CE-222 STRUCTURAL MECHANICS I

DEPARTMENT OF CIVIL ENGINEERING

Tutorial Assignment # 7: Deflection by Catigliano's Theorem and Conjugate Beam Method

<u>Problem 1</u>: Find the vertical component of deflection of joint *C* using Castigliano's theorem. Assume *EA* as axial stiffness for all members.

<u>Problem 2</u>: Find the vertical component of deflection of point *B* using Castigliano's theorem. Assume *EA*, *EI* as axial and bending stiffness wherever required.

<u>Problem 3</u>: Find the rotation of point *D* using Castigliano's theorem. Assume *EI* as bending stiffness.

<u>Problem 4</u>: Find the position and magnitude of the maximum vertical deflection using Conjugate Beam Method. Assume *EI* as bending stiffness.

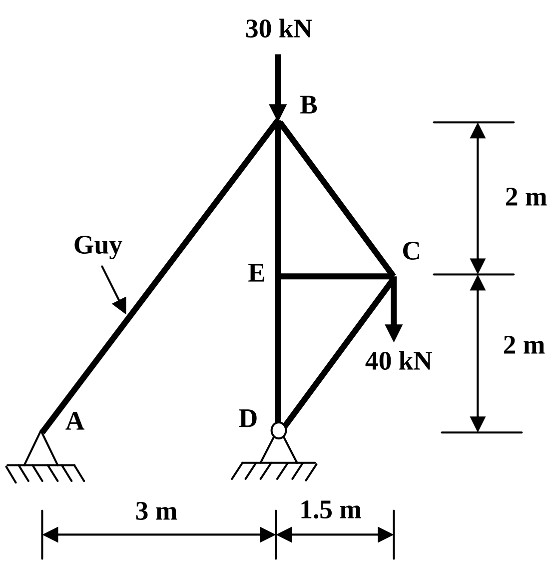

Fig. 1

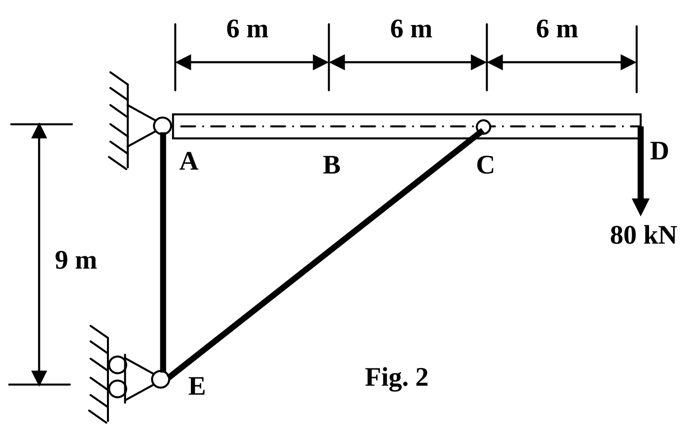

Fig. 2

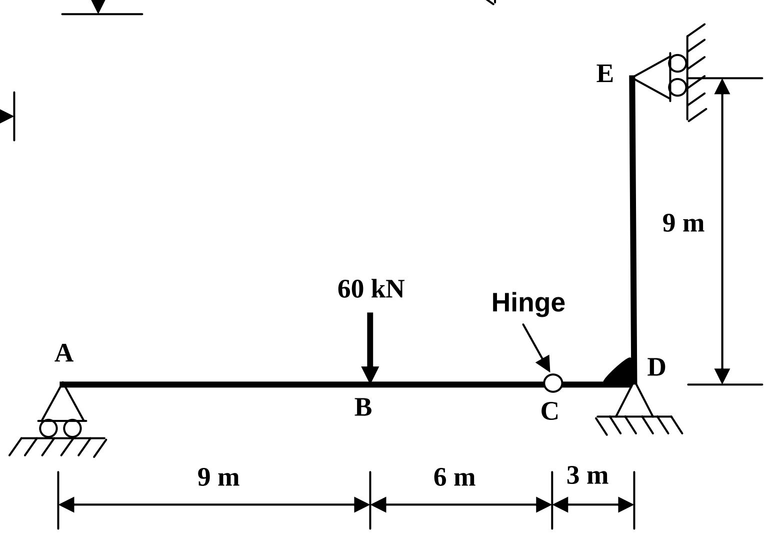

Fig. 3

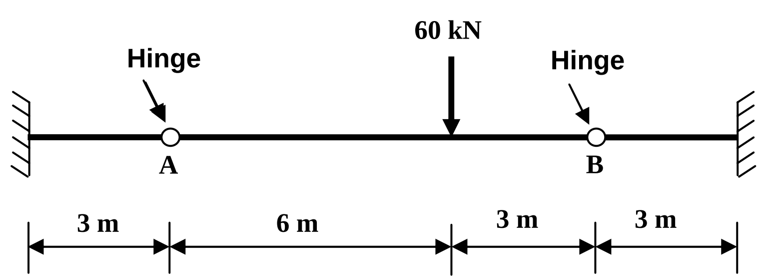

