DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE-317 STRUCTURAL MECHANICS II

Endsem exam 18/11/11

Notes

1. Common data for problems 2, 3, 4: $E = 200\,\text{GPa}$, $G = 80\,\text{GPa}$, $I_y = I_z = 8000\,\text{cm}^4$, $A = 200\,\text{cm}^2$, $J = 10000\,\text{cm}^4$
2. Numerical answers must be accurate to 2 or more places after decimal.
3. Must: use member end (local) coordinate system as done in class; use convention for all forces and displacements (linear and angular) as in class; use numbering sequence of structure's nodal forces and displacements as in class.

Problem 1

Data: $EI_z = 10$, $A/I_z = 5$, $L = 2$, $P = 1$. Rotational spring constant $k = 5$. Settlement of support node 3 is 0.1m downward. Member 23 is heated 20 ^{o}C above ambient temperature, coefficient of linear expansion is 0.000012/^{o}C. All data provided in units of N, m.

Find the numerical values of the stiffness matrix $\mathbf{K_{II}}$ and the load vector $\mathbf{P_I}$ that are required to solve for the displacements. You do not need to invert $\mathbf{K_{II}}$ and do not need to solve for displacements using $\mathbf{K_{II}^{-1}P_I}$. Settlement must be handled by including it in load vector $\mathbf{P_I}$.

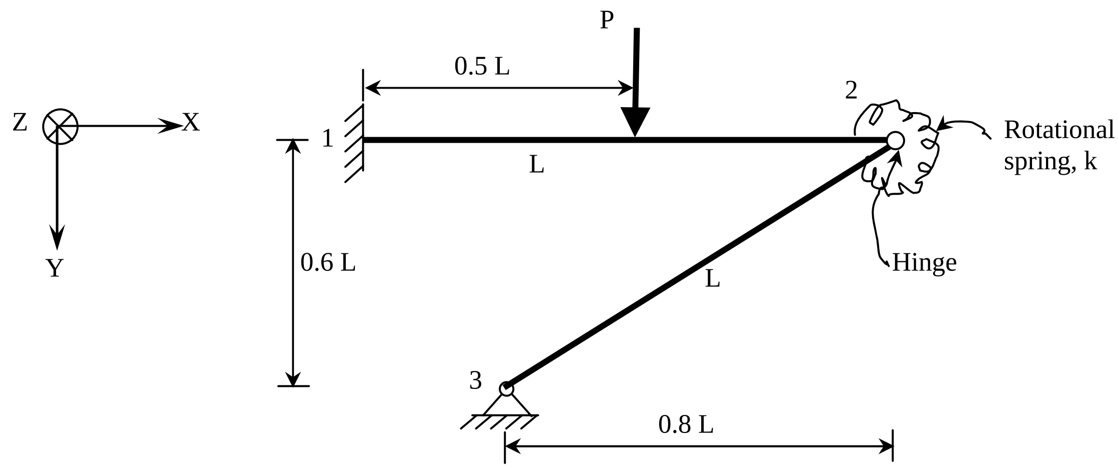

Fig. 1

Problem 2

Taking the redundant at node 4. **Use Flexibility matrix method** to **find redundant.**

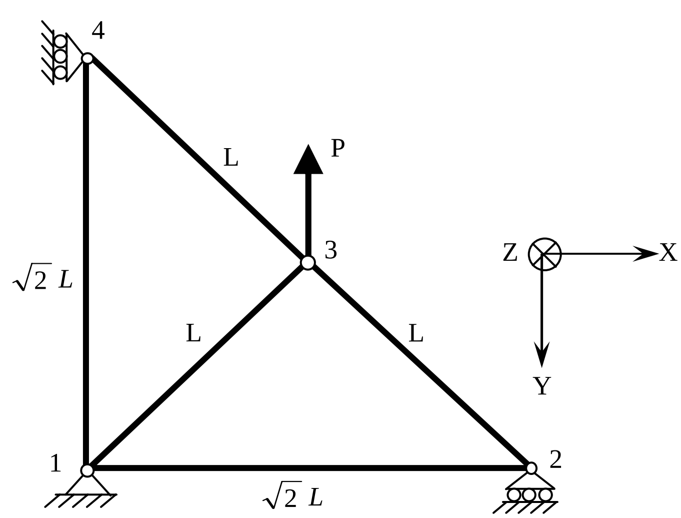

Fig. 2

Problem 3

Data: $L = 10$ m

Plan view of a grid is shown in **Fig. 3**. Thus, the vertical downward direction (i.e., Z) acts into the plane of the paper. All 16 members of the grid are subjected to vertically downward uniformly distributed load of magnitude **20 kN/m.** Further, the grid supports a vertically downward load of magnitude **100 kN** at joint 1.

