DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem exam 16/11/13 PAPER CODE: A

Note: Write your name & roll no. on answerbook and on summary answer sheet provided.

You must submit the summary-answer-sheet along with the answerbook.

Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.

All questions carry equal marks. All answers should be given upto at least three significant digits.

Must use: global coordinate system provided with the problem; member end (local) coordinate system as done in class; convention for all forces and displacements (linear and angular) as done in class; numbering sequence of structure's nodal forces and displacements as done in class.

Problem 1

For this question: Use only Stiffness Method. Settlement must be handled only through self straining by including it in the load vector $\underline{\mathbf{P}_{\mathbf{I}}}$. Thus, settlement must not be handled through $\underline{\boldsymbol{\Delta}_{\mathbf{II}}}$ term for this question.

Fig.1 shows a pin-jointed truss with joints 1, 2, 3, 4. The truss has six members, i.e., members 24 and 13 are not connected to each other.

Member 34 is **cooled** to 100^0C below ambient temperature.

Support at joint-4 settles down by 0.5 cm

Assume $AE = 40000$ kN, $\alpha = 0.000012/{}^0C$, for all members.

Find: (i) $\mathbf{K}_{\mathbf{II}}$; (ii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector) ; (iii) $\Delta_{\mathbf{I}}$ (joint displacement vector)

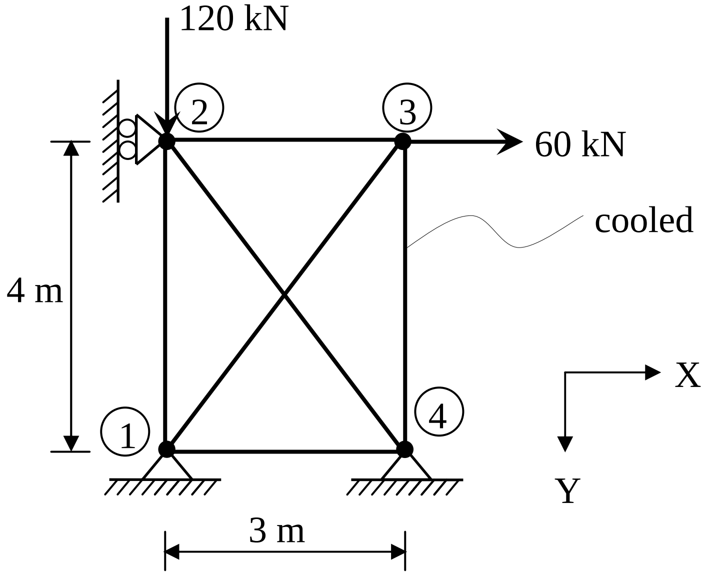

Fig. 1

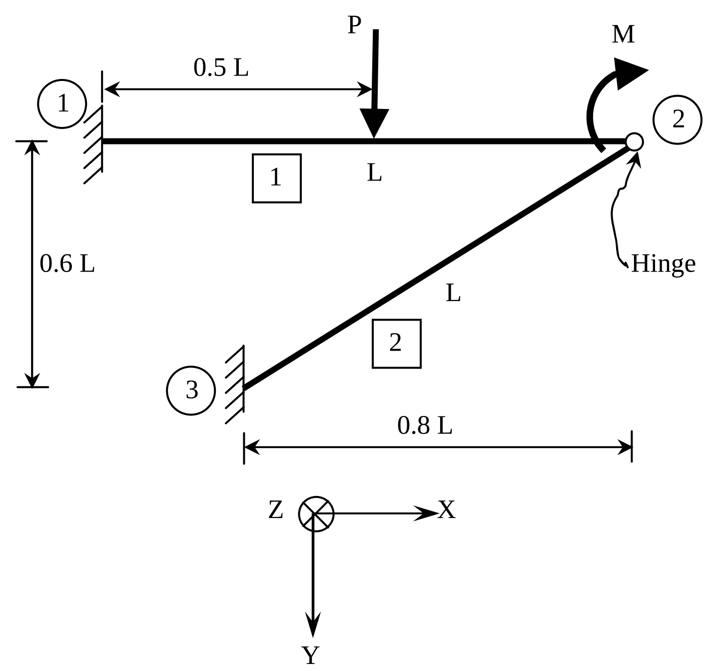

Fig. 2

Problem 2

For this question use only Stiffness Method.

The frame shown in Fig. 2 has an internal hinge. It is loaded as shown. The moment is applied to member-1 at joint-2.

Data: $EI = 10$, $A/I = 3$, $L = 2$, $P = 20$, $M = 3$. All data provided in units of kN, m.

Find: (i) $\mathbf{K}_{\mathbf{II}}$; (ii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector)

Problem 3

Data: $E = 200\text{ GPa},\ G = 80\text{ GPa},\ I_y = I_z = 8000\text{ cm}^4,\ A = 200\text{ cm}^2,\ J = 10000\text{ cm}^4,\ L = 10\text{ m}$

Find: deflections at joint-2, i.e., Δ_2 **using Stiffness Method**.

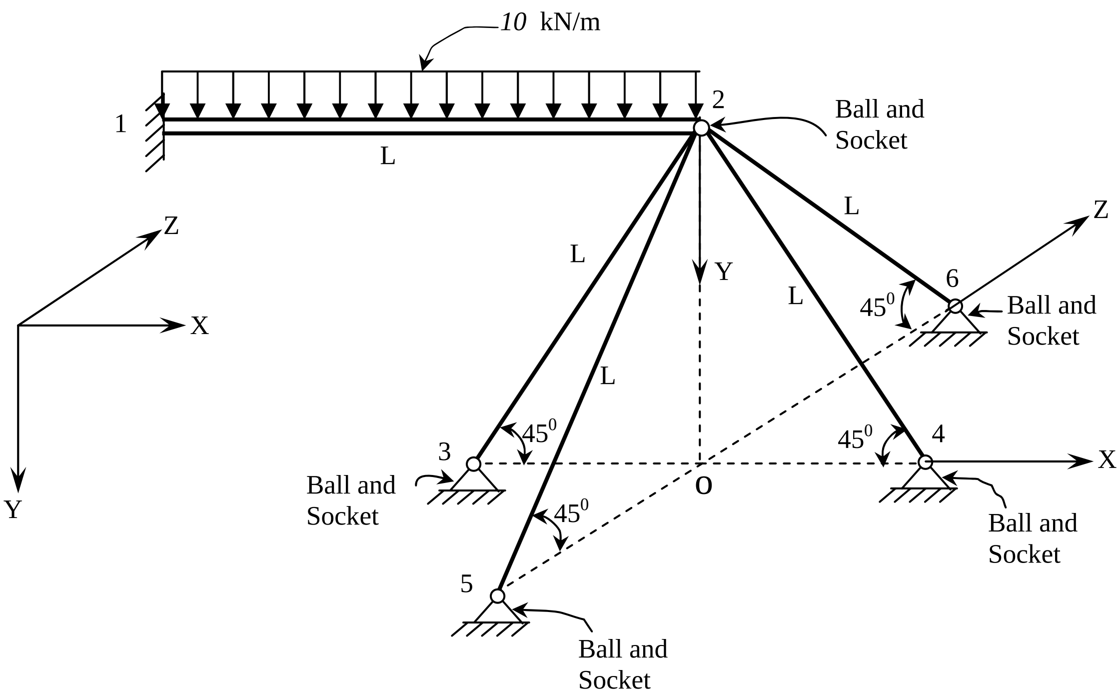

Fig. 3

Problem 4

For this question: Use only Flexibility Method. Settlement must be handled only through Δ_{II} term. Thus, settlement must not be handled through self straining by including it in the load vector P_I.

Fig.4 shows a pin-jointed truss. **You must consider redundant as reaction at joint-4.**
Support at joint-4 settles down by 0.6 cm
Assume $EA = 40000$ kN.

Find: (i) $\mathbf{D_{II,II}}$; (ii) $\mathbf{D_{II,I}}$; (iii) $\mathbf{P_I}$ (joint load vector); (iv) Reaction at joint-4

reaction at joint-4 **using Flexibility Method**.

