DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

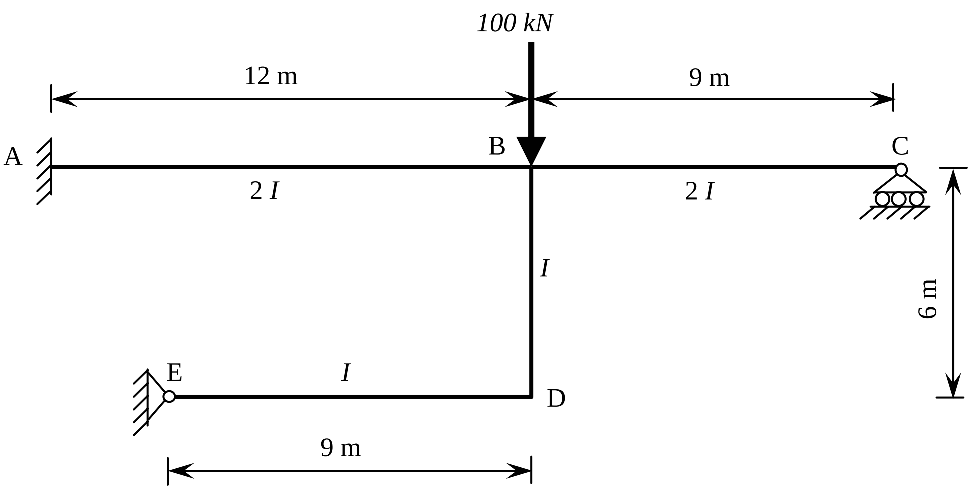
CE-317 STRUCTURAL MECHANICS II

Midsem 13/9/11

Problem 1

Using the Slope Deflection method, **determine** the **reactions** at the supports for the frame shown in **Fig. 1**

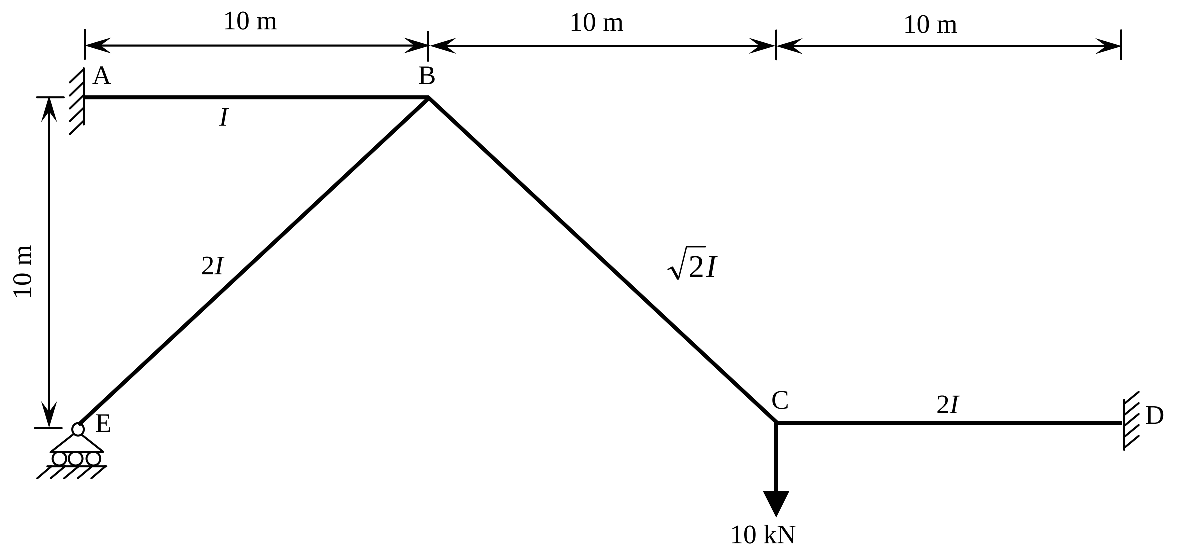
Fig. 1



Problem 2

Using the Moment Distribution method, determine the **bending moment at** the **fixed supports** and the **reaction at** the **roller** for the frame shown in **Fig. 2.** You must use only two iterations for moment distribution at each joint and the sequential method (Method-1)**,** and stiffness modifications wherever possible**.**