Find deflections at joint 1 and reactions at joint 2 **using Stiffness Method**.

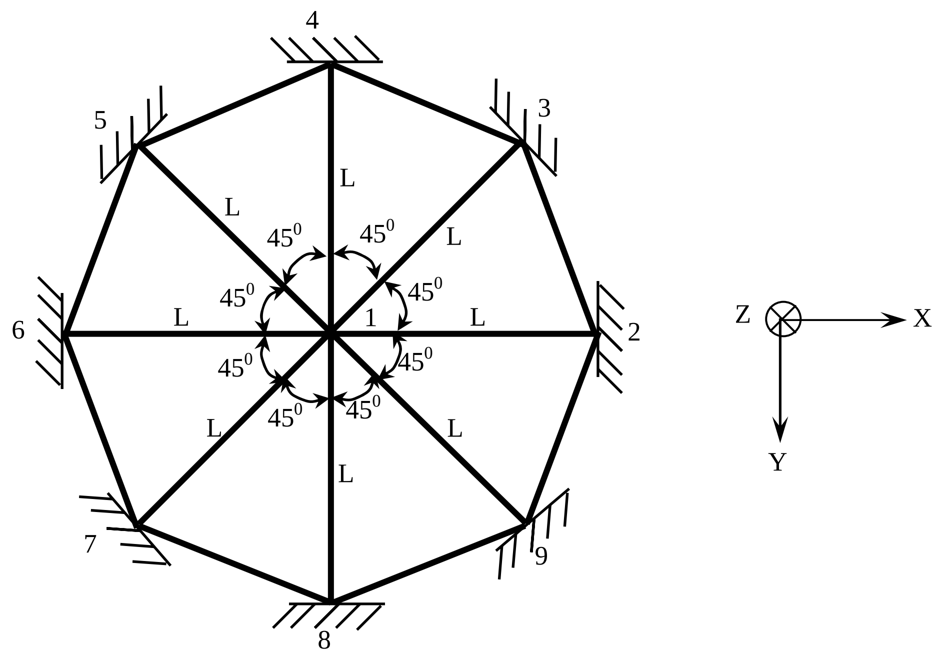

Fig. 3

Problem 4

Data: $L = 10$ m

Find deflections at joint 2 **using Stiffness Method**.

