DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem 20/11/12

Problem 1

Use only Matrix Stiffness Method.

Members of a raft foundation are modeled as the rigid jointed grid lying in the horizontal plane as shown in **Fig. 1**. The grid consists of two orthogonal members having identical length, properties and loading. The grid is subjected to vertical soil loads and carries a vertical column load of 500 kN at ***O***. A pile, which is modeled as a vertical spring, provides support to the grid at ***O***.

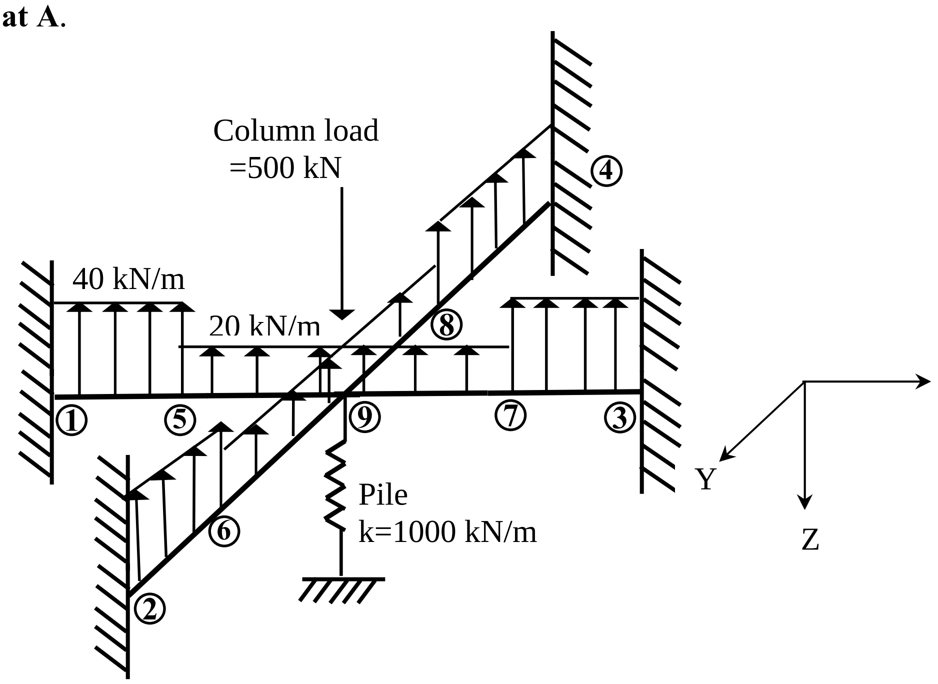
Data: $AM = MO = OP = PC = BN = NO = OQ = QD = 2$ m.

40 kN/m acts on *AM*, *PC*, *BN*, *QD* portions of the members

20 kN/m acts on *MO*, *OP*, *NO*, *OQ* portions of the members

Find: Reactions at A.

Fig. 1



Problem 2

Use only Matrix Stiffness Method.

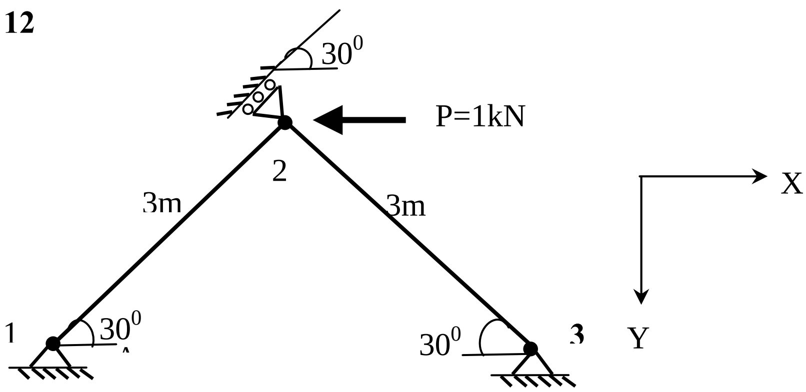
Data: $EI_y = EI_z = 9, \quad A/I_y = 2/3, \quad L = 3, \quad GJ/EI_y = 0.5, \quad P = 2.$

Settlement of support node 3: 0.2m downward.

Member 12 is heated 20 ^{o}C above ambient temperature, coefficient of linear expansion is 0.01/^{o}C.