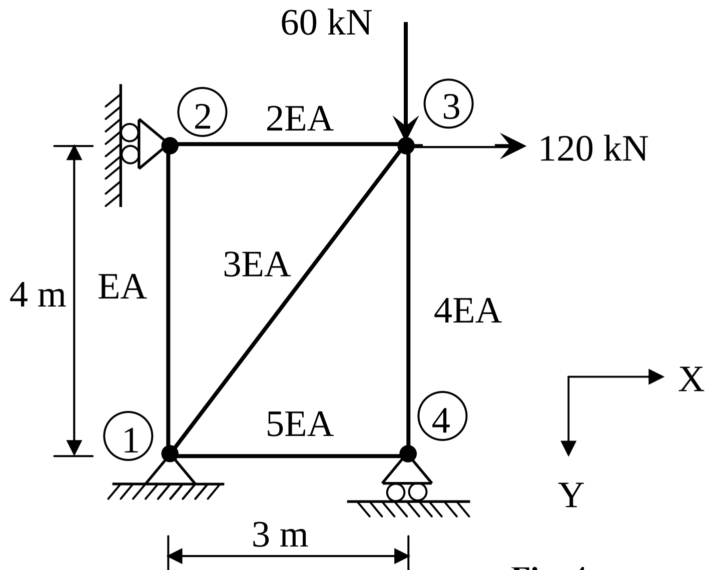

Fig. 4

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem exam 16/11/13 PAPER CODE: A

SUMMARY ANSWER SHEET

NAME:

ROLL No:

Problem 1

(i) $\mathbf{K_{II}}$ =

(ii) $\mathbf{P_{I}}$ =

(iii) Δ_{I} =

Problem 2

(i) $\mathbf{K_{II}}$ =

(ii) $\mathbf{P_{I}}$ =

Problem 3

Δ_2 =

Problem 4

(i) $\mathbf{D_{II,II}}$ =

(ii) $\mathbf{D_{II,I}}$ =

(iii) $\mathbf{P_{I}}$ =

(iv) Reaction at joint-4 =

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem exam 16/11/13 PAPER CODE: B

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Must use: global coordinate system provided with the problem; member end (local) coordinate system as done in class; convention for all forces and displacements (linear and angular) as done in class; numbering sequence of structure's nodal forces and displacements as done in class.

Problem 1

For this question: Use only Stiffness Method. Settlement must be handled only through self straining by including it in the load vector $\underline{\mathbf{P}_{\mathbf{I}}}$. Thus, settlement must not be handled through $\underline{\boldsymbol{\Delta}_{\mathbf{II}}}$ term for this question.

Fig.1 shows a pin-jointed truss with joints 1, 2, 3, 4. The truss has six members, i.e., members 24 and 13 are not connected to each other.

Member 34 is **cooled** to 75^0C below ambient temperature.

Support at joint-4 settles down by 1 cm

Assume $AE = 40000$ kN, $\alpha = 0.000012/^0C$, for all members.

Find: (i) $\mathbf{K}_{\mathbf{II}}$; (ii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector) ; (iii) $\Delta_{\mathbf{I}}$ (joint displacement vector)

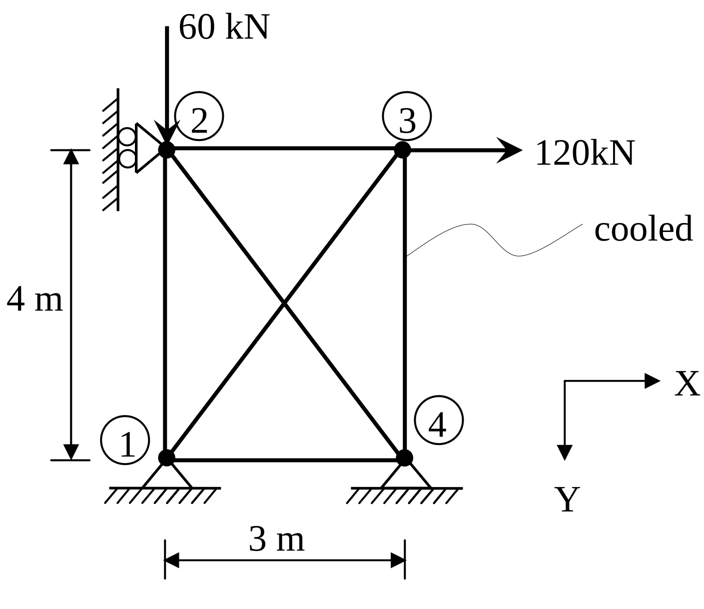

Fig. 1

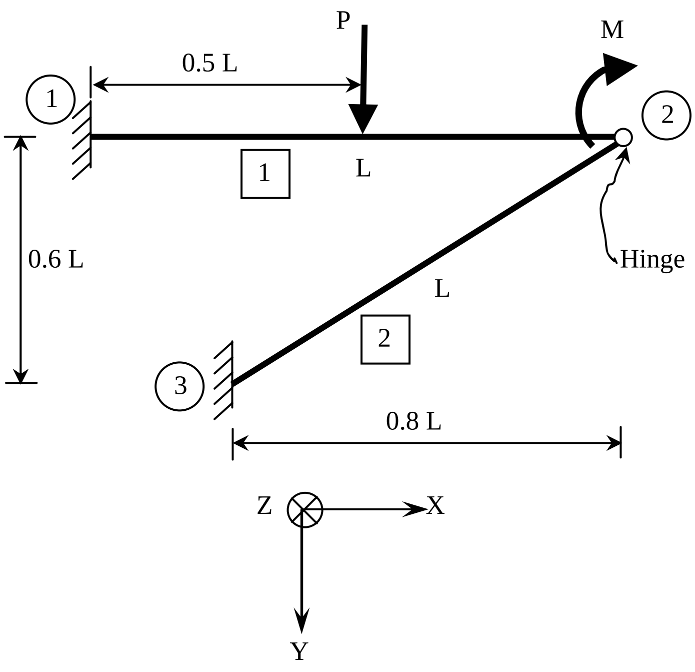

Fig. 2

Problem 2

For this question use only Stiffness Method.

The frame shown in Fig. 2 has an internal hinge. It is loaded as shown. The moment is applied to member-1 at joint-2.

Data: $EI = 10$, $A/I = 4$, $L = 2$, $P = 15$, $M = 3$. All data provided in units of kN, m.

Find: (i) $\mathbf{K}_{\mathbf{II}}$; (ii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector)

Problem 3

Data: $E = 200\,\text{GPa},\; G = 80\,\text{GPa},\; I_y = I_z = 8000\,\text{cm}^4,\; A = 200\,\text{cm}^2,\; J = 10000\,\text{cm}^4,\; L = 10\,\text{m}$

Find: deflections at joint-2, i.e., Δ_2 **using Stiffness Method**.