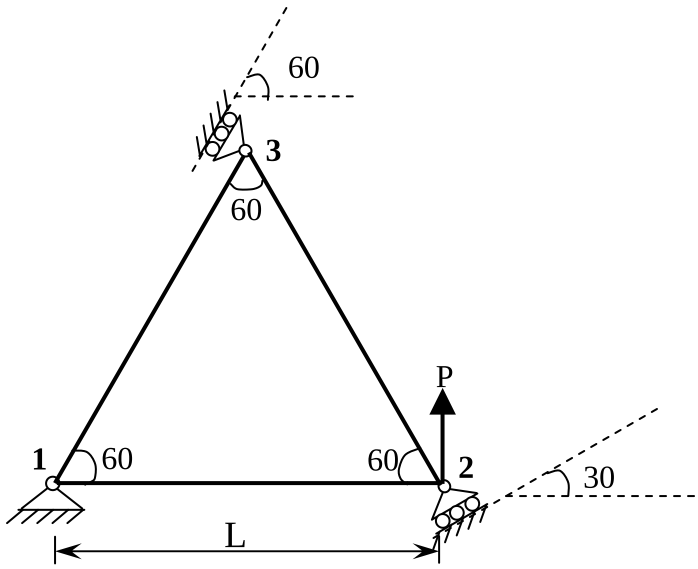
Fig. 2



Problem 3

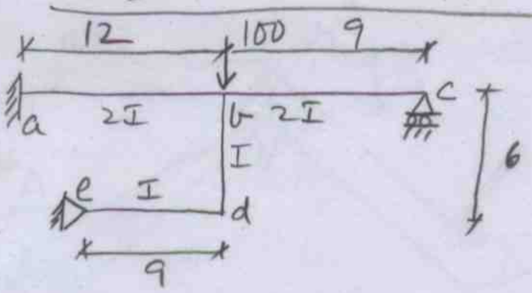
Using the Stiffness matrix method, determine the **displacements and reactions at** the **supports** for the truss shown in **Fig. 3**

Fig. 3



CE317 2011 MIDSEM SOLN.

P1



$$M_{ab} = EI\left(2\left(\tfrac{2}{12}\right)\theta_b - 6\left(\tfrac{2}{12}\right)\tfrac{\Delta}{12}\right) = -337.43$$

$$M_{ba} = EI\left(4\left(\tfrac{2}{12}\right)\theta_b - 6\left(\tfrac{2}{12}\right)\tfrac{\Delta}{12}\right) = -339.62$$

$$M_{bc} = EI\left(3\left(\tfrac{2}{9}\right)\theta_b + 3\left(\tfrac{2}{9}\right)\tfrac{\Delta}{9}\right) = 293.61$$

$$M_{bd} = EI\left(4\left(\tfrac{1}{6}\right)\theta_b + 2\left(\tfrac{1}{6}\right)\theta_d\right) = 46.01$$

$$M_{db} = EI\left(2\left(\tfrac{1}{6}\right)\theta_b + 4\left(\tfrac{1}{6}\right)\theta_d\right) = 98.60$$

$$M_{de} = EI\left(3\left(\tfrac{1}{9}\right)\theta_d - 3\left(\tfrac{1}{9}\right)\tfrac{\Delta}{9}\right) = -98.60$$

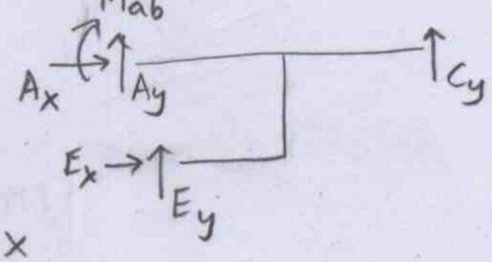
$$M_{ba} + M_{bc} + M_{bd} = 0 \;;\; M_{db} + M_{de} = 0.$$