Find: Force in member 12

Fig. 2

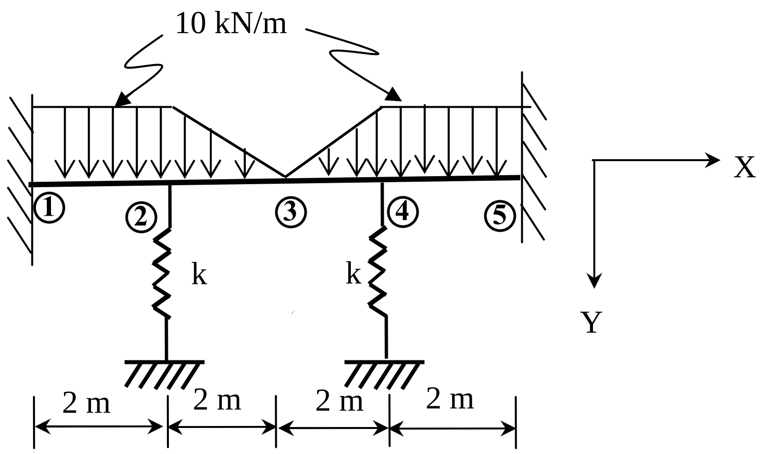


Problem 3

Use only Stiffness Matrix Method.
Data: vertical displacement of ***B*** measured as 1.909 mm;
$k = 5000$ kN/m; $EI = 10000$ kN.m^2

Find: Vertical displacement at *C* and rotation at *D*

Fig. 3

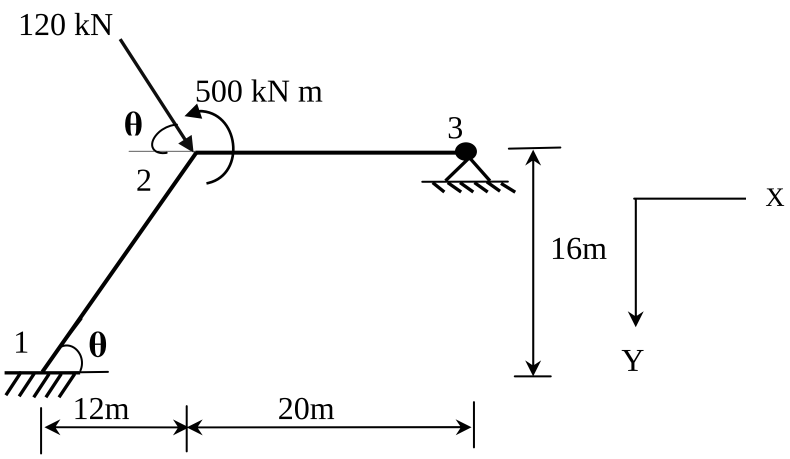


Problem 4

Use only Flexibility Matrix Method.
Data: $EI / EA = 100$ **c**m^2

Find: Reactions at node 3

Fig. 4



P1 Symmetry $\Rightarrow (\Delta_4)_q = (\Delta_5)_q = 0$ (ie $(\theta_X)_q = (\theta_Y)_q = 0$)

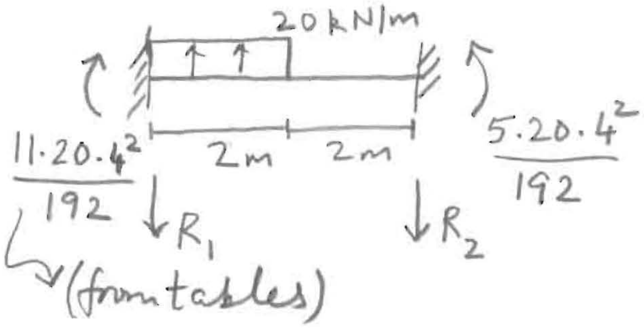
$K_{II} = K_{qq}$ (3x3 matrix).

$\because (\Delta_4)_o = (\Delta_5)_o = 0$, we need only $K_{qq}(1,1)$ to find $(\Delta_3)_q$

$\because a_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \boxed{} & \\ 0 & & \end{pmatrix}$ for grid, $K_{qq}(1,1) = \sum_{j=1}^{4} k_{qj}(1,1) + k$

$$K_{qq}(1,1) = K_{II}(1,1) = \frac{12EI}{4^3} * 4 + k = \frac{12 \cdot 200E6 \cdot 10^{-3}}{16} + 1000 = 151000 \text{ kN/m}$$