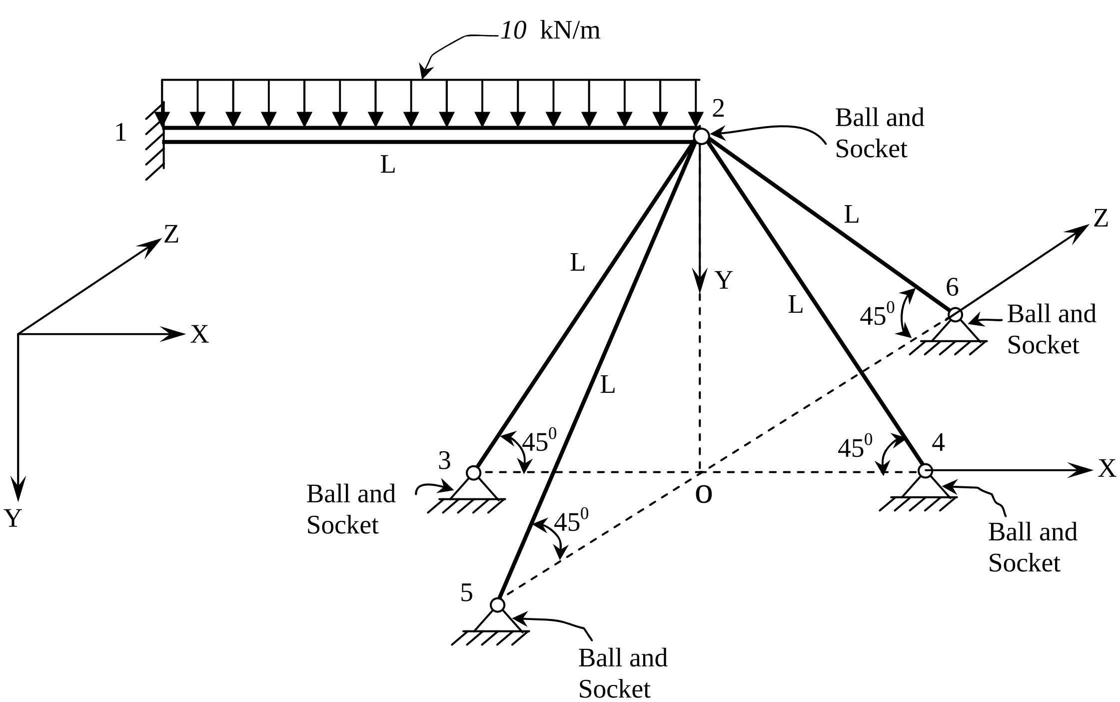

Fig. 4

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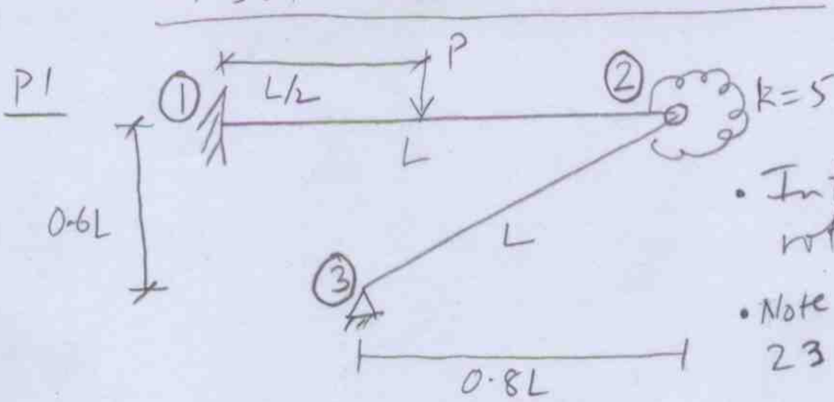
P1



- Internal hinge with partial rotational restraint.
- Note that due to spring, member 23 behaves as frame & not truss.

$$a_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad a_{23} = \begin{bmatrix} 0.8 & -0.6 & 0 \\ 0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Total 10 displs (3 at 1, 4 at 2, 3 at 3)
- dof's are (4, 5, 6, 7, 10).

$K_{22}\,(4\times4)$, $K_{23}\,(4\times3)$, $K_{32}\,(3\times4)$, $K_{33}\,(3\times3)$

$$K_{II} = \left[\begin{array}{c|c} K_{22} & K_{23}\,(\text{last col}) \\ \hline K_{32}\,(\text{last row}) & K_{33}(3,3) \end{array}\right] \rightarrow \text{not required for element wise assembly}$$

$$k^1_{22} = k^3_{22} = k^2_{33} = \frac{EI}{L}\begin{bmatrix} A/I & 0 & 0 \\ 0 & 12/L^2 & -6/L \\ 0 & -6/L & 4 \end{bmatrix} = 5\begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 4 \end{bmatrix}; \quad k_{23} = k_{32} = 5\begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 2 \end{bmatrix}$$

$$K^1_{22} = a^T_{21} k^1_{22} a_{21} = k^1_{22} = 5\;\begin{array}{c c} \begin{array}{ccc} 4 & 5 & 6 \end{array} & \\ \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 4 \end{bmatrix} & \begin{array}{c} 4 \\ 5 \\ 6 \end{array} \end{array}$$

$$K^3_{22} = a^T_{23} k^3_{22} a_{23} = 5\begin{bmatrix} 4 & 1.8 & -1.8 \\ -3 & 2.4 & -2.4 \\ 0 & -3 & 4 \end{bmatrix}\begin{bmatrix} 0.8 & -0.6 & 0 \\ 0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 5\;\begin{array}{c c} \begin{array}{ccc} 4 & 5 & 7 \end{array} & \\ \begin{bmatrix} 4.28 & -0.96 & -1.8 \\ -0.96 & 3.72 & -2.4 \\ -1.8 & -2.4 & 4 \end{bmatrix} & \begin{array}{c} 4 \\ 5 \\ 7 \end{array} \end{array}$$