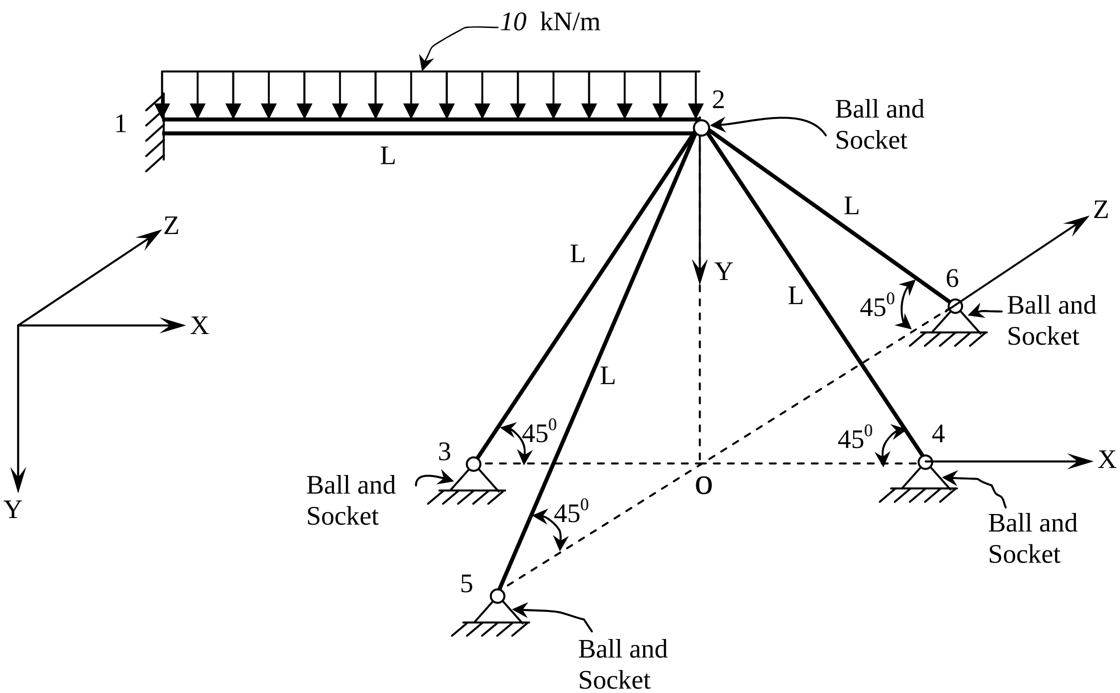

Fig. 3

Problem 4

For this question: Use only Flexibility Method. Settlement must be handled only through $\underline{\Delta_{II}}$ term. Thus, settlement must not be handled through self straining by including it in the load vector $\underline{P_I}$.

Fig.4 shows a pin-jointed truss. **You must consider redundant as reaction at joint-4.**
Support at joint-4 settles down by 1cm
Assume $EA = 40000\,\text{kN}$.
Find: (i) $\mathbf{D}_{\mathbf{II,II}}$; (ii) $\mathbf{D}_{\mathbf{II,I}}$; (iii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector) ; (iv) Reaction at joint-4
reaction at joint-4 **using Flexibility Method**.

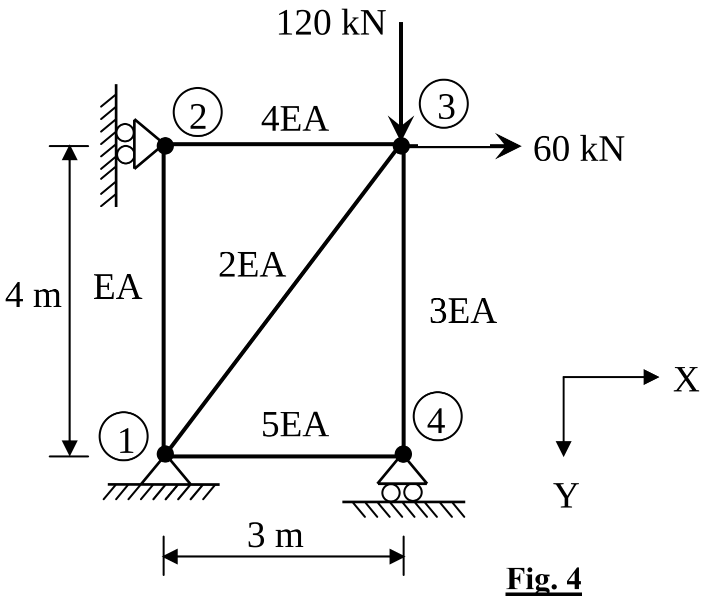

Fig. 4

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem exam 16/11/13 PAPER CODE: B

SUMMARY ANSWER SHEET

NAME:

ROLL No:

Problem 1

(i) $\mathbf{K_{II}}$ =

(ii) $\mathbf{P_I}$ =

(iii) Δ_I =

Problem 2

(i) $\mathbf{K_{II}}$ =

(ii) $\mathbf{P_I}$ =

Problem 3

Δ_2 =

Problem 4

(i) $\mathbf{D_{II,II}}$ =

(ii) $\mathbf{D_{II,I}}$ =

(iii) $\mathbf{P_I}$ =

(iv) Reaction at joint-4 =

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem exam 16/11/13 PAPER CODE: C

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All questions carry equal marks. All answers should be given upto at least three significant digits.

Must use: global coordinate system provided with the problem; member end (local) coordinate system as done in class; convention for all forces and displacements (linear and angular) as done in class; numbering sequence of structure's nodal forces and displacements as done in class.

Problem 1

For this question: Use only Stiffness Method. Settlement must be handled only through self straining by including it in the load vector $\underline{\mathbf{P}_{\mathbf{I}}}$. Thus, settlement must not be handled through $\underline{\boldsymbol{\Delta}_{\mathbf{II}}}$ term for this question.

Fig.1 shows a pin-jointed truss with joints 1, 2, 3, 4. The truss has six members, i.e., members 24 and 13 are not connected to each other.

Member 34 is **heated** to 75^0C above ambient temperature.

Support at joint-4 settles down by 1cm

Assume $AE = 40000$ kN, $\alpha = 0.000012/^0C$, for all members.

Find: (i) $\mathbf{K}_{\mathbf{II}}$; (ii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector) ; (iii) $\Delta_{\mathbf{I}}$ (joint displacement vector)

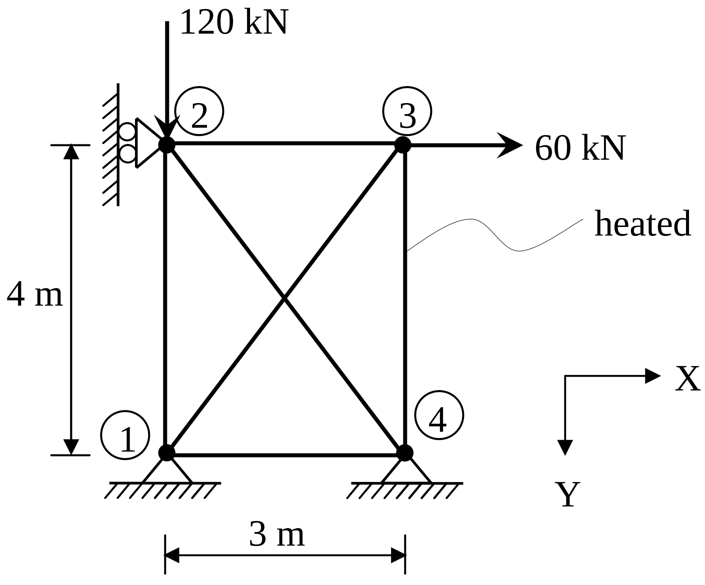

Fig. 1

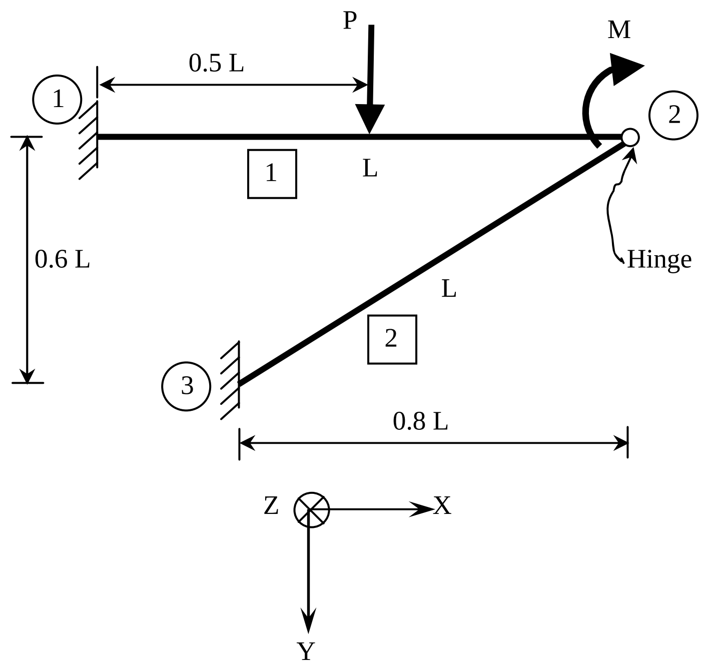

Fig. 2

Problem 2

For this question use only Stiffness Method.

