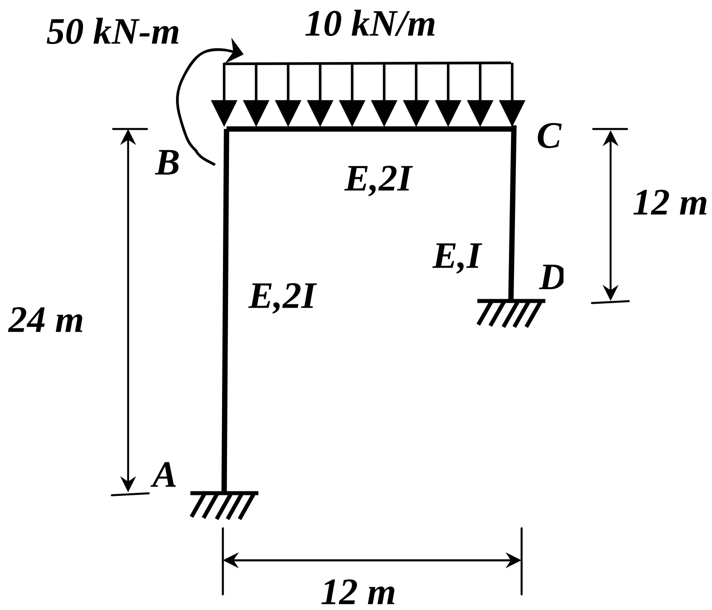
DEPARTMENT OF CIVIL ENGINEERING
CE-317 STRUCTURAL MECHANICS II
Midsem 11/9/12

Problem 1

Use only Slope Deflection Method.
Determine the reactions at supports *A* and *D*, for the frame in **Fig. 1**.

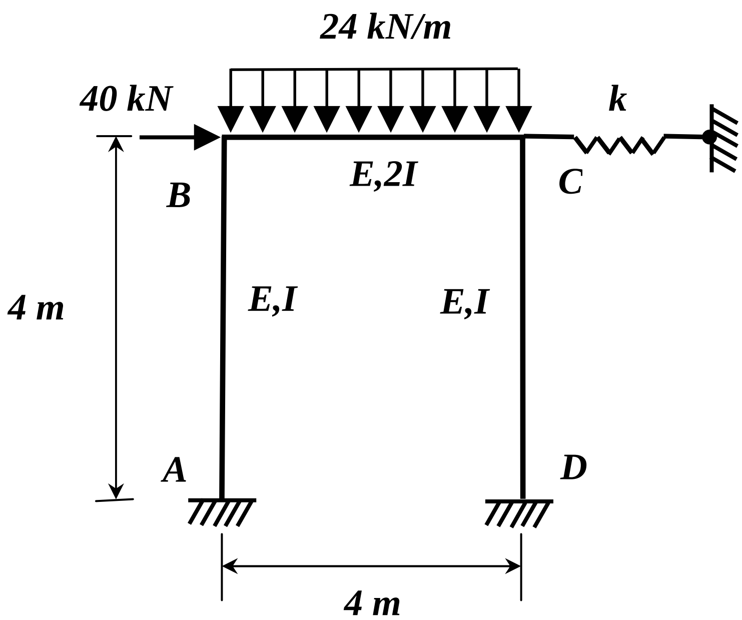
Fig. 1



Problem 2

Use only Moment Distribution Method with modified stiffnesses wherever possible.
Draw the bending moment diagram for the frame in **Fig. 2**. Data: $k = 100$ kN/m and $EI = \frac{4000}{3}$ kN.m^2.

Fig. 2

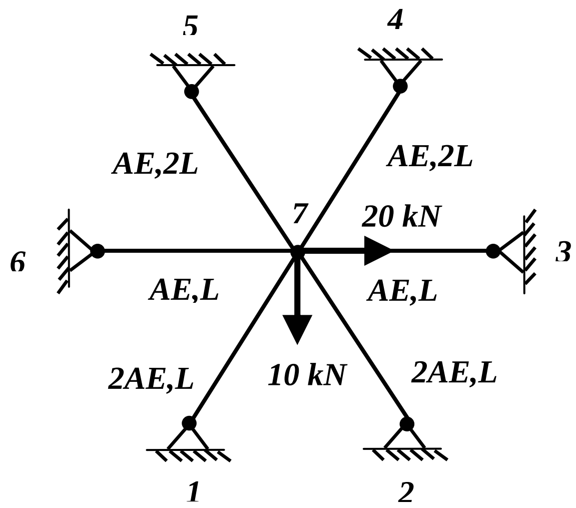


Problem 3

Use only Stiffness Matrix Method.

Determine nodal displacements and member forces for the 6-member truss shown in **Fig. 3**. Assume the truss is an equal angle truss, i.e., angle between adjacent members is 60^0.

