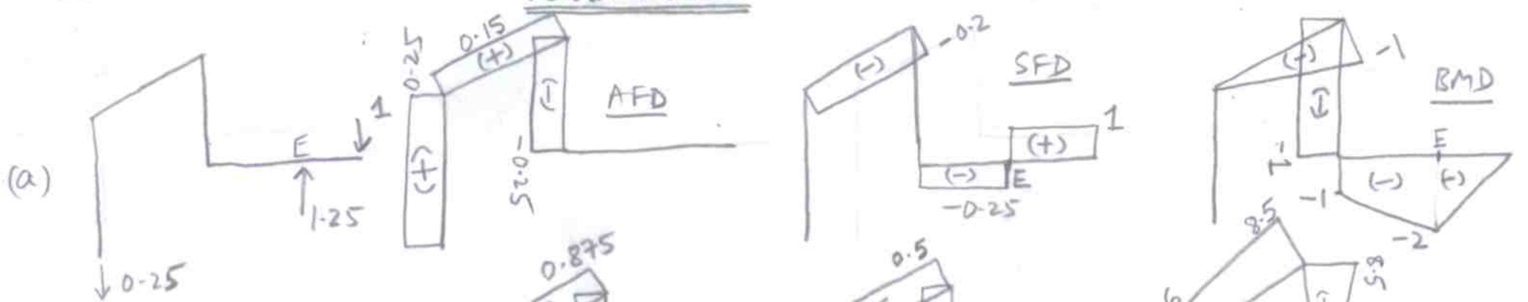
5) **Actual Load**

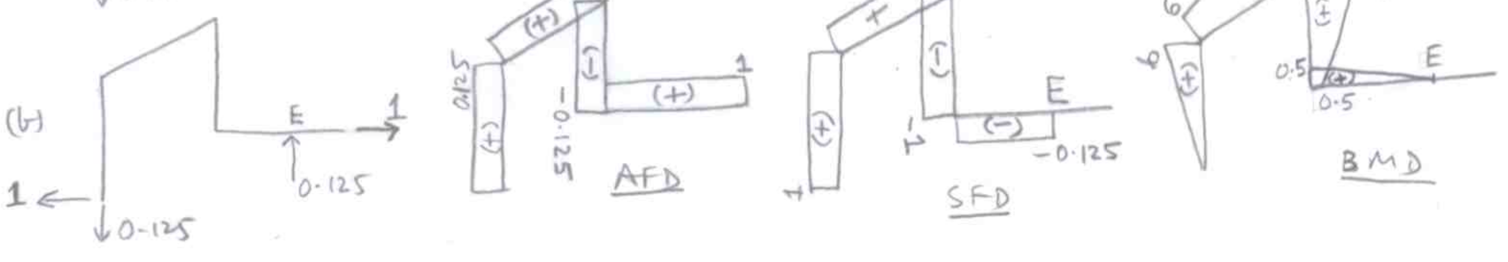


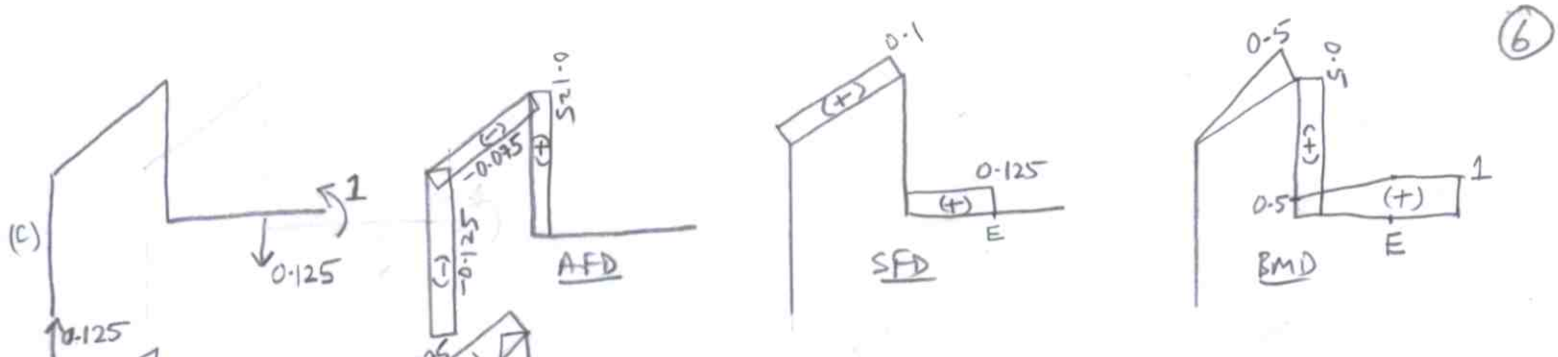
$$E_y = \frac{(60)(6)+(120)(9)+(60)(10)}{8} = 255$$
$$A_y = 255 - 60 = 195$$