$$K^{spring}_{22} = 5\;\begin{array}{c c} \begin{array}{cc} 6 & 7 \end{array} & \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{array}{c} 6 \\ 7 \end{array} \end{array}$$

$$a^T_{23} k_{23} a_{32} = \hat{K}_{23} = 5\begin{bmatrix} 4 & 1.8 & -1.8 \\ -3 & 2.4 & -2.4 \\ 0 & -3 & 2 \end{bmatrix}\begin{bmatrix} -0.8 & 0.6 & 0 \\ -0.6 & -0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 5\;\begin{array}{c c} \begin{array}{ccc} 8 & 9 & 10 \end{array} & \\ \begin{bmatrix} -4.28 & 0.96 & -1.8 \\ 0.96 & -3.72 & -2.4 \\ 1.8 & 2.4 & 2 \end{bmatrix} & \begin{array}{c} 4 \\ 5 \\ 7 \end{array} \end{array}$$

$$\hat{K}_{32} = a^T_{32} k_{32} a_{23} = \begin{array}{c c} \begin{array}{c} 4\,5\,7 \end{array} & \\ \begin{bmatrix} \hat{K}^T_{23} \end{bmatrix} & \begin{array}{c} 8 \\ 9 \\ 10 \end{array} \end{array}$$

$$K_{33} = a^T_{32} k^2_{33} a_{32} = 5\begin{bmatrix} -4 & -1.8 & 1.8 \\ 3 & -2.4 & 2.4 \\ 0 & -3 & 4 \end{bmatrix}\begin{bmatrix} -0.8 & 0.6 & 0 \\ -0.6 & -0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 5\;\begin{array}{c c} \begin{array}{ccc} 8 & 9 & 10 \end{array} & \\ \begin{bmatrix} 4.28 & -0.96 & 1.8 \\ -0.96 & 3.72 & 2.4 \\ 1.8 & 2.4 & 4 \end{bmatrix} & \begin{array}{c} 8 \\ 9 \\ 10 \end{array} \end{array}$$

Node-wise assembly

From above $\hat{K}_{23}$,

$$K_{23} = 5\;\begin{array}{c c} \begin{array}{ccc} 8 & 9 & 10 \end{array} & \\ \begin{bmatrix} -4.28 & 0.96 & -1.8 \\ 0.96 & -3.72 & -2.4 \\ 0 & 0 & 0 \\ 1.8 & 2.4 & 2 \end{bmatrix} & \begin{array}{c} 4 \\ 5 \\ 6 \\ 7 \end{array} \end{array} \qquad K_{32} = K^T_{23}$$

$$K_{22} = \left[\begin{array}{ccc|c} & K^{1}_{22} & & 0 \\ & & & 0 \\ \hline & & & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] + \left[\begin{array}{c|c} K^{3}_{22} & \\ \hline & \end{array}\right] \begin{array}{l} \text{insert 2nd last,} \\ \text{ie 3rd, row} \\ \{0000\} \end{array} + 5\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

insert 2nd last, ie 3rd, col$\{0000\}^T$

contribution of mem 1-2 — contribution of mem 2-3 — contribution of spring

$$K_{22} = 5\begin{bmatrix} 5+4.28+0, & 0-0.96+0 & 0+0+0 & 0-1.8+0 \\ 0-0.96+0 & 3+3.72+0 & -3+0+0 & 0-2.4+0 \\ 0+0+0 & -3+0+0 & 4+0+1 & 0+0-1 \\ 0-1.8+0 & 0-2.4+0 & 0+0-1 & 0+4+1 \end{bmatrix}$$

$$K_{II} = \begin{bmatrix} 46.4 & -4.8 & 0 & -9 & -9 \\ -4.8 & 33.6 & -15 & -12 & -12 \\ 0 & -15 & 25 & -5 & 0 \\ -9 & -12 & -5 & 25 & 10 \\ -9 & -12 & 0 & 10 & 20 \end{bmatrix} \blacktriangleleft$$