Fig. 4

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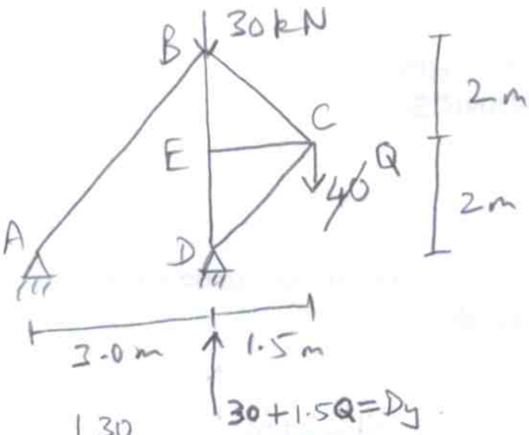
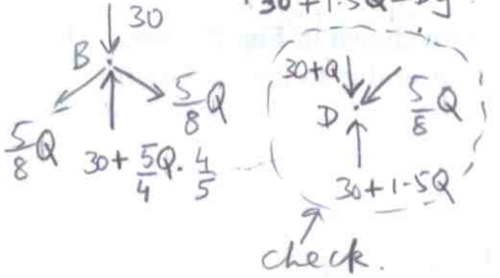
(1)

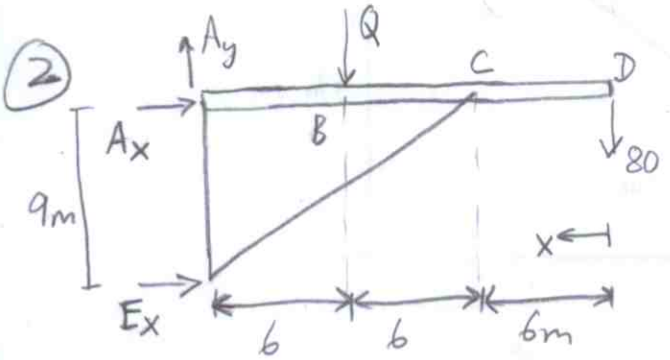




mem	P_k	$\partial P_k/\partial Q$	L_k	$P_k \frac{\partial P_k}{\partial Q} L_k \Big\vert_{Q=40}$
EC	0	0	1.5	0
BC	$\frac{Q}{2}\cdot\frac{5}{4}$	5/8	2.5	39.0625
DC	$-\frac{Q}{2}\cdot\frac{5}{4}$	-5/8	2.5	39.0625
DE	$-(30+Q)$	-1	2	140
BE	$-(30+Q)$	-1	2	140
AB	$\frac{5}{8}Q$	5/8	5	78.125
				436.25

$$\boxed{D_{CV} = \frac{436.25}{AE}\,(\downarrow)}$$

(2)



$$E_x = \frac{(80)(18)+6Q}{9} = 160 + \frac{2Q}{3} = -A_x$$

$$EC = -\left(160+\frac{2}{3}Q\right)\left(\frac{5}{4}\right) = -\left(200+\frac{5}{6}Q\right);\ \frac{\partial EC}{\partial Q} = -\frac{5}{6}$$

$$AE = \left(200+\frac{5}{6}Q\right)\left(\frac{3}{5}\right) = 120 + \frac{Q}{2};\ \frac{\partial AE}{\partial Q} = \frac{1}{2}$$