$$V_a + V_e - V_c - 100 = 0$$

$$EI\begin{bmatrix} 4\cdot\frac{2}{12} + 3\cdot\frac{2}{9} + 4\cdot\frac{1}{6} & 2\cdot\frac{1}{6} & -6\cdot\frac{2}{12^2} + 3\cdot\frac{2}{9^2} \\ 2\cdot\frac{1}{6} & 4\cdot\frac{1}{6} + 3\cdot\frac{1}{9} & -3\cdot\frac{1}{9^2} \\ \left[-\left(2\cdot\frac{2}{12} + 4\cdot\frac{2}{12}\right)\cdot\frac{1}{12} + 3\cdot\frac{2}{9}\cdot\frac{1}{9}\right] & -3\left(\frac{1}{9}\right)\cdot\frac{1}{9} & \left[6\cdot\frac{2}{12^2}\cdot\frac{2}{12} + 3\cdot\frac{1}{9^2}\cdot\frac{1}{9} + 3\cdot\frac{2}{9^2}\cdot\frac{1}{9}\right] \end{bmatrix}\begin{Bmatrix}\theta_b \\ \theta_d \\ \Delta\end{Bmatrix} = \begin{Bmatrix}0 \\ 0 \\ 100\end{Bmatrix}$$

$$\{\theta_b \;\; \theta_d \;\; \Delta\} = \frac{1}{EI}\{-6.5733 \quad 151.1869 \quad 4022.885\}$$

$$V_c = -\frac{M_{bc}}{9} = -\frac{293.61}{9} = -32.62 = -C_y$$

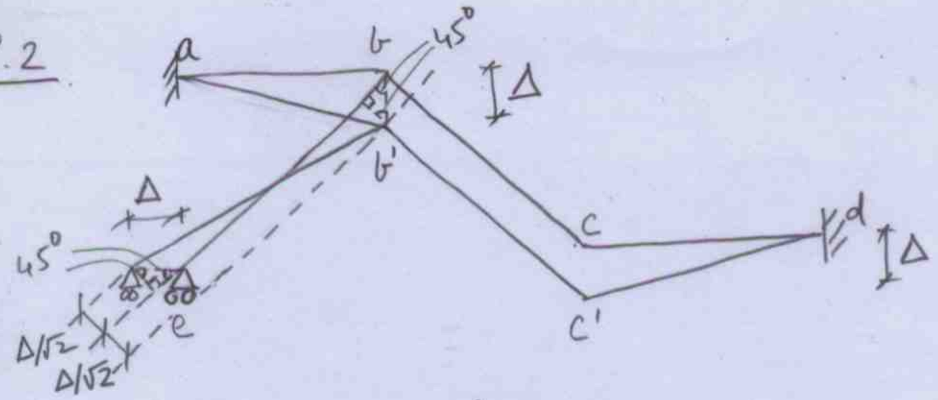


$$V_a = A_y = -\frac{(M_{ab} + M_{ba})}{12} = 56.42 (\uparrow)$$

$$A_x = V_{bd} = -\frac{(M_{bd} + M_{db})}{6} = -24.10 = -E_x$$

$$E_y = V_E = -\frac{M_{de}}{9} = 10.96$$

P.2



No no-sway solution.

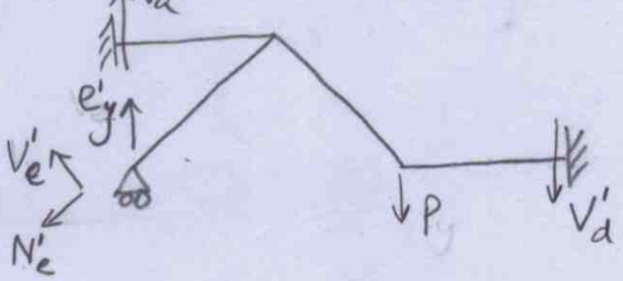
	a	b			c		d
mem end	ab	ba	be	bc	cb	cd	dc
rel stiff	k	k	$\sqrt{2}k$	k	k	$2k$	$2k$
mod stiff	k	k	$\frac{3}{2\sqrt{2}}k$	k	k	$2k$	$2k$
df	0	0.3267	0.3465	0.3268	1/3	2/3	0
fem	-100	-100	$-\frac{100}{\sqrt{2}}$	0	0	200	200
	27.89 ←	55.77	59.15	55.79 →	27.90		
				-37.99 ←	-75.97	-151.93 →	-75.97
	6.21 ←	12.41	13.16	12.42 →	6.21		
					-2.07	-4.14	→ Stop.
convg BM'	-65.9	-31.82	1.6	30.22	-43.93	43.93	124.03
BM	-24.95	-12.05	0.61	11.44	-16.64	16.64	46.97