The frame shown in Fig. 2 has an internal hinge. It is loaded as shown. The moment is applied to member-1 at joint-2.

Data: $EI = 10$, $A/I = 5$, $L = 2$, $P = 20$, $M = 4$. All data provided in units of kN, m.

Find: (i) $\mathbf{K}_{\mathbf{II}}$; (ii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector)

Problem 3

Data: $E = 200\,\text{GPa},\ G = 80\,\text{GPa},\ I_y = I_z = 8000\,\text{cm}^4,\ A = 200\,\text{cm}^2,\ J = 10000\,\text{cm}^4,\ L = 10\,\text{m}$

Find: deflections at joint-2, i.e., Δ_2 **using Stiffness Method**.

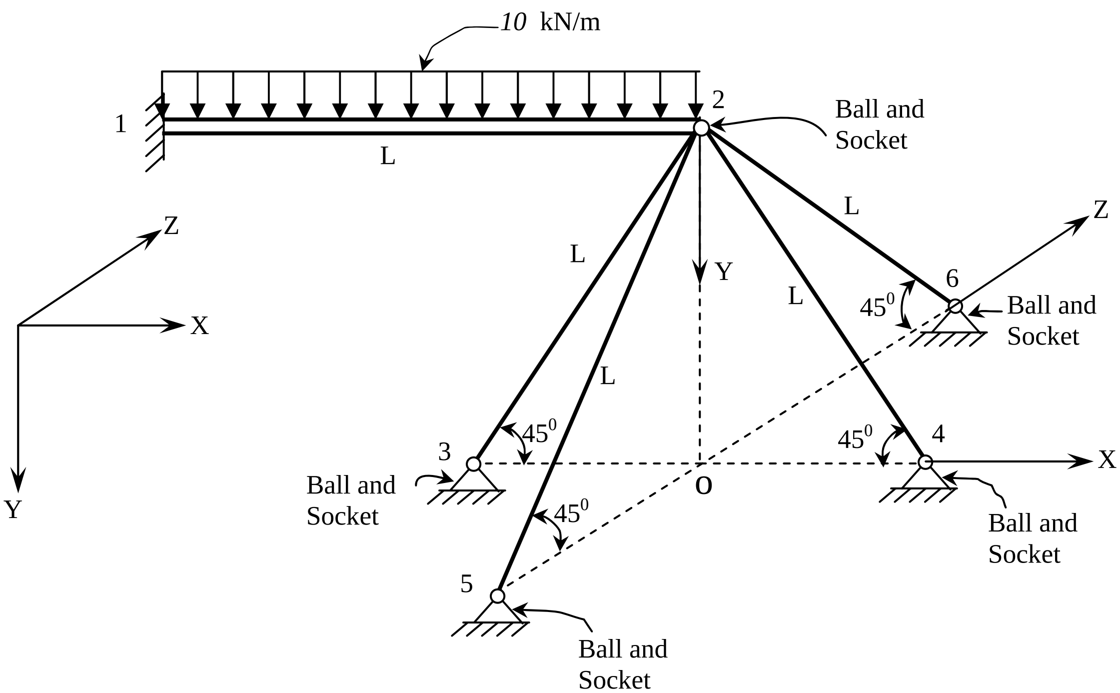

Fig. 3

Problem 4

For this question: Use only Flexibility Method. Settlement must be handled only through Δ_{II} term. Thus, settlement must not be handled through self straining by including it in the load vector P_I :

Fig.4 shows a pin-jointed truss. **You must consider redundant as reaction at joint-4.**
Support at joint-4 settles down by 0.5 cm
Assume $EA = 40000\,\text{kN}$.
Find: (i) $\mathbf{D_{II,II}}$; (ii) $\mathbf{D_{II,I}}$; (iii) $\mathbf{P_I}$ (joint load vector) ; (iv) Reaction at joint-4
reaction at joint-4 **using Flexibility Method**.

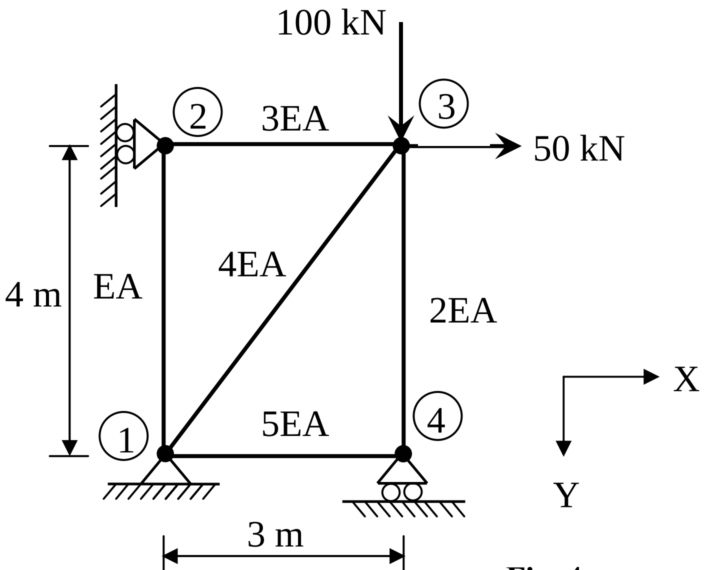

Fig. 4

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem exam 16/11/13 PAPER CODE: C

SUMMARY ANSWER SHEET

NAME:

ROLL No:

Problem 1

(i) $\mathbf{K}_{\mathbf{II}}$ =

(ii) $\mathbf{P}_{\mathbf{I}}$ =

(iii) $\Delta_{\mathbf{I}}$ =

Problem 2

(i) $\mathbf{K}_{\mathbf{II}}$ =

(ii) $\mathbf{P}_{\mathbf{I}}$ =

Problem 3

Δ_2 =

Problem 4

(i) $\mathbf{D}_{\mathbf{II,II}}$ =

(ii) $\mathbf{D}_{\mathbf{II,I}}$ =

(iii) $\mathbf{P}_{\mathbf{I}}$ =

(iv) Reaction at joint-4 =

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem exam 16/11/13 PAPER CODE: D

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Must use: global coordinate system provided with the problem; member end (local) coordinate system as done in class; convention for all forces and displacements (linear and angular) as done in class; numbering sequence of structure's nodal forces and displacements as done in class.

Problem 1

For this question: Use only Stiffness Method. Settlement must be handled only through self straining by including it in the load vector $\underline{\mathbf{P_I}}$. Thus, settlement must not be handled through $\underline{\boldsymbol{\Delta}_{\mathbf{II}}}$ term for this question.

Fig.1 shows a pin-jointed truss with joints 1, 2, 3, 4. The truss has six members, i.e., members 24 and 13 are not connected to each other.

Member 34 is **cooled** to 50^0C below ambient temperature.

Support at joint-4 settles down by 0.5 cm

Assume $AE = 40000$ kN, $\alpha = 0.000012/^0C$, for all members.