For beam:

$$M = -80x,\quad 0 \le x \le 6 \longrightarrow \frac{\partial M}{\partial Q} = 0$$

$$= -80x + \left(120+\frac{Q}{2}\right)(x-6),\quad 6 \le x \le 12 \longrightarrow \frac{\partial M}{\partial Q} = (x-6)/2$$

$$= -80x + \left(120+\frac{Q}{2}\right)(x-6) - Q(x-12),\quad 12 \le x \le 18 \longrightarrow \frac{\partial M}{\partial Q} = \frac{(x-6)}{2} - (x-12) = 9 - \frac{x}{2}$$

$$AF = N = 160 + \frac{2}{3}Q,\quad \frac{\partial N}{\partial Q} = \frac{2}{3},\ \text{for } AC.$$

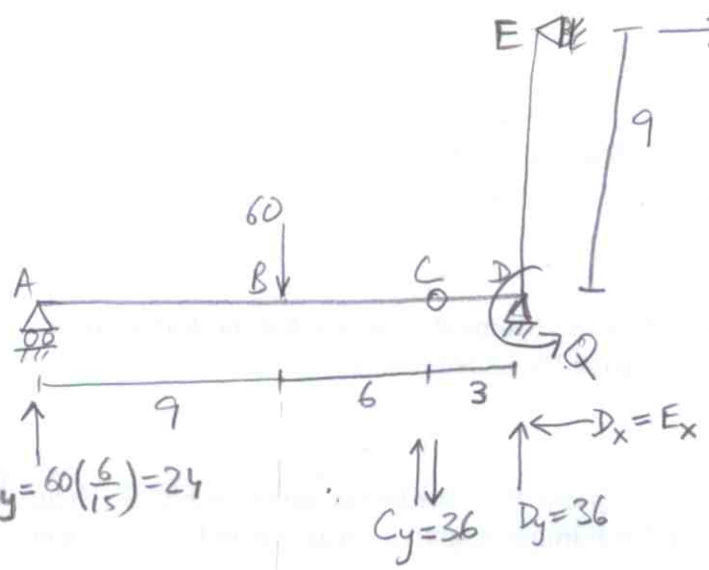
$$\Delta_{Bv} = \left\{\frac{1}{AE}\left[(EC)\frac{\partial(EC)}{\partial Q}L_{EC} + (AE)\frac{\partial(AE)}{\partial Q}L_{AE} + N\frac{\partial N}{\partial Q}L_{AC}\right] + \frac{1}{EI}\int M\frac{\partial M}{\partial Q}\,dx\right\}_{Q=0}$$

$$= \frac{1}{AE}\left[(-200)\left(-\frac{5}{6}\right)(15) + (120)\left(\frac{1}{2}\right)(9) + (160)\left(\frac{2}{3}\right)(12)\right]$$

$$+\frac{1}{EI}\left[\int_0^6(-80x)(0)\,dx + \int_6^{12}(40x-720)\frac{(x-6)}{2}\,dx + \int_{12}^{18}(40x-720)\left(9-\frac{x}{2}\right)dx\right]$$

$$\Delta_{BV} = \frac{4320}{AE} + \frac{1}{EI}\left[\frac{20}{3}\cdot(12^3-6^3) + (2160)(6) - \frac{480}{2}\cdot(12^2-6^2) - \frac{20}{3}(18^3-12^3) - (6480)(6) + \frac{720}{2}\cdot(18^2-12^2)\right] = \frac{4320}{AE} - \frac{4320}{EI}$$

(3)