Element-wise assembly – Direct stiffness method

Add element contributions consistently using row-col node number pairs. Only those element submatrices will participate which have at least one entry whose row & col numbers correspond to dof's (ie unconstrained nodes). Thus

$$K_{II} = K^{1}_{22} + K^{3}_{22} + K^{spring}_{22} + \hat{K}_{23} + \hat{K}_{32} + K_{33}\text{, added consistently.}$$

$$= 5\;\begin{array}{c|c} \begin{array}{ccccc} 4 & 5 & 6 & 7 & 10 \end{array} & \\ \hline \begin{bmatrix} 5+4.28 & -0.96 & — & -1.8 & -1.8 \\ -0.96 & 3+3.72 & -3 & -2.4 & -2.4 \\ — & -3 & 4+1 & -1 & — \\ -1.8 & -2.4 & -1 & 4+1 & 2 \\ -1.8 & -2.4 & — & 2 & 4 \end{bmatrix} & \begin{array}{c} 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \end{array}$$

⟨(—) dashes are zeros⟩

→ same as above result

Loads

$$p^a = \underline{0} \; ; \quad \tilde{P}_M^e = \{0 \quad -P/2 \quad PL/8 \quad 0 \quad 0\}^T$$

Cut at 3 for temp, ∵ settlement given at 3.

$$\{\delta_s\}_3 = \begin{Bmatrix} -\alpha T_{av} L \\ 0 \\ 0 \end{Bmatrix} + \underbrace{\begin{bmatrix} -0.8 & 0.6 & 0 \\ -0.6 & -0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{a_{32}} \underbrace{\begin{Bmatrix} 0 \\ 0.1 \\ 0 \end{Bmatrix}}_{\text{settlement}} = \begin{Bmatrix} -4.8E-4 + 0.06 \\ -0.08 \\ 0 \end{Bmatrix}$$

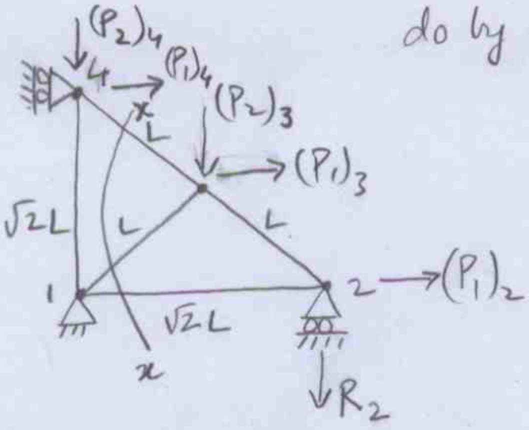
$$\{F_s\}_{32}^f = k_{33}^2 \{\delta_s\}_3 = 5 \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 4 \end{bmatrix} \begin{Bmatrix} 0.05952 \\ -0.08 \\ 0 \end{Bmatrix} = 5 \begin{Bmatrix} 0.2976 \\ -0.24 \\ 0.24 \end{Bmatrix}$$

$$\{F_s\}_{23}^f = k_{23} \{\delta_s\}_3 = 5 \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 2 \end{bmatrix} \begin{Bmatrix} 0.05952 \\ -0.08 \\ 0 \end{Bmatrix} = 5 \begin{Bmatrix} 0.2976 \\ -0.24 \\ 0.24 \end{Bmatrix}$$

$$\left.\begin{aligned} \{P_s\}_3^e &= a_{32}^T \{F_s\}_{32}^f = 5 \begin{Bmatrix} - \\ - \\ 0.24 \end{Bmatrix} \\ \{P_s\}_2^e &= a_{23}^T \{F_s\}_{23}^f = 5 \begin{Bmatrix} 0.09408 \\ -0.37056 \\ 0.24 \end{Bmatrix} \end{aligned}\right\} \Rightarrow \tilde{P}_S^e = 5 \begin{Bmatrix} 0.09408 \\ -0.37056 \\ 0 \\ 0.24 \\ 0.24 \end{Bmatrix}$$

$$P_I = P^a - \tilde{P}_M^e - \tilde{P}_S^e = \begin{Bmatrix} -0.4704 \\ 2.3528 \\ -0.25 \\ -1.2 \\ -1.2 \end{Bmatrix}$$

P2 do by Flexibility matrix method.