$k = E \cdot \frac{100}{10} = 10E$

$\frac{6EI\Delta}{10^2} = 100$

$\frac{3(2EI)(\sqrt{2}\Delta)}{(10\sqrt{2})^2} = \frac{100}{\sqrt{2}}$

$\Sigma F_y = 0 = V_a' - V_d' + e_y' - P$



$V_a' = -\frac{(-65.9 - 31.82)}{10} = 9.772$

$V_d' = -\left(\frac{43.93 + 124.03}{10}\right) = -16.796$

$V_e' = -\frac{(1.6)}{10\sqrt{2}} = -0.1131$

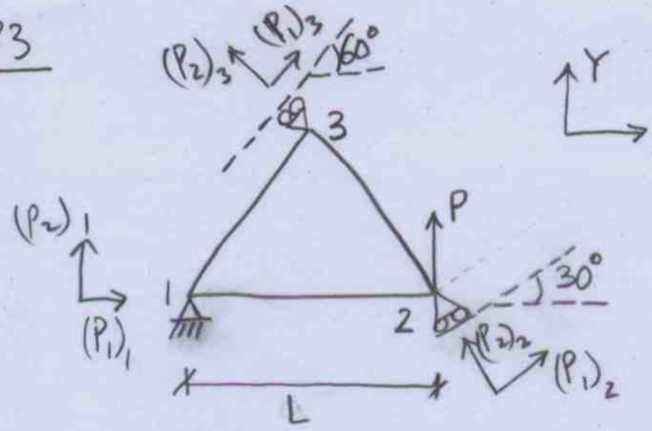
$\because$ e is roller, $\frac{V_e'}{\sqrt{2}} + \frac{N_e'}{\sqrt{2}} = 0 \Rightarrow N_e' = 0.1131$

$e_y' = \frac{V_e'}{\sqrt{2}} - \frac{N_e'}{\sqrt{2}} = -\sqrt{2}(0.1131) = -0.16$

$\Rightarrow P = 26.408$

$\Rightarrow BM = 10/26.408 * BM'$. $\Rightarrow e_y = e_y' * \frac{10}{26.408} = 0.06$

P3



Transformation matrices:

$$\underline{a}_{12} = [-1 \;\; 0], \quad \underline{a}_{13} = \left[-0.5 \;\; -\tfrac{\sqrt{3}}{2}\right]; \quad \underline{a}_{21} = \left[\tfrac{\sqrt{3}}{2} \;\; -0.5\right]; \quad a_{23} = [0 \;\; -1]$$

$$\underline{a}_{31} = [1 \;\; 0], \quad a_{32} = \left[0.5 \;\; \tfrac{\sqrt{3}}{2}\right]$$

Member stiffness matrices $\quad \underline{\underline{k}}_{ii}^{j} = \underline{\underline{k}}_{jj}^{i} \; ; \; \underline{\underline{k}}_{ij} = \underline{\underline{k}}_{ji}$

$$\underline{\underline{k}}_{11}^{2} = \underline{\underline{k}}_{12} = \underline{\underline{k}}_{11}^{3} = \underline{\underline{k}}_{13} = \underline{\underline{k}}_{22}^{3} = \underline{\underline{k}}_{23} = EA/L$$

Structure stiffness matrix

$$\underline{\underline{K}} = \begin{bmatrix} \underline{\underline{K}}_{11} & \underline{\underline{K}}_{12} & \underline{\underline{K}}_{13} \\ \underline{\underline{K}}_{21} & \underline{\underline{K}}_{22} & \underline{\underline{K}}_{23} \\ \underline{\underline{K}}_{31} & \underline{\underline{K}}_{32} & \underline{\underline{K}}_{33} \end{bmatrix}$$ — only this required.