$$E_x = \frac{Q + (36)(3)}{9} = \frac{Q}{9} + 12$$

$M = 24x$, $\frac{\partial M}{\partial Q} = 0$, in AB

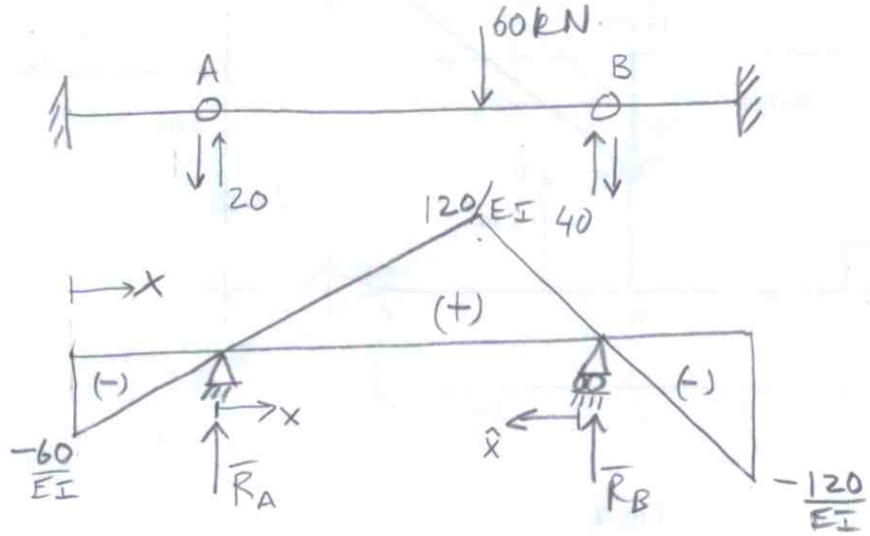
$M = 36x$, $\frac{\partial M}{\partial Q} = 0$, in CB

$M = -36x$, $\frac{\partial M}{\partial Q} = 0$ in CD

$M = -\left(\frac{Q}{9} + 12\right)x$, $\frac{\partial M}{\partial Q} = -\frac{x}{9}$ in ED

$$\theta_D = \frac{1}{EI}\int_0^9 (12x)\left(\frac{x}{9}\right)dx = \frac{1}{EI}\frac{(12)}{(9)(3)}(9)^3 = \frac{324}{EI}$$

(4)



Conjugate beam with loading.

$$\bar{R}_A(9) - \frac{1(60)}{2EI}(3)(2+9) + \frac{1}{2}\left(\frac{120}{EI}\right)(6)(2+3) + \frac{1}{2}\left(\frac{120}{EI}\right)(3)(2) + \frac{1}{2}\left(\frac{120}{EI}\right)(3)(2) = 0$$

$\bar{R}_A = -170/EI$

$\bar{R}_A + \bar{R}_B + \left[\left(\frac{120}{2}\right)(9) - \left(\frac{60}{2}\right)(3) - \left(\frac{120}{2}\right)(3)\right]\frac{1}{EI} = 0 \Rightarrow \bar{R}_B = -\frac{100}{EI}$

For max displ, we need $\bar{M}$ max, ie $\bar{V} = 0$.

$\bar{V}_{A-} = -\frac{1}{2}\left(\frac{60}{EI}\right)(3) = -\frac{90}{EI}$, $\bar{V}_{A+} = -\frac{90}{EI} - \frac{170}{EI} = -260/EI$

$\bar{V} = \left\{-260 + \frac{1}{2}(x)\left(\frac{120}{6}x\right)\right\}\frac{1}{EI} = \frac{-260 + 10x^2}{EI}$, $0 < x < 6 \rightarrow \bar{V} = 0$ for $x = \sqrt{26}$ lies in region

$\bar{V}_{B+} = \frac{1}{2}\left(\frac{120}{EI}\right)(3) = \frac{180}{EI}$, $\bar{V}_{B-} = \frac{180 + 100}{EI} = \frac{280}{EI}$

$\bar{V} = \left\{280 - \frac{1}{2}(\hat{x})\left(\frac{120}{3}\hat{x}\right)\right\}\frac{1}{EI} = \frac{280 - 20\hat{x}^2}{EI}$, $0 < \hat{x} < 3 \rightarrow \bar{V} = 0$ for $x = \sqrt{14}$ lies outside region

$\Rightarrow \bar{M}_{max}$ occurs at $x = \sqrt{26}$

at $x = \sqrt{26}$ from A $\leftarrow$ $v_{max} = \bar{M}_{max} = -\frac{1}{2}(3)\left(\frac{60}{EI}\right)(2) + \int_0^{\sqrt{26}} \frac{(10x^2 - 260)\,dx}{EI} = -\frac{1}{EI}\left[180 - 10\frac{(\sqrt{26})^3}{3} + 260\sqrt{26}\right] = -1063.83/EI$