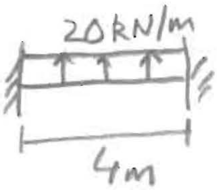
$$(P_3)_q = K_{II}(1,1)(\Delta_3)_q \quad (\because (\Delta_4)_q = (\Delta_5)_q = 0)$$



$$R_2 = \frac{1}{4}\left[(5-11)\frac{20 \cdot 4^2}{192} + 20 \cdot \frac{2^2}{2}\right] = 7.5$$

$$(P_3)_q = 500 - 4\left(7.5 + \frac{20 \cdot 4}{2}\right) = 310 \text{kN}$$

due to

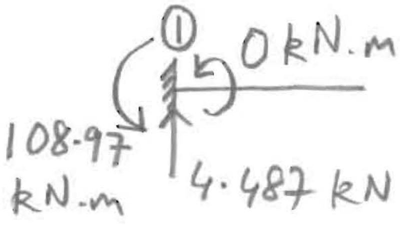


$$(\Delta_3)_q = \frac{310}{151000} \text{m} = 2.053 \text{ mm}$$

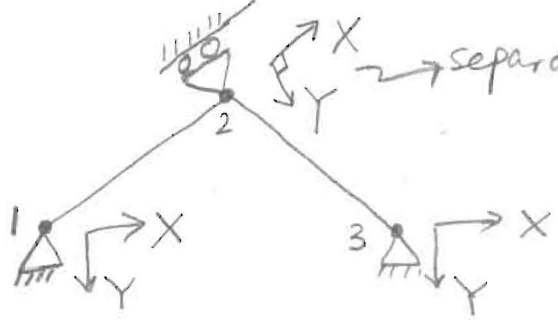
$$F_{1q} = k_{1q}\,\delta_{q1} + F_{1q}^{f} \quad (\because \delta_{1q} = \underset{\sim}{0})$$

$$= \frac{EI}{L}\begin{bmatrix} -12/L^2 & 0 & -6/L \\ 0 & GJ/EI & \\ -6/L & 0 & -2 \end{bmatrix}\begin{Bmatrix} 2.053\text{E}-3 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} \frac{20 \cdot 4}{2} + (20 \cdot 2 - 7.5) \\ 0 \\ \frac{11 \cdot 20 \cdot 4^2}{192} + \frac{20 \cdot 4^2}{12} \end{Bmatrix}$$

$$= \begin{Bmatrix} -4.487 \\ 0 \\ -108.97 \end{Bmatrix}$$



P2



$\to$ separate global coord for node ②

$$a_{12} = \left(-\frac{\sqrt{3}}{2} \;\; \frac{1}{2}\right), \quad a_{32} = \left(\frac{\sqrt{3}}{2} \;\; \frac{1}{2}\right)$$

$$a_{21} = (1 \;\; 0), \quad a_{23} = \left(-\frac{1}{2} \;\; -\frac{\sqrt{3}}{2}\right)$$

②

$$K_{II} = K_{22}(1,1)$$

$$K_{22} = \frac{EA}{L}\left(\begin{bmatrix}1\\0\end{bmatrix}[1 \;\; 0] + \begin{bmatrix}-1/2\\-\sqrt{3}/2\end{bmatrix}\left[-\frac{1}{2} \;\; -\frac{\sqrt{3}}{2}\right]\right) = \frac{EA}{L}\begin{pmatrix}5/4 & \sqrt{3}/4\\ \sqrt{3}/4 & 3/4\end{pmatrix}$$

$$K_{II} = \frac{EA}{L}\left(\frac{5}{4}\right) = 9\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{5}{4} = \frac{5}{2}$$

$$P_{I,mech} = P_a = -P\frac{\sqrt{3}}{2} = -\sqrt{3}/2$$

$$P_{2,\text{temp + settle}} = a_{23}^T \underbrace{\left[\frac{EA}{L}(-\alpha\Delta T L) + \frac{EA}{L}a_{32}\underbrace{\begin{Bmatrix}0\\s\end{Bmatrix}}_{\text{settlement}}\right]}_{[\,] = F^f_{s23}}$$

$$= 9\cdot\frac{2}{3}\cdot\frac{1}{3}\begin{bmatrix}-1/2\\-\sqrt{3}/2\end{bmatrix}\left[-0.01\cdot 20\cdot 3 \;\; [\sqrt{3}/2 \;\; 1/2]\begin{bmatrix}0\\0.2\end{bmatrix}\right]$$

$$= \underbrace{\begin{bmatrix}0.5\\ \sqrt{3}/2\end{bmatrix}}_{\tilde{p}^e}$$

$$P_I = P_a - \tilde{p}^e = -\frac{\sqrt{3}}{2} - \frac{1}{2} = -1.366 \text{ kN}$$

$$\Delta_I = K_{II}^{-1} P_I = -\frac{2}{5}(1.366) = -0.5464$$

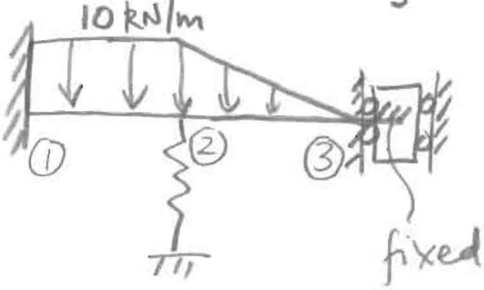
$$F_{21} = \frac{EA}{L}a_{21}\Delta_2 = 2\,[1 \;\; 0]\begin{bmatrix}-0.5464\\0\end{bmatrix} = -1.0928 \text{ kN}$$ ◀