Find: (i) $\mathbf{K_{II}}$; (ii) $\mathbf{P_I}$ (joint load vector); (iii) $\Delta_{\mathbf{I}}$ (joint displacement vector)

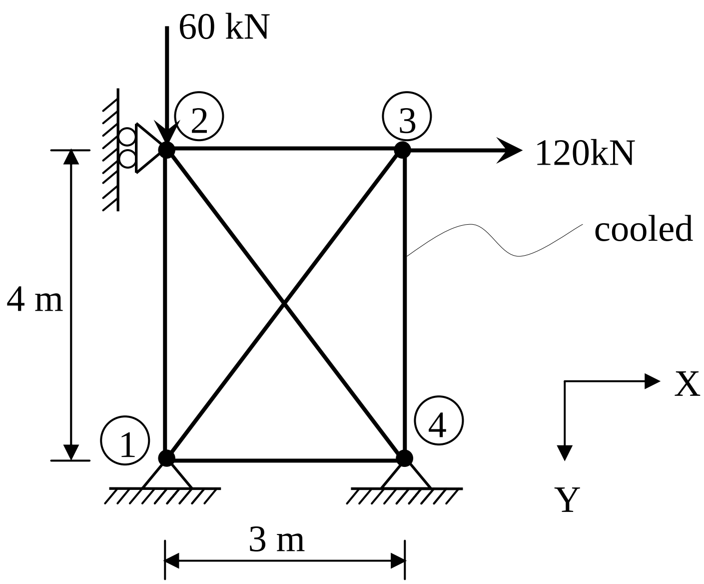


Fig. 1

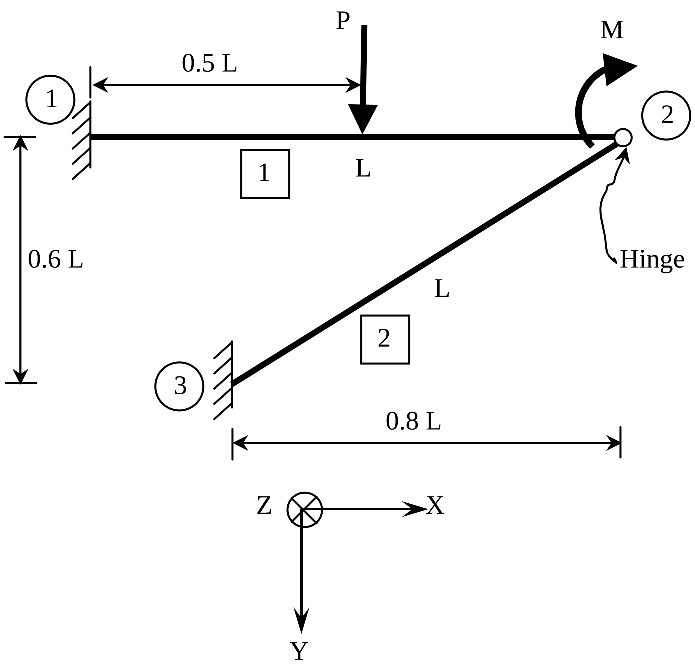


Fig. 2

Problem 2

For this question use only Stiffness Method.

The frame shown in Fig. 2 has an internal hinge. It is loaded as shown. The moment is applied to member-1 at joint-2.

Data: $EI = 10$, $A/I = 6$, $L = 2$, $P = 15$, $M = 4$. All data provided in units of kN, m.

Find: (i) $\mathbf{K_{II}}$; (ii) $\mathbf{P_I}$ (joint load vector)

Problem 3

Data: $E = 200\,\text{GPa},\ G = 80\,\text{GPa},\ I_y = I_z = 8000\,\text{cm}^4,\ A = 200\,\text{cm}^2,\ J = 10000\,\text{cm}^4,\ L = 10\,\text{m}$

Find: deflections at joint-2, i.e., Δ_2 **using Stiffness Method**.

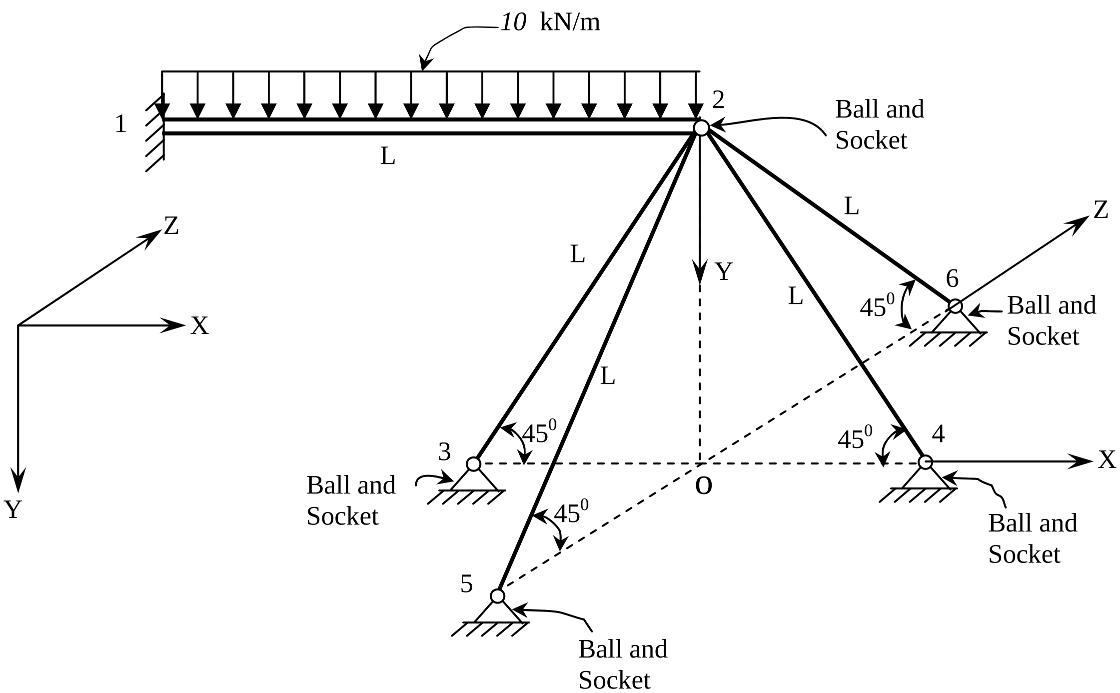


Fig. 3

Problem 4

For this question: Use only Flexibility Method. Settlement must be handled only through $\underline{\Delta_{II}}$ term. Thus, settlement must not be handled through self straining by including it in the load vector $\underline{P_I}$.

Fig.4 shows a pin-jointed truss. **You must consider redundant as reaction at joint-4.**
Support at joint-4 settles down by 0.4 cm
Assume $EA = 40000\,\text{kN}$.
Find: (i) $\mathbf{D_{II,II}}$; (ii) $\mathbf{D_{II,I}}$; (iii) $\mathbf{P_I}$ (joint load vector) ; (iv) Reaction at joint-4
reaction at joint-4 **using Flexibility Method**.

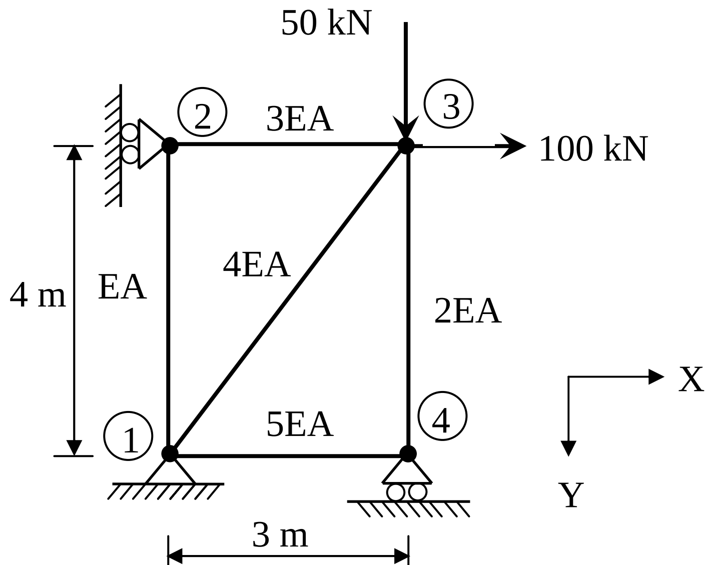


Fig. 4

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Endsem exam 16/11/13 PAPER CODE: D

SUMMARY ANSWER SHEET

NAME:

ROLL No:

Problem 1

(i) $\mathbf{K}_{\mathbf{II}}$ =

(ii) $\mathbf{P}_{\mathbf{I}}$ =

(iii) $\Delta_{\mathbf{I}}$ =

Problem 2

(i) $\mathbf{K}_{\mathbf{II}}$ =

(ii) $\mathbf{P}_{\mathbf{I}}$ =

Problem 3

Δ_2 =

Problem 4

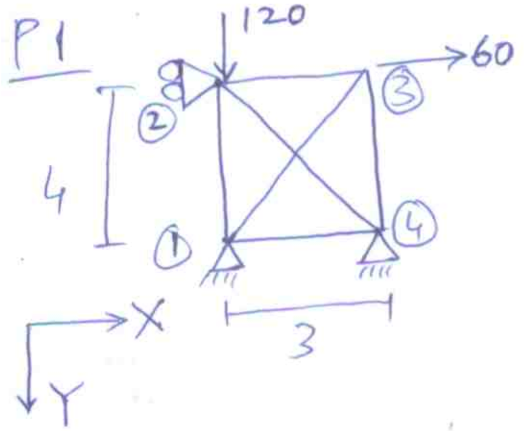
(i) $\mathbf{D}_{\mathbf{II,II}}$ =

(ii) $\mathbf{D}_{\mathbf{II,I}}$ =

(iii) $\mathbf{P}_{\mathbf{I}}$ =

(iv) Reaction at joint-4 =

P1



dof's : 4, 5, 6

$$K_{II} = \begin{array}{c} \begin{array}{cc} 3\,4 & 5\,6 \end{array} \\ \begin{bmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{bmatrix} \end{array} \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$$= \begin{bmatrix} K_{22}(2,2) & K_{23}(2,1) & K_{23}(2,2) \\ K_{32}(1,2) & \multicolumn{2}{c}{\boxed{K_{33}}} \\ K_{32}(2,2) & & \end{bmatrix}$$

$$K_{22} = EA\left[\frac{1}{4}\begin{pmatrix}0\\-1\end{pmatrix}\begin{bmatrix}0 & -1\end{bmatrix} + \frac{1}{3}\begin{bmatrix}-1\\0\end{bmatrix}\begin{bmatrix}-1 & 0\end{bmatrix} + \frac{1}{5}\begin{bmatrix}-0.6\\-0.8\end{bmatrix}\begin{bmatrix}-0.6 & -0.8\end{bmatrix}\right]$$

$$= EA\begin{bmatrix} x & x \\ x & \frac{1}{4} + \frac{0.64}{5} \end{bmatrix} = EA\begin{pmatrix} x & x \\ x & 0.378 \end{pmatrix}$$

$$K_{33} = EA\begin{bmatrix} \frac{1}{3} + \frac{0.36}{5} & -\frac{0.48}{5} \\ -\frac{0.48}{5} & \frac{1}{4} + \frac{0.64}{5} \end{bmatrix} = EA\begin{bmatrix} \cancel{152/375} \rightarrow 0.4053 & -0.096 \\ -0.096 & 0.378 \end{bmatrix}$$

$$K_{23} = \frac{EA}{3}\begin{bmatrix}-1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} = EA\begin{bmatrix} -1/3 & 0 \\ 0 & 0 \end{bmatrix}$$

$$K_{II} = \underset{40000}{\cancel{EA}}\begin{bmatrix} 0.378 & 0 & 0 \\ 0 & 0.4053 = \frac{152}{375} & -0.096 \\ 0 & -0.096 & 0.378 \end{bmatrix} = 10^4\begin{bmatrix} 1.5120 & 0 & 0 \\ 0 & 1.6213 & -0.3840 \\ 0 & -0.3840 & 1.5120 \end{bmatrix}$$

$$P^e_{②} = a^T_{24}\, k_{24}\, a_{42}\, \Delta^s_{④} = \begin{bmatrix}-0.6\\-0.8\end{bmatrix}\frac{EA}{5}\begin{bmatrix}0.6 & 0.8\end{bmatrix}\begin{bmatrix}0\\s\end{bmatrix} = EA\begin{bmatrix} -\frac{0.48}{5}s \\ -\frac{0.64}{5}s \end{bmatrix}$$

$$P^e_{③} = a^T_{34}\, k_{34}\left(a_{43}\,\Delta^s_{④} - \delta^T_{④}\right) = \begin{bmatrix}0\\-1\end{bmatrix}\frac{EA}{4}\left[\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}0\\s\end{bmatrix} - \alpha\,\Delta T.4\right]$$

$$= EA\begin{bmatrix} 0 \\ -\frac{s}{4} + \alpha\Delta T \end{bmatrix}$$

$$P_I = P_a - EA\begin{Bmatrix} -\frac{0.64}{5}s \\ 0 \\ -\frac{s}{4} + \alpha\Delta T \end{Bmatrix}$$

$\alpha = 0.000012/°c$

Code A $s = 0.5\text{ cm},\ \Delta T = -100°c,\ P_a = [120\ \ 60\ \ 0]^T$

$$P_I = \begin{bmatrix}120\\60\\0\end{bmatrix} - EA\begin{bmatrix}-6.4E-4\\0\\-2.45E-3\end{bmatrix} = \begin{bmatrix}120\\60\\0\end{bmatrix} - \begin{bmatrix}-25.6\\0\\-98\end{bmatrix} = \begin{bmatrix}145.6\\60\\98\end{bmatrix}$$

$$\Delta_I = K_{II}^{-1} P_I = \begin{bmatrix}0.0096\\0.0056\\0.0079\end{bmatrix} m$$

Code B $s = 1\text{cm},\ \Delta T = -75°c,\ P_a = [60\ \ 120\ \ 0]^T$

$$P_I = \begin{bmatrix}60\\120\\0\end{bmatrix} - EA\begin{bmatrix}-1.28E-3\\0\\-3.4E-3\end{bmatrix} = \begin{bmatrix}60\\120\\0\end{bmatrix} - \begin{bmatrix}-51.2\\0\\-136\end{bmatrix} = \begin{bmatrix}111.2\\120\\136\end{bmatrix}$$

$$\Delta_I = K_{II}^{-1} P_I = \begin{bmatrix}0.0074\\0.0101\\0.0116\end{bmatrix} m$$

Code C $s = 1\text{cm},\ \Delta T = 75°c,\ P_a = [120\ \ 60\ \ 0]^T$

$$P_I = \begin{bmatrix}120\\60\\0\end{bmatrix} - EA\begin{bmatrix}-1.28E-3\\0\\-1.6E-3\end{bmatrix} = \begin{bmatrix}120\\60\\0\end{bmatrix} - \begin{bmatrix}-51.2\\0\\-64\end{bmatrix} = \begin{bmatrix}171.2\\60\\64\end{bmatrix}$$

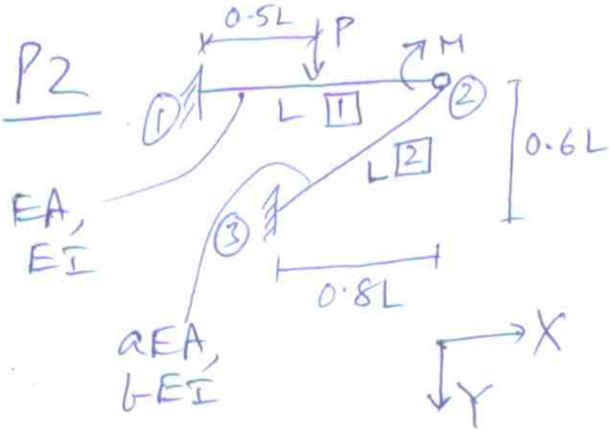
$$\Delta_I = K_{II}^{-1} \cdot P_I = \begin{bmatrix}0.0113\\0.0050\\0.0055\end{bmatrix} m$$

Code D $s = 0.5\text{ cm},\ \Delta T = -50°c,\ P_a = [60\ \ 120\ \ 0]^T$

$$P_I = \begin{bmatrix}60\\120\\0\end{bmatrix} - EA\begin{bmatrix}-6.4E-3\\0\\-1.85E-3\end{bmatrix} = \begin{bmatrix}60\\120\\0\end{bmatrix} - \begin{bmatrix}-25.6\\0\\-74\end{bmatrix} = \begin{bmatrix}85.6\\120\\74\end{bmatrix}$$

$$\Delta_I = K_{II}^{-1} P_I = \begin{bmatrix}0.0057\\0.0091\\0.0072\end{bmatrix} m$$