$$P_{II} = \{(P_1)_4\}$$

$$P_I = \{(P_1)_2 \ (P_1)_2 \ (P_2)_2 \ (P_2)_4\}$$

Equilibrium matrices

$$\sum M_{1} = 0 \Rightarrow R_2 \sqrt{2}\cancel{L} + (P_1)_3 \frac{\cancel{L}}{\sqrt{2}} + (P_2)_3 \frac{\cancel{L}}{\sqrt{2}} + (P_1)_4 \sqrt{2}\cancel{L} = 0$$

$$F_{32} = F_{23} = R_2\sqrt{2} = -(P_1)_3 \frac{1}{\sqrt{2}} - (P_2)_3 \frac{1}{\sqrt{2}} - (P_1)_4\sqrt{2}$$

$$F_{21} = F_{12} = (P_1)_2 - \frac{F_{23}}{\sqrt{2}} = (P_1)_2 + (P_1)_3 \cdot \frac{1}{2} + (P_2)_3 \frac{1}{2} + (P_1)_4$$

Section xx $\rightarrow \sum M_{2} = 0 \Rightarrow \underbrace{F_{31}}_{=F_{13}} \cdot L - (P_1)_3 \frac{L}{\sqrt{2}} + (P_2)_3 \frac{L}{\sqrt{2}} = 0$

$$\Rightarrow F_{31} = F_{13} = (P_1)_3 \frac{1}{\sqrt{2}} - \frac{(P_2)_3}{\sqrt{2}}$$

$$\sum M_{1} = 0 \Rightarrow \underbrace{F_{43}}_{=F_{34}} \cdot L + (P_1)_4 \sqrt{2}L = 0 \Rightarrow F_{34} = F_{43} = -\sqrt{2}(P_1)_4$$

$$F_{41} = F_{14} = -(P_2)_4 - F_{43} \cdot \frac{1}{\sqrt{2}} = -(P_2)_4 + (P_1)_4$$

$$\begin{Bmatrix} F_{21} \\ F_{32} \\ F_{31} \\ F_{43} \\ F_{41} \end{Bmatrix} = \Bigg[\underbrace{\begin{matrix} 1 & 1/2 & 1/2 & 0 \\ 0 & -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{matrix}}_{b_I} \ \Bigg| \ \underbrace{\begin{matrix} 1 \\ -\sqrt{2} \\ 0 \\ -\sqrt{2} \\ 1 \end{matrix}}_{b_{II}} \Bigg] \begin{Bmatrix} \left.\begin{matrix} (P_1)_2 \\ (P_1)_3 \\ (P_2)_3 \\ (P_2)_4 \end{matrix}\right\} \rightarrow P_I \\ \hline \left.(P_1)_4\right\} \rightarrow P_{II} \end{Bmatrix}$$

$$d_u = \frac{L}{EA} \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} \end{bmatrix}$$

$$P_{II} = -D_{II\,II}^{-1}\, D_{II\,I}\, P_I$$

$$D_{II\,II} = b_{II}^T d_u b_{II} = \frac{L}{EA}\left\{\sqrt{2} \quad -\sqrt{2} \quad 0 \quad -\sqrt{2} \quad \sqrt{2}\right\} b_{II}$$

$$= \sqrt{2} + 2 + 2 + \sqrt{2} = 4 + 2\sqrt{2}$$

$$D_{II\,I} = b_{II}^T d_u b_I = \left\{\sqrt{2} \quad 1+\tfrac{1}{\sqrt{2}} \quad 1+\tfrac{1}{\sqrt{2}} \quad -\sqrt{2}\right\} \frac{L}{EA}$$

$$P_{II} = -\frac{1}{(4+2\sqrt{2})}\left\{\sqrt{2} \quad 1+\tfrac{1}{\sqrt{2}} \quad 1+\tfrac{1}{\sqrt{2}} \quad -\sqrt{2}\right\}\begin{Bmatrix} 0 \\ 0 \\ -P \\ 0 \end{Bmatrix} = \frac{\sqrt{2}+1}{(4\sqrt{2}+4)} P = \frac{P}{4}$$

6

P3 Grid $\Rightarrow \{(\Delta_1)_1\ (\Delta_2)_1\ (\Delta_6)_1\} = \{0\ \ 0\ \ 0\}$

Symmetry $\Rightarrow \{(\Delta_4)_1\ (\Delta_5)_1\} = \{0\ \ 0\}$

ie no out-of-plane rotation of joint 1, due to symmetry.