(ie, Compressive)

$$F_{23} = \frac{EA}{L}a_{23}\Delta_2 + F^f_{s23} = 2\left[-\frac{1}{2} \;\; -\frac{\sqrt{3}}{2}\right]\begin{bmatrix}-0.5464\\0\end{bmatrix} + [-1]$$

$$= -0.4536$$ (ie compressive) ◀

P3 Symmetry $\Rightarrow (\Delta_6)_3 = 0$ (ie $(\theta)_3 = 0$).



fixed-roller.

Equivalent half-structure

Can use beam formulation ($\because$ no horizontal loads)

$$K_{II} = \text{keep rows/cols } (3,4,5) = \begin{bmatrix} K_{22}(1,1)+k & K_{22}(1,2) & K_{23}(1,1) \\ K_{22}(2,1) & K_{22}(2,2) & K_{23}(2,1) \\ K_{32}(1,1) & K_{32}(1,2) & K_{33}(1,1) \end{bmatrix}$$

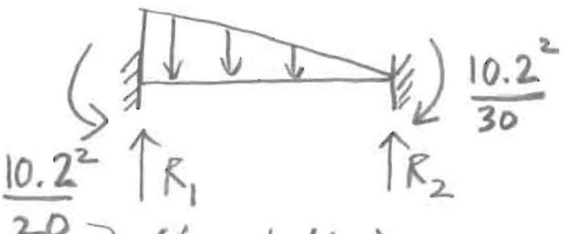
$$a_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = a_{32} \quad , \quad a_{23} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K_{22} = \frac{EI}{L}\left(\begin{bmatrix} 12/L^2 & -6/L \\ -6/L & 4 \end{bmatrix} + \begin{bmatrix} 12/L^2 & 6/L \\ 6/L & 4 \end{bmatrix} \right) = \frac{2EI}{L}\begin{bmatrix} 12/L^2 & 0 \\ 0 & 4 \end{bmatrix}$$

$$K_{33} = k_{33}^2 = \frac{EI}{L}\begin{bmatrix} 12/L^2 & -6/L \\ -6/L & 4 \end{bmatrix}$$

$$K_{23} = \frac{EI}{L}\begin{bmatrix} -12/L^2 & 6/L \\ -6/L & 2 \end{bmatrix} \quad , \quad K_{32} = K_{23}^T$$

$$K_{II} = \frac{EI}{L}\begin{bmatrix} 24/L^2 + \frac{L}{EI}k & 0 & -12/L^2 \\ 0 & 8 & -6/L \\ -12/L^2 & -6/L & 12/L^2 \end{bmatrix} = 5000\begin{bmatrix} 7 & 0 & -3 \\ 0 & 8 & -3 \\ -3 & -3 & 3 \end{bmatrix}$$



(from tables)

$$R_2 = \frac{1}{2}\left(\frac{10.2^2}{30} - \frac{10.2^2}{20} + \frac{1}{2}.2.10.\frac{1}{3}.2 \right) = 3$$

$$R_1 = \frac{1}{2}.2.10 - 3 = 7 \text{ kN}$$

$$P_I = \begin{Bmatrix} \frac{10.2}{2} + 7 \\ -\frac{10.2^2}{12} + \frac{10.2^2}{20} \\ 3 \end{Bmatrix} = \begin{Bmatrix} 17 \\ -4/3 \\ 3 \end{Bmatrix} \; ; \; \Delta_I = K_{II}^{-1} P_I = \begin{Bmatrix} 1.909 \\ 1.212 \\ -3.321 \end{Bmatrix} 10^{-3}$$