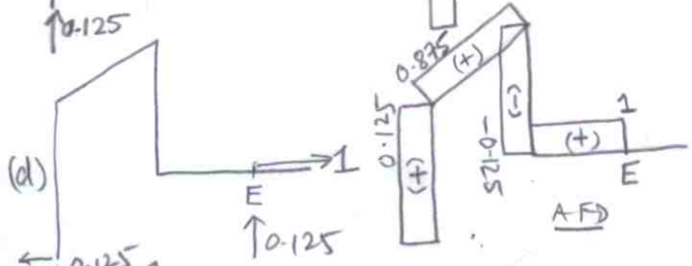


Unit Load

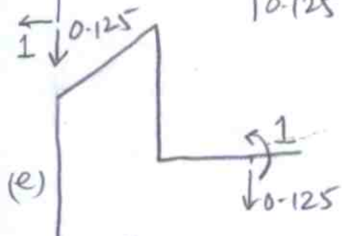




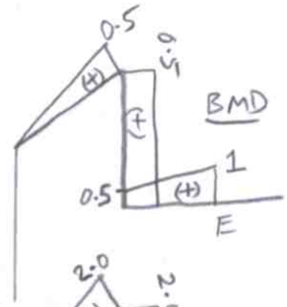


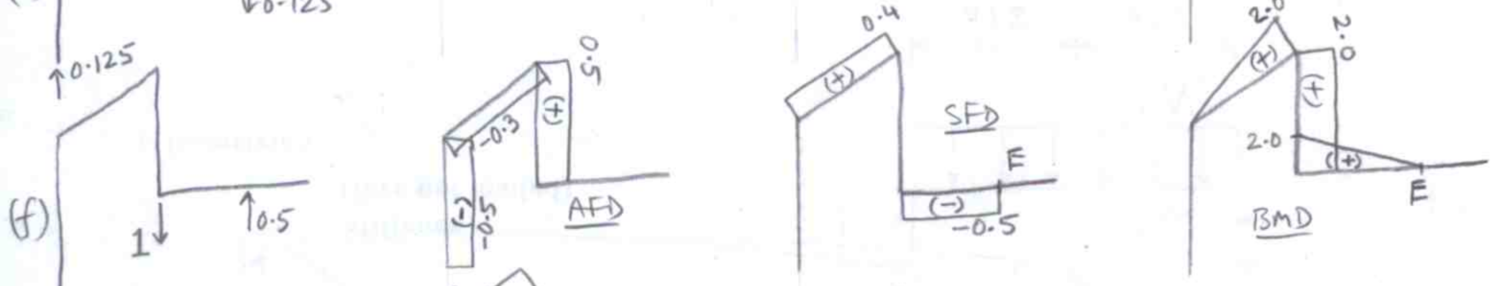


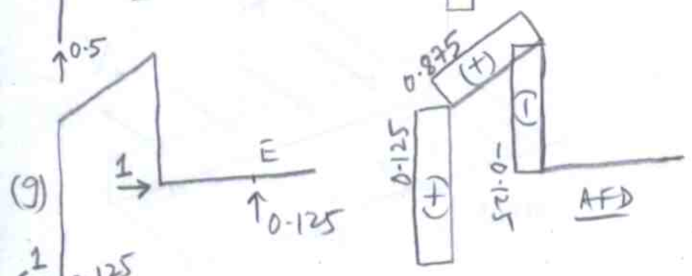
SFD, BMD as in (b).