P2



$$K_{\boxed{1}} = \begin{array}{c|cc} & 1\,2\,3 & 4\,5\,6 \\ \hline \begin{matrix}1\\2\\3\end{matrix} & K_{11} & K_{12} \\ \begin{matrix}4\\5\\6\end{matrix} & K_{21} & K_{22}^{1} \end{array} \; ; \; K_{\boxed{2}} = \begin{array}{cc|c} 4\,5\,7 & 8\,9\,10 & \\ \hline K_{22}^{3} & K_{23} & \begin{matrix}4\\5\\7\end{matrix} \\ K_{32} & K_{33} & \begin{matrix}8\\9\\10\end{matrix} \end{array}$$

$X \rightarrow$, $Y \downarrow$

$$a_{23} = \begin{bmatrix} 0.8 & -0.6 & 0 \\ 0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$K_{22}^{1} = \underbrace{a_{21}^{T}}_{I} k_{22}^{1} a_{21} = \frac{EI}{L} \begin{array}{ccc|c} 4 & 5 & 6 & \\ \gamma & 0 & 0 & 4 \\ 0 & 12/L^2 & -6/L & 5 \\ 0 & -6/L & 4 & 6 \end{array}$$

$$K_{22}^{3} = a_{23}^{T} k_{22}^{3} a_{23} = b\frac{EI}{L} \begin{bmatrix} 0.8 & 0.6 & 0 \\ -0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{a}{b}\gamma & 0 & 0 \\ 0 & 12/L^2 & -6/L \\ 0 & -6/L & 4 \end{bmatrix} \begin{bmatrix} 0.8 & -0.6 & 0 \\ 0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= b\frac{EI}{L} \begin{bmatrix} 0.8\frac{a}{b}\gamma & 0.6 * 12/L^2 & -0.6 * 6/L \\ -0.6\frac{a}{b}\gamma & 0.8 * 12/L^2 & -0.8 * 6/L \\ 0 & -6/L & 4 \end{bmatrix} \begin{bmatrix} 0.8 & -0.6 & 0 \\ 0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= b\frac{EI}{L} \begin{array}{ccc|c} (4) & (5) & (7) & \\ 0.64\frac{a}{b}\gamma + 0.36 * 12/L^2 & -0.48\frac{a}{b}\gamma + 0.48 * \frac{12}{L^2} & \frac{-3.6}{L} & (4) \\ & 0.36\frac{a}{b}\gamma + 0.64 * \frac{12}{L^2} & -4.8/L & (5) \\ (\text{symm}) & & 4 & (7) \end{array}$$

$$K_{II} = \frac{EI}{L} \begin{array}{c|cccc} & (4) & (5) & (6) & (7) \\ \hline (4) & \left(\gamma + 0.64\frac{a}{\cancel{b}}\gamma\cancel{b} + 0.36 * \frac{12}{L^2} b\right) & \left(0 - 0.48\frac{a}{\cancel{b}}\cdot\gamma\cancel{b} + 0.48 * \frac{12}{L^2} b\right) & 0 & \frac{-3.6}{L} b \\ (5) & & \left(\frac{12}{L^2} + 0.36 a\gamma + 0.64\frac{12}{L^2}\cdot b\right) & -\frac{6}{L} & \frac{-4.8 b}{L} \\ (6) & & & 4 & 0 \\ (7) & (\text{symm}) & & & 4 \end{array}$$

dof's

$$P_I = \begin{bmatrix} 0 \\ 0 \\ M \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -P/2 \\ PL/8 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ P/2 \\ M - PL/8 \\ 0 \end{bmatrix}$$

$a = b = 1$, $L = 2$, $EI = 10$, for all paper codes.

<u>Code A</u>

$\gamma = 3$, $P = 20$, $M = 3$

$$K_{II} = \begin{bmatrix} 30 & 0 & 0 & 9 \\ 0 & 30 & -15 & -12 \\ 0 & -15 & 20 & 0 \\ -9 & -12 & 0 & 20 \end{bmatrix}, \quad P_I = \begin{bmatrix} 0 \\ 10 \\ -2 \\ 0 \end{bmatrix}$$

<u>Code B</u> : $\gamma = 4$, $P = 15$, $M = 3$

$$K_{II} = \begin{bmatrix} 38.2 & -2.4 & 0 & -9 \\ & 31.8 & -15 & -12 \\ & & 20 & 0 \\ \text{(symm)} & & & 20 \end{bmatrix}, \quad P_I = \begin{bmatrix} 0 \\ 7.5 \\ -0.75 \\ 0 \end{bmatrix}$$

<u>Code C</u> : $\gamma = 5$, $P = 20$, $M = 4$

$$K_{II} = \begin{bmatrix} 46.4 & -4.8 & 0 & -9 \\ & 33.6 & -15 & -12 \\ & & 20 & 0 \\ \text{(symm)} & & & 20 \end{bmatrix}, \quad P_I = \begin{bmatrix} 0 \\ 10 \\ -1 \\ 0 \end{bmatrix}$$

<u>Code D</u> : $\gamma = 6$, $P = 15$, $M = 4$

$$K_{II} = \begin{bmatrix} 54.6 & -7.2 & 0 & -9 \\ & 35.4 & -15 & -12 \\ & & 20 & 0 \\ \text{(symm)} & & & 20 \end{bmatrix}, \quad P_I = \begin{bmatrix} 0 \\ 7.5 \\ 0.25 \\ 0 \end{bmatrix}$$

P3 Symmetry $\Rightarrow (\Delta_3)_2 = (\Delta_4)_2 = (\Delta_5)_2 = 0$ at joint 2, ie members 12, 23, 24 deform in XY plane only. Actually as we see later even 25, 26 deform in XY plane.

Thus,

$$a_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad a_{23} = \left\{ \tfrac{1}{\sqrt{2}} \;\; -\tfrac{1}{\sqrt{2}} \;\; 0 \right\} ; \quad a_{24} = \left\{ -\tfrac{1}{\sqrt{2}} \;\; -\tfrac{1}{\sqrt{2}} \;\; 0 \right\}$$

$$a_{25} = \left\{ 0 \;\; -\tfrac{1}{\sqrt{2}} \;\; \tfrac{1}{\sqrt{2}} \right\} ; \quad a_{26} = \left\{ 0 \;\; -\tfrac{1}{\sqrt{2}} \;\; -\tfrac{1}{\sqrt{2}} \right\}$$

$$k^1_{22} = \frac{EI_z}{L} \begin{bmatrix} A/I_z & 0 & 0 \\ 0 & 12/L^2 & -6/L \\ 0 & -6/L & 4 \end{bmatrix} = 1600 \begin{bmatrix} 250 & 0 & 0 \\ 0 & 0.12 & -0.6 \\ 0 & -0.6 & 4 \end{bmatrix},$$

where we used

$$\frac{EI_z}{L} = \frac{200E6 \cdot 8000E-8}{10} = 1600$$

$$\frac{A}{I_z} = \frac{200E-4}{8000E-8} = 250$$

$$k^3_{22} = k^4_{22} = k^5_{22} = k^6_{22} = [EA/L] = [4E5]$$

$$K^1_{22} = a^T_{21} k^1_{22} a_{21} = k^1_{22} = 1600 \begin{bmatrix} 250 & 0 & 0 \\ 0 & 0.12 & -0.6 \\ 0 & -0.6 & 4 \end{bmatrix}$$

$$K^3_{22} = a^T_{23} k^3_{22} a_{23} = \begin{Bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{Bmatrix} [4E5] \{ 1/\sqrt{2} \;\; -1/\sqrt{2} \;\; 0 \} = \begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 \end{matrix} \\ \begin{bmatrix} 2E5 & -2E5 & 0 \\ -2E5 & 2E5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$$K^4_{22} = \begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 \end{matrix} \\ \begin{bmatrix} 2E5 & 2E5 & 0 \\ 2E5 & 2E5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \quad K^5_{22} = \begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2E5 & -2E5 \\ 0 & -2E5 & 2E5 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \quad K^6_{22} = \begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2E5 & 2E5 \\ 0 & 2E5 & 2E5 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$$K_{22} = \begin{array}{c} \begin{matrix} 1 & \qquad\qquad 2 & \qquad\qquad 6 \end{matrix} \\ \begin{bmatrix} 4E5+2E5+2E5 & -2E5+2E5 & 0 \\ -2E5+2E5 & 192+2E5+2E5+2E5+2E5 & -960 \\ 0 & -960 & 6400 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 6 \end{matrix} = K_{II} = \begin{bmatrix} 8E5 & 0 & 0 \\ 0 & (8E5+192) & -960 \\ 0 & -960 & 6400 \end{bmatrix}$$

$$P_I = \begin{Bmatrix} 0 \\ 10*10/2 \\ -10*10^2/12 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 50 \\ \frac{-1000}{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 50 \\ -83.33 \end{Bmatrix}$$