So only $(\Delta_3)_1$ is $\neq 0$, ie only one-dof activated.

As always, $K_{11} = \sum_{i=1}^{9} K_{11}^{i}$

ie K_{11} contains entries pertaining to dof's (3,4,5) at node ①. But ∵ dof's $(4,5)_①$ are known zero from symmetry, we need only $K_{11}(1,1)$ to solve for $(\Delta_3)_1$.

Symmetry, ie 1-dof, $\Rightarrow K_{II} = K_{11}(1,1)$

eg $K_{11}^{3}(1,1) = a_{13}^{T}\, k_{11}^{3}\, a_{13}$, $a_{13} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$, $k_{11}^{3} = \frac{EI}{L}\begin{bmatrix} 12/L^2 & 0 & 6/L \\ 0 & 0.5 & 0 \\ 6/L & 0 & 4 \end{bmatrix}$

where we used $\frac{GJ}{EI_y} = \frac{80 * 10000}{200 * 8000} = 0.5$

$\Rightarrow K_{11}^{3}(1,1) = \frac{EI}{L}\frac{12}{L^2} \Rightarrow K_{II} = K_{11}(1,1) = 8 * K_{11}^{3}(1,1)$

$= 8 * \frac{12}{L^2}\frac{EI_y}{L} = \frac{96}{10^2} \cdot \frac{200E6 \cdot 8000 \cdot E\text{-}8}{10}$

$K_{II} = 1536$

$P_I = \left[100 + \frac{20 * 10}{2} * 8\right] = [900]$ kN

$\Delta_I = \{(\Delta_3)_1\} = \frac{1}{1536} \cdot 900 = 0.5859$ m

$F_{21} = k_{22}^{1}\underbrace{\{\delta\}_2}_{0} + k_{21}\{\delta\}_1 + \underbrace{\begin{Bmatrix} -wL_1 \\ 0 \\ -\frac{wL_1^2}{12}\cos(67.5) * 2 \end{Bmatrix}}_{\text{Fef's of circumferential members, ie } \{F\}_{23}^{f} + \{F\}_{29}^{f}} + \underbrace{\begin{Bmatrix} -wL/2 \\ 0 \\ -wL^2/12 \end{Bmatrix}}_{\{F\}_{21}^{f}}$

$= \frac{EI_y}{L}\begin{bmatrix} -12/L^2 & 0 & -6/L \\ 0 & 0.5 & 0 \\ -6/L & 0 & -2 \end{bmatrix}\begin{Bmatrix} 900/1536 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -20 * 2 * 10\cos(67.5) \\ 0 \\ -20 * \frac{(2 * 10\cos(67.5))^2}{12} * \cos(67.5) * 2 \end{Bmatrix} + \begin{Bmatrix} -20 * 10/2 \\ 0 \\ -20 * 10^2/12 \end{Bmatrix}$

$= 1600\begin{bmatrix} -0.12 & 0 & -0.6 \\ 0 & 0.5 & 0 \\ -0.6 & 0 & -2 \end{bmatrix}\begin{Bmatrix} 900/1536 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -153.07 \\ 0 \\ -74.72 \end{Bmatrix} + \begin{Bmatrix} -100 \\ 0 \\ -166.67 \end{Bmatrix}$

$= \begin{Bmatrix} -365.57 \\ 0 \\ -803.89 \end{Bmatrix}$

NOTE: Symmetry of structure & loading $\Rightarrow (\Delta_4)_1 = (\Delta_5)_1 = 0$, ie when $(P_4)_1 = (P_5)_1 = 0$ & $(P_3)_1 \neq 0$

(extra, not reqd) Thus K_{II} check that $K_{II}^{-1} P_I$ give $(\Delta_4)_1 = (\Delta_5)_1 = 0$ for $(P_4)_1 = (P_5)_1 = 0$ & $(P_3)_1 \neq 0$

Thus K_{II} is diagonal for this symmetric structure.

P4 Symmetry $\Rightarrow (\Delta_3)_2 = (\Delta_4)_2 = (\Delta_5)_2 = 0$ at joint 2, ie members 12, 23, 24 deform in XY plane only. Actually as we see later even 25, 26 deform in XY plane.