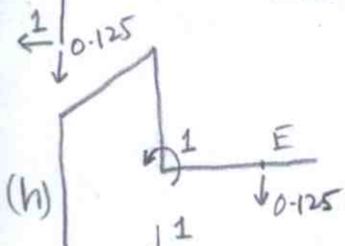
AFD, SFD as in (c)



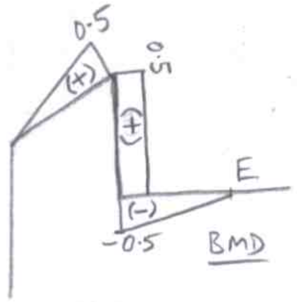


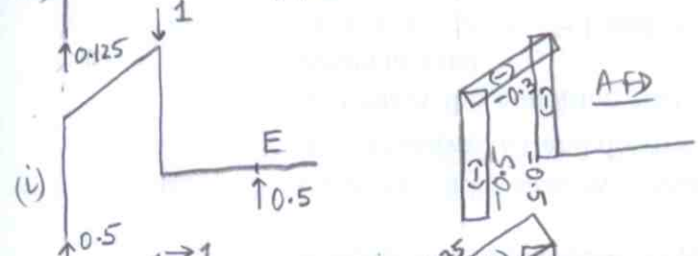


SFD, BMD as in (b)

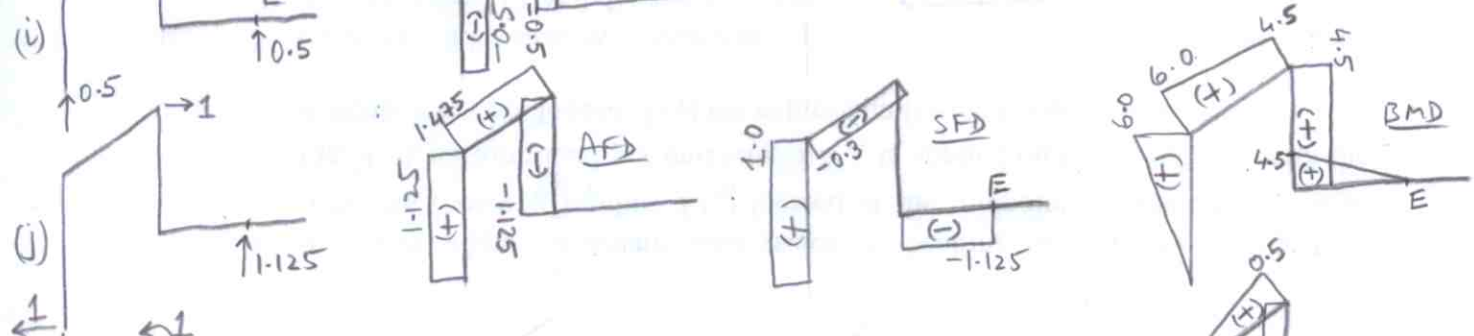


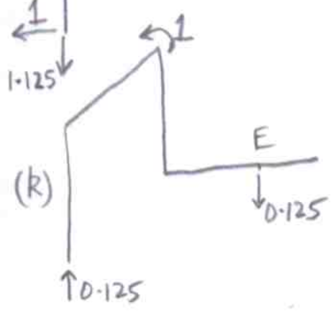
AFD, SFD as in (c)



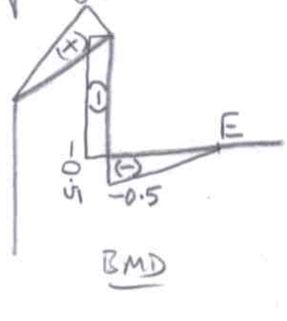


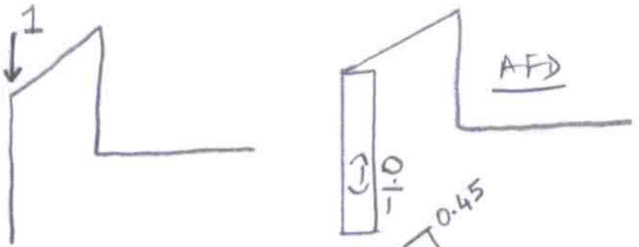
SFD, BMD as in (f)