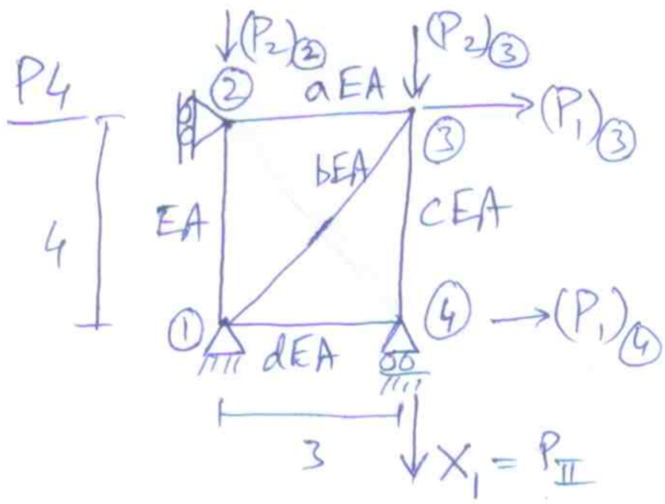
$$\Delta_I = K^{-1}_{II} P_I = \begin{bmatrix} 1/8E5 & & \\ 0 & \frac{1}{det} \begin{bmatrix} 6400 & 960 \\ 960 & (8E5+192) \end{bmatrix} \\ 0 & \end{bmatrix} \begin{Bmatrix} 0 \\ 50 \\ \frac{-1000}{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 240000 \\ 66634666.67 \end{Bmatrix} \frac{1}{det}$$

$$det = 5120307200$$

$$= \begin{Bmatrix} 0 \\ 4.69E-5 \\ 0.01301 \end{Bmatrix}$$

$\Rightarrow (\Delta_2)_1 = 0 \Rightarrow$ can use beam formulation and 2x2 transf for truss members. This is expected ∵ beam 12 is just resting on 4 truss members (and no X-dir loading), so no X-dir mem force in mem 21, ie no X-dir reactions at nodes 1, 3, 4, 5, 6, $\Rightarrow$ no X-dir displ at node 2.

P4



$$\begin{bmatrix} F_{21} \\ F_{32} \\ F_{31} \\ F_{43} \\ F_{41} \end{bmatrix} = \begin{array}{c} \left[\begin{array}{cccc|c} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & -\sqrt{2} & 0 & -\sqrt{2} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \\ \underbrace{(P_2)_2 \quad (P_1)_3 \quad (P_2)_3 \quad (P_1)_4}_{b_I} \quad \underbrace{X_1}_{b_{II}} \end{array} \begin{bmatrix} (P_2)_2 \\ (P_1)_3 \\ (P_2)_3 \\ (P_1)_4 \\ X_1 = P_{II} \end{bmatrix}$$

$$d_u = \frac{1}{EA}\begin{bmatrix} 4 & & & & \\ & 3/a & & 0 & \\ & & 5/b & & \\ & 0 & & 4/c & \\ & & & & 3/d \end{bmatrix}$$

$$D_{II\,II} = b_{II}^T d_u b_{II} = \frac{1}{EA}\left(0 + \frac{3}{a} + \frac{5(-\sqrt{2})^2}{b} + \frac{4 \times (1)^2}{c} + 0\right)$$

$$= \frac{1}{EA}\left(\frac{3}{a} + \frac{10}{b} + \frac{4}{c}\right)$$

$$D_{II\,I} = b_{II}^T d_u b_I = \frac{1}{EA}\begin{bmatrix} 0 & \frac{3}{a} & -\frac{5\sqrt{2}}{b} & \frac{4}{c} & 0 \end{bmatrix} b_I$$

$$= \frac{1}{EA}\begin{bmatrix} 0 & \frac{3}{a} & \left(\frac{3}{a} + \frac{10}{b}\right) & 0 \end{bmatrix}$$

$$P_I = \begin{bmatrix} 0 & (P_1)_3 & (P_2)_3 & 0 \end{bmatrix}^T$$

$$X_1 = P_{II} = D_{II\,II}^{-1}\left(\Delta_{II} - D_{II\,I} P_I\right)$$
$$= EA\left(\frac{3}{a}+\frac{10}{b}+\frac{4}{c}\right)^{-1}\left(S - \frac{1}{EA}\left[\frac{3}{a}(P_1)_{③} + \left\{\frac{3}{a}+\frac{10}{b}\right\}(P_2)_{③}\right]\right)$$

Code A $a=2,\ b=3,\ c=4,\ d=5,\ (P_1)_{③}=120,\ (P_2)_{③}=60,\ S=0.6\text{cm}$

$D_{II\,II} = \frac{1}{40000} * \frac{35}{6} = 1.4583E-4$, $D_{II\,I} = \frac{1}{40000}\left[0\ \ \frac{3}{2}\ \ \frac{29}{6}\ \ 0\right]$

5.833   4.833

$P_I = \left[0\ \ 120\ \ 60\ \ 0\right]^T$ $= \left[0\ \ 3.75E-5\ \ 1.2083E-4\ \ 0\right]^T$

$P_{II} = X_1 = \frac{EA}{5.833}\left(0.006 - 0.0118\right) = -39.4286$

Code B : $a=4,\ b=2,\ c=3,\ d=5,\ (P_1)_{③}=60,\ (P_2)_{③}=120,\ S=1\text{cm}$

$D_{II\,II} = \frac{1}{40000} * \frac{85}{12} = 1.7708E-4$, $D_{II\,I} = \frac{1}{40000}\left[0\ \ \frac{3}{4}\ \ 5.75\ \ 0\right]$

7.0833

$= \left[0\ \ 1.875E-5\ \ 1.4375E-4\ \ 0\right]$

$P_I = \left(0\ \ 60\ \ 120\ \ 0\right]^T$

$P_{II} = X_1 = \frac{EA}{7.0833}\left(0.01 - 0.0184\right) = -47.2941$

Code C : $a=3,\ b=4,\ c=2,\ d=5,\ (P_1)_{③}=50,\ (P_2)_{③}=100,\ S=0.5\text{cm}$

$D_{II\,II} = \frac{1}{40000} * 5.5 = 1.375E-4$, $D_{II\,I} = \frac{1}{40000}\left[0\ \ 1\ \ 3.5\ \ 0\right]$

$P_I = \left[0\ \ 50\ \ 100\ \ 0\right]^T$ $= \left[0\ \ 2.5E-5\ \ 8.75E-5\ \ 0\right]$

$P_{II} = X_1 = \frac{EA}{5.5}\left(0.005 - 0.01\right) = -36.3636$

Code D : $a=3,\ b=4,\ c=2,\ d=5,\ (P_1)_{③}=100,\ (P_2)_{③}=50,\ S=0.4$

$D_{II\,II} = \frac{1}{40000} * 5.5 = 1.375E-4$, $D_{II\,I} = \frac{1}{40000}\left[0\ \ 1\ \ 3.5\ \ 0\right]$

$P_I = \left(0\ \ 100\ \ 50\ \ 0\right]^T$ $= \left[0\ \ 2.5E-5\ \ 8.75E-5\ \ 0\right]$

$P_{II} = X_1 = \frac{EA}{5.5}\left(0.004 - 0.0069\right) = -20.9091$