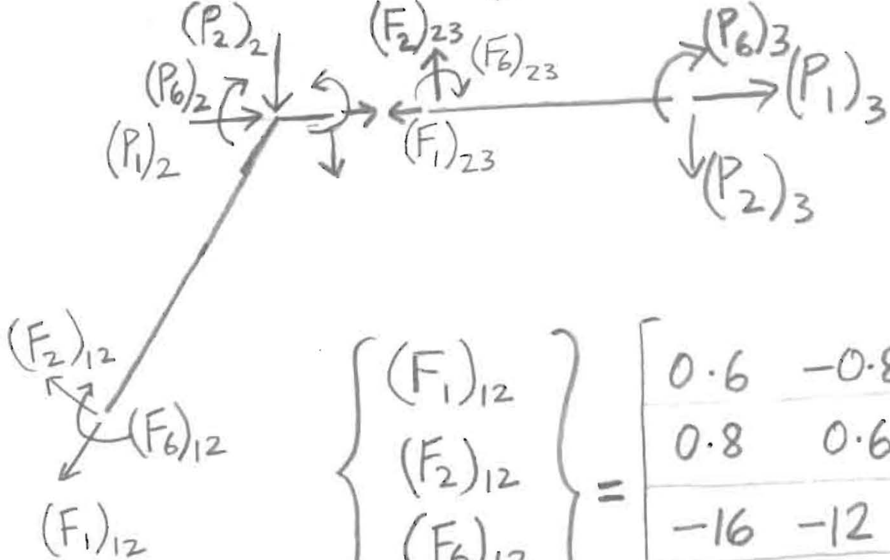
$\downarrow$ 3

$\circlearrowleft$ 4

P4 Reactions at node 3 are redundants.
Only external redundants.

$$P_{II} = \begin{Bmatrix} (P_1)_3 \\ (P_2)_3 \end{Bmatrix} = D_{II\,II}^{-1}\left(-D_{II\,I}\, P_I\right)$$

$$P_I = \left\{120 \cdot \frac{3}{5} \quad 120 \cdot \frac{4}{5} \quad -500 \quad 0\right\}^T$$



$$\begin{Bmatrix} (F_1)_{12} \\ (F_2)_{12} \\ (F_6)_{12} \\ (F_1)_{23} \\ (F_2)_{23} \\ (F_6)_{23} \end{Bmatrix} = \Bigg[\underbrace{\begin{matrix} 0.6 & -0.8 & 0 & 0 \\ 0.8 & 0.6 & 0 & 0 \\ -16 & -12 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{matrix}}_{b_I} \Bigg| \underbrace{\begin{matrix} 0.6 & -0.8 \\ 0.8 & 0.6 \\ -16 & -32 \\ 1 & 0 \\ 0 & 1 \\ 0 & -20 \end{matrix}}_{b_{II}}\Bigg] \begin{Bmatrix} (P_1)_2 \\ (P_2)_2 \\ (P_6)_2 \\ (P_6)_3 \\ (P_1)_3 \\ (P_2)_3 \end{Bmatrix}$$

$\beta = \frac{EI}{EA}$, $L_{12} = L_{23} = L$

$$d_u = \frac{L}{EI}\begin{bmatrix} \boxed{\begin{matrix} \beta & 0 & 0 \\ 0 & L^2/3 & L/2 \\ 0 & L/2 & 1 \end{matrix}} & 0 \\ 0 & \square \end{bmatrix} = \frac{20}{EI}\begin{bmatrix} \boxed{\begin{matrix} 0.01 & 0 & 0 \\ 0 & 400/3 & 10 \\ 0 & 10 & 1 \end{matrix}} & 0 \\ 0 & \square \end{bmatrix}$$

$$b_{II}^T d_u = \begin{bmatrix} 0.006 & -160/3 & -8 & 0.01 & 0 & 0 \\ -0.008 & -240 & -26 & 0 & -200/3 & -10 \end{bmatrix}$$

$$D_{II\,II} = b_{II}^T d_u b_{II} = \begin{bmatrix} 85.347 & 223.995 \\ 223.995 & 821.340 \end{bmatrix} \cdot \frac{L}{EI} \cdot 20$$

$$D_{II\,I} = b_{II}^T d_u b_I = \begin{bmatrix} 85.337 & 63.995 & 8 & 8 \\ 223.995 & 168.006 & 26 & 36 \end{bmatrix} \cdot \frac{L}{EI} \cdot 20$$

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$$D_{II\,I}\,P_I = \left\{8287.784 \quad 19256.216\right\}^T \cdot \frac{L}{EI} \cdot 20$$

$$D_{II\,II}^{-1} = \begin{bmatrix} 0.0412 & -0.0112 \\ -0.0112 & 0.0043 \end{bmatrix} \cdot \frac{EI}{L} \cdot \frac{1}{20}$$

$$P_{II} = \begin{Bmatrix} -125.158 \\ 10.688 \end{Bmatrix}$$