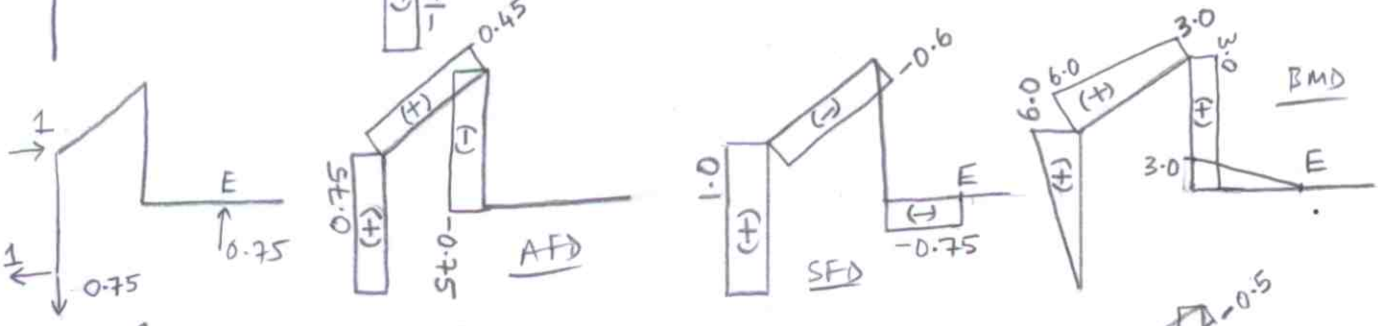


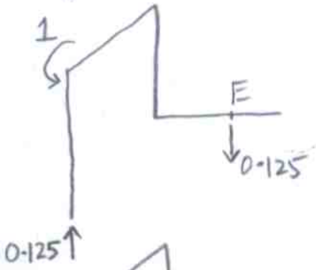
AFD, SFD as in (c)



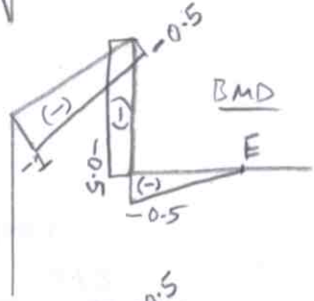


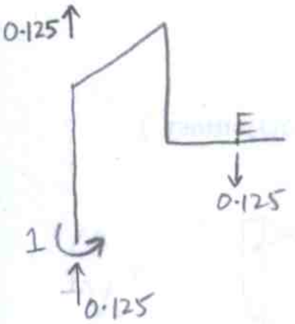
SF, BM is zero.



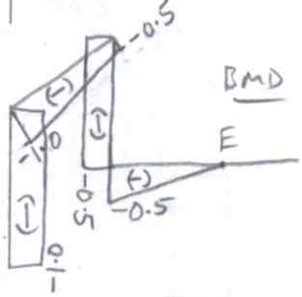


AFD, SFD as in (C)





AFD, SFD as in (C)



$$\Delta_{FV} = \frac{1}{EI}\left\{ \frac{1}{6}(-1)(1080+2*660)(5) + (660)(-1)(8) + \frac{1}{6}(780)((2)(-1)+(-2))(4) + \frac{1}{6}(-2)(120)(2) - (120)\left(\frac{(-1-2)}{2}(4) + \frac{(-2)(2)}{2}\right)\right\}$$

$$+ \frac{1}{EA}\left\{ (0.25)(195)(6) + (0.15)(213)(5) + (-0.25)(-195)(8)\right\}$$

$$= \frac{-8480}{EI} + \frac{842.25}{EA} \quad (\downarrow)$$

$$\Delta_{FH} = \frac{1}{EI}\left\{ \frac{1}{3}(6)(1080)(6) + \frac{1}{6}\left[(6)(2*1080+660) + (8.5)(1080+2*660)\right](5) + \frac{1}{2}(660)(8.5+0.5)(8) + \frac{1}{6}(780)(2*0.5+0.167)(4) + \frac{1}{6}(120)(0.167)(2) - (120)\frac{(0.5)(6)}{2}\right\}$$