Thus,

$$a_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad a_{23} = \left\{ \tfrac{1}{\sqrt{2}} \;\; -\tfrac{1}{\sqrt{2}} \;\; 0 \right\} ; \quad a_{24} = \left\{ -\tfrac{1}{\sqrt{2}} \;\; -\tfrac{1}{\sqrt{2}} \;\; 0 \right\}$$

$$a_{25} = \left\{ 0 \;\; -\tfrac{1}{\sqrt{2}} \;\; \tfrac{1}{\sqrt{2}} \right\} ; \quad a_{26} = \left\{ 0 \;\; -\tfrac{1}{\sqrt{2}} \;\; -\tfrac{1}{\sqrt{2}} \right\}$$

$$k_{22}^{1} = \frac{EI_z}{L} \begin{bmatrix} A/I_z & 0 & 0 \\ 0 & 12/L^2 & -6/L \\ 0 & -6/L & 4 \end{bmatrix} = 1600 \begin{bmatrix} 250 & 0 & 0 \\ 0 & 0.12 & -0.6 \\ 0 & -0.6 & 4 \end{bmatrix},$$

where we used

$$\boxed{\frac{EI_z}{L} = \frac{200E6 \cdot 8000E{-8}}{10} = 1600 \qquad \frac{A}{I_z} = \frac{200E{-4}}{8000E{-8}} = 250}$$

$$k_{22}^{3} = k_{22}^{4} = k_{22}^{5} = k_{22}^{6} = [EA/L] = [4E5]$$

$$K_{22}^{1} = a_{21}^{T} k_{22}^{1} a_{21} = k_{22}^{1} = 1600 \begin{bmatrix} 250 & 0 & 0 \\ 0 & 0.12 & -0.6 \\ 0 & -0.6 & 4 \end{bmatrix}$$

$$K_{22}^{3} = a_{23}^{T} k_{22}^{3} a_{23} = \begin{Bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{Bmatrix} [4E5] \{1/\sqrt{2} \;\; -1/\sqrt{2} \;\; 0\} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 2E5 & -2E5 & 0 \\ -2E5 & 2E5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

$$K_{22}^{4} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 2E5 & 2E5 & 0 \\ 2E5 & 2E5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \quad K_{22}^{5} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2E5 & -2E5 \\ 0 & -2E5 & 2E5 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \quad K_{22}^{6} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2E5 & 2E5 \\ 0 & 2E5 & 2E5 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

$$K_{22} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 6 \end{array} \\ \begin{bmatrix} 4E5+2E5+2E5 & -2E5+2E5 & 0 \\ -2E5+2E5 & 192+2E5+2E5+2E5+2E5 & -960 \\ 0 & -960 & 6400 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 6 \end{array} = K_{II} = \begin{bmatrix} 8E5 & 0 & 0 \\ 0 & (8E5+192) & -960 \\ 0 & -960 & 6400 \end{bmatrix}$$

$$P_I = \begin{Bmatrix} 0 \\ 10*10/2 \\ -10*10^2/12 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 50 \\ -\frac{1000}{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 50 \\ -83.33 \end{Bmatrix}$$

$$\Delta_I = K_{II}^{-1} P_I = \begin{bmatrix} 1/8E5 & 0 & 0 \\ 0 & \frac{1}{det}\begin{bmatrix} 6400 & 960 \\ 960 & (8E5+192) \end{bmatrix} \\ 0 & \end{bmatrix} \begin{Bmatrix} 0 \\ 50 \\ -\frac{1000}{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 240000 \\ 66634666.67 \end{Bmatrix} \frac{1}{det}$$

$$det = 5120307200$$

$$= \begin{Bmatrix} 0 \\ 4.69E{-5} \\ 0.01301 \end{Bmatrix} \Rightarrow (\Delta_2)_1 = 0 \Rightarrow \text{can use beam formulation and } 2\times2 \text{ transf for truss members.}$$

This is expected ∵ beam 12 is just resting on 4 truss members, and no X-dir loading, so no X-dir memforce in mem 21, ie no X-dir reactions at nodes 1, 3, 4, 5, 6, ⇒ no X-dir displ at node 2.