$\underline{\underline{K}}_{II} \rightarrow$ delete rows and columns 1, 2, 4, 6

$K_{II\,I} \rightarrow$ delete rows 3, 5 & cols 1, 2, 4, 6.

$$\underline{\underline{K}}_{12} = \underline{\underline{a}}_{12}^{T}\, \underline{\underline{k}}_{12}\, \underline{\underline{a}}_{21} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \left[\tfrac{\sqrt{3}}{2} \;\; -0.5\right] \frac{EA}{L} = \frac{EA}{L} \begin{bmatrix} -\frac{\sqrt{3}}{2} & 0.5 \\ 0 & 0 \end{bmatrix}$$

$$\underline{\underline{K}}_{13} = \underline{\underline{a}}_{13}^{T}\, \underline{\underline{k}}_{13}\, a_{31} = \begin{bmatrix} -0.5 \\ -\sqrt{3}/2 \end{bmatrix} [1 \;\; 0] \frac{EA}{L} = \frac{EA}{L} \begin{bmatrix} -0.5 & 0 \\ -\sqrt{3}/2 & 0 \end{bmatrix}$$

$$\underline{\underline{K}}_{23} = \underline{\underline{a}}_{23}^{T}\, \underline{\underline{k}}_{23}\, \underline{\underline{a}}_{32} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \left[0.5 \;\; \tfrac{\sqrt{3}}{2}\right] \frac{EA}{L} = \begin{bmatrix} 0 & 0 \\ -0.5 & -\frac{\sqrt{3}}{2} \end{bmatrix} \frac{EA}{L} = \underline{\underline{K}}_{32}^{T}$$

$$\underline{\underline{K}}_{22} = \underline{\underline{a}}_{21}^{T}\, \underline{\underline{k}}_{22}^{1}\, \underline{\underline{a}}_{21} + a_{23}^{T}\, k_{22}^{3}\, a_{23} = \frac{EA}{L} \left[\begin{bmatrix} \frac{\sqrt{3}}{2} \\ -0.5 \end{bmatrix} \left[\tfrac{\sqrt{3}}{2} \;\; -0.5\right] + \begin{bmatrix} 0 \\ -1 \end{bmatrix} [0 \;\; -1] \right]$$

$$= \frac{EA}{L} \begin{bmatrix} 3/4 & -\sqrt{3}/4 \\ -\sqrt{3}/4 & 1.25 \end{bmatrix}$$

$$\underline{\underline{K}}_{33} = \underline{\underline{a}}_{31}^{T}\, \underline{\underline{k}}_{33}^{1}\, \underline{\underline{a}}_{31} + \underline{\underline{a}}_{32}^{T}\, \underline{\underline{k}}_{33}^{2}\, \underline{\underline{a}}_{32} = \frac{EA}{L} \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} [1 \;\; 0] + \begin{bmatrix} 0.5 \\ \sqrt{3}/2 \end{bmatrix} \left[0.5 \;\; \tfrac{\sqrt{3}}{2}\right] \right] = \frac{EA}{L} \begin{bmatrix} 1.25 & \sqrt{3}/4 \\ \sqrt{3}/4 & 3/4 \end{bmatrix}$$

$$\underline{\underline{K}}_{II} = \frac{EA}{L}\begin{bmatrix} 3/4 & 0 \\ 0 & 1.25 \end{bmatrix} \quad ; \quad \underline{\underline{K}}_{II\,I} = \frac{EA}{L}\begin{bmatrix} -\frac{\sqrt{3}}{2} & -0.5 \\ 0 & -\sqrt{3}/2 \\ -\frac{\sqrt{3}}{4} & -0.5 \\ 0 & \sqrt{3}/4 \end{bmatrix}$$

Displacements & reactions.

$$\underline{P}_I = \left\{\frac{P}{2} \quad 0\right\}^T$$

$$\underline{\Delta}_I = \underline{\underline{K}}_{II}^{-1}\,\underline{P}_I = \frac{L}{EA}\begin{bmatrix} 4/3 & 0 \\ 0 & 4/5 \end{bmatrix}\begin{Bmatrix} P/2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 2/3 \\ 0 \end{Bmatrix}\frac{PL}{AE} = \begin{Bmatrix} (\Delta_1)_2 \\ (\Delta_1)_3 \end{Bmatrix}$$

$$\underline{P}_{II} = \underline{\underline{K}}_{II\,I}\,\underline{\Delta}_I = \left\{-1/\sqrt{3} \quad 0 \quad \left(-\frac{1}{2\sqrt{3}} - \frac{\sqrt{3}}{2}\right) \quad 0\right\}^T$$

(due to direct reaction of $P\cos 30$ at support at jt. 2).