$$+ \frac{1}{EA}\left\{(0.125)(195)(6) + (0.875)(213)(5) + (-0.125)(-195)(8)\right\} = \frac{68253.52}{EI} + \frac{1273.125}{EA} \quad (\rightarrow)$$

$$\theta_F = \frac{1}{EI}\left\{\frac{1}{6}(0.5)(1080+2*660)(5) + (0.5)(660)(8) + \frac{(780)}{6}(2*0.5+1)(4) + \frac{1}{2}(120)(1)(2) - (120)\left[\frac{(1+0.5)}{2}(4) + (1)(2)\right]\right\}$$

$$+ \frac{1}{EA}\left\{(-0.125)(195)(6) + (-0.075)(213)(5) + (0.125)(-195)(8)\right\}$$

$$= \frac{4200}{EI} - \frac{421.125}{EA} \quad (\circlearrowleft)$$

$$\Delta_{EH} = \Delta_{FH}$$

$$\theta_E = \frac{1}{EI}\left[4200 - \frac{1}{2}(120)(1)(2) + (120)(1)(2)\right] - \frac{421.125}{EA} = \frac{4320}{EI} - \frac{421.125}{EA} \quad (\circlearrowleft)$$

$$\Delta_{DV} = \frac{1}{EI}\left\{ \frac{1}{6}(2)(1080+2*660)(5) + (2)(660)(8) + \frac{1}{3}(2)(780)(4) - (120)\frac{(2)(4)}{2} \right\}$$

$$+ \frac{1}{EA}\left\{ (-0.5)(195)(6) + (-0.3)(213)(5) + (0.5)(-195)(8) \right\} = \frac{16160}{EI} - \frac{1684.5}{EA} \quad (\downarrow)$$

$$\Delta_{DH} = \Delta_{FH}$$

$$\theta_D = \frac{1}{EI}\left\{ \frac{1}{6}(0.5)(1080+2*660)(5) + (0.5)(660)(8) + \frac{1}{3}(780)(-0.5)(4) - (120)\left(\frac{-0.5)(4)}{2}\right) \right\} - \frac{421.125}{EA}$$

$$= \frac{3240}{EI} - \frac{421.125}{EA} \quad (\circlearrowleft)$$

$$\Delta_{CV} = \frac{16160}{EI} + \frac{1}{EA}\left(-1684.5 + 2*(0.5)(195)(8)\right) = \frac{16160}{EI} - \frac{124.5}{EA} \quad (\downarrow)$$

$$\Delta_{CH} = \frac{1}{EI}\left\{ \frac{1}{3}(6)(1080)(6) + \frac{1}{6}\left[(6)(2*1080+660) + (4.5)(1080+2*660)\right](5) + (4.5)(660)(8) \right.$$

$$\left. + \frac{1}{3}(4.5)(780)(4) - (120)\frac{(4.5)(4)}{2} \right\}$$

$$+ \frac{1}{EA}\left\{ (1.125)(195)(6) + (1.475)(213)(5) + (-1.125)(-195)(8) \right\}$$

$$= \frac{63420}{EI} + \frac{4642.125}{EA} \quad (\longrightarrow)$$

$$\theta_C = \frac{1}{EI}\left(3240 - 2*(0.5)(660)(8)\right) - \frac{421.125}{EA} = -\frac{2040}{EI} - \frac{421.125}{EA} \quad (\circlearrowleft)$$

$$\Delta_{BV} = \frac{1}{EA}(-1)(195)(6) = -\frac{1170}{EA} \quad (\downarrow)$$

$$\Delta_{BH} = \frac{1}{EI}\left\{ \frac{1}{3}(6)(1080)(6) + \frac{1}{6}\left[(6)(2*1080+660) + (3)(1080+2*660)\right](5) + (3)(660)(8) \right.$$

$$\left. + \frac{1}{3}(3)(780)(4) - (120)\frac{(3)(4)}{2} \right\}$$

$$+ \frac{1}{EA}\left\{ (0.75)(195)(6) + (0.45)(213)(5) + (-0.75)(-195)(8) \right\} = \frac{51300}{EI} + \frac{2526.75}{EA} \quad (\longrightarrow)$$

$$\theta_B = \frac{1}{EI}\left(-2040 - \frac{1}{6}(0.5)(1080+2*660)(5) + \frac{1}{6}\left[(-1)(2*1080+660) + (-0.5)(1080+2*660)\right](5)\right)$$

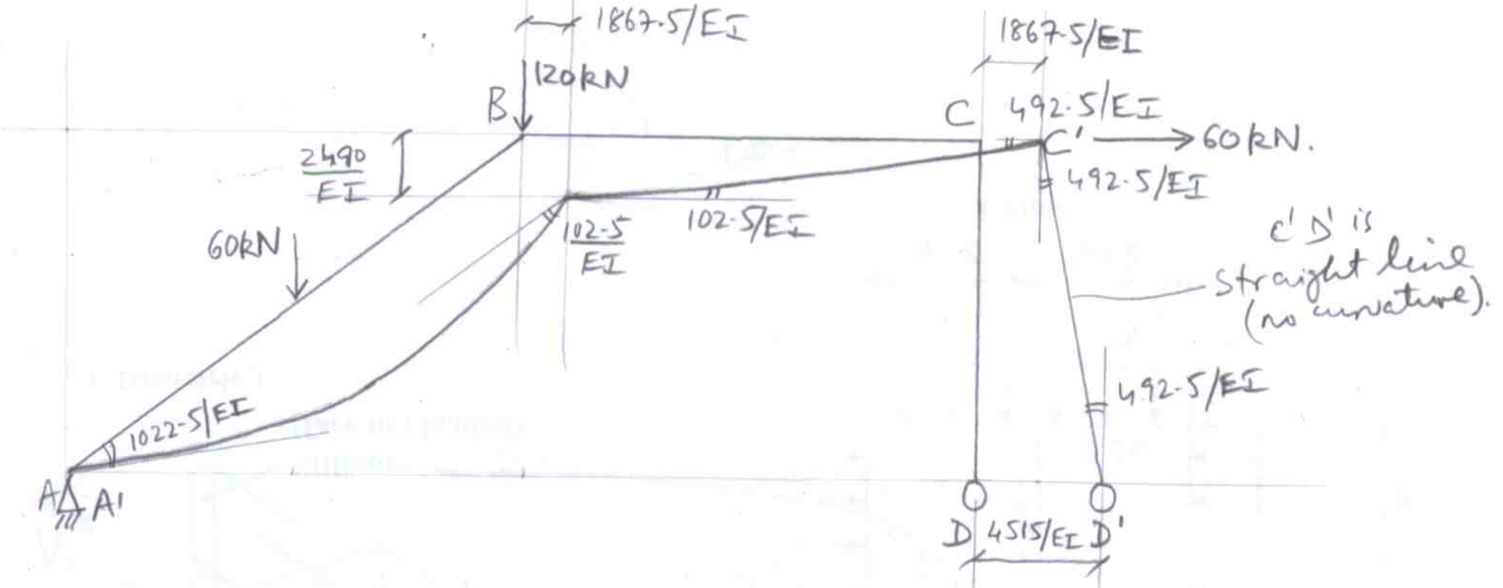
$$- \frac{421.125}{EA} = -\frac{6390}{EI} - \frac{421.125}{EA} \quad (\circlearrowleft)$$

$$\theta_A = \frac{1}{EI}\left(-6390 + \frac{1}{2}(-1)(1080)(6)\right) - \frac{421.125}{EA} = -\frac{9630}{EI} - \frac{421.125}{EA} \quad (\circlearrowleft)$$

Deflected shapes of P.4, P5 with flexural deflections only.

4)

EI*	Δ_H	Δ_V	θ
A	—	—	−1022.5
B	1867.5	2490	102.5
C	1867.5	0	492.5
D	4515	0	492.5



5)

EI*	Δ_H	Δ_V	θ
A	—	—	−9630
B	51300	0	−6390
C	63420	16160	−2040
D	68253	16160	3240
E	68253	0	4320
F	68253	−8480	4200