Fig. 3



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P1

$$\frac{EI}{12}\begin{bmatrix} \frac{4.2}{2}+\frac{4.2}{1} & \frac{2.2}{1} & -\frac{6.2}{2}\cdot\frac{1}{24} \\ \frac{2.2}{1} & \frac{4.2}{1}+\frac{4.1}{1} & -\frac{6.1}{1}\cdot\frac{1}{12} \\ \frac{6.2}{2}\cdot\frac{1}{2} & \frac{6.1}{1} & \left(-2.\frac{6.2}{2}\cdot\frac{1}{24}\cdot\frac{1}{2} - 2.\frac{6.1}{1}\cdot\frac{1}{12}\right) \end{bmatrix} \begin{Bmatrix} \theta_b \\ \theta_c \\ \Delta \end{Bmatrix} = \begin{Bmatrix} \frac{10.12^2}{12}+50 \\ -\frac{10.12^2}{12} \\ 0 \end{Bmatrix}$$

In above we have used,

$$M_{BA}+M_{BC}=50 \quad ; \quad M_{CB}+M_{CD}=0$$

$$V_A+V_D=0=-\frac{(M_{AB}+M_{BA})}{~~24~~\ 2}-\frac{(M_{CD}+M_{DC})}{~~12~~}$$

$$\begin{bmatrix} 12 & 4 & -0.25 \\ 4 & 12 & -0.5 \\ 3 & 6 & -1.25 \end{bmatrix} \begin{Bmatrix} \theta_b \\ \theta_c \\ \Delta \end{Bmatrix} = \frac{12}{EI}\begin{Bmatrix} 170 \\ -120 \\ 0 \end{Bmatrix}$$

$$EI\theta_b=\frac{29520}{127}=232.44 \;;\; EI\theta_c=-\frac{27660}{127}=-217.80$$

$$EI\Delta=-\frac{61920}{127}=-487.56$$

$$M_{AB}=\frac{2.2}{24}.(232.44)-\frac{6.2}{24}\cdot\frac{1}{24}.(-487.56)=\frac{6210}{127}=48.90$$

$$M_{BA}=\frac{4.2}{24}(232.44)-\frac{6.2}{24}\cdot\frac{1}{24}(-487.56)=\frac{11130}{127}=87.64$$

$$M_{BC}=\frac{4.2}{12}(232.44)+\frac{2.2}{12}(-217.80)-\frac{10.12^2}{12}=-\frac{4780}{127}=-37.64$$

$$M_{CB}=\frac{2.2}{12}(232.44)+\frac{4.2}{12}(-217.80)+\frac{10.12^2}{12}=\frac{6640}{127}=52.28$$

$$M_{CD}=\frac{4.1}{12}(-217.80)-\frac{6.1}{12}\cdot\frac{1}{12}(-487.56)=-\frac{6640}{127}=-52.28$$

$$M_{DC}=\frac{2.1}{12}(-217.80)-\frac{6.1}{12}\cdot\frac{1}{12}(-487.56)=-\frac{2030}{127}=-15.98$$

$$V_A = -\frac{(M_{AB} + M_{BA})}{24} = -\frac{1445}{254} = -5.69 \text{ kN} = 5.69 (\rightarrow)$$

$$V_D = -\frac{(M_{CD} + M_{DC})}{12} = \frac{1445}{254} = 5.69 \text{ kN} (\leftarrow)$$

$M_{AB} = 48.90$ kN.m (↻), $M_{DC} = 15.98$ kN.m (↺)

$$A_y = -\frac{\left(M_{BC} + M_{CB} - \frac{10.12^2}{2}\right)}{12} = \frac{7465}{127} = 58.78$$

$$D_y = \frac{\left(M_{BC} + M_{CB} + \frac{10.12^2}{2}\right)}{12} = \frac{7775}{127} = 61.22$$

(check that they add up to $120 = 10 \times 12$)

P2 No sway soln. Restrain with roller at C (◁B ← R_c)
Symmetric deformation.

mem end	AB	BA	BC	CB	CD	DC
mod stiff	1	1	1			
df	0	0.5	0.5			
Fem	0	0	-32			
dist, co	8	16	16			
BM1	8	16	-16	16	-16	-8

$\frac{4EI}{L} \equiv 1$

$\frac{2E2I}{L} \equiv 1$

$\frac{24.4^2}{12} = 32$

ΣF_x : $R_c - 40 = 0$, $R_c (\leftarrow)$

Sway soln. Apply $R_c' (\rightarrow)$. Antisymmetric deformation.

mem end	AB	BA	BC	CB	CD	DC
mod stiff	1	1	3			
df	0	0.25	0.75			
Fem	-100	-100	0			
dist, co	12.5	25	75			
BM2	-87.5	-75	75	75	-75	-87.5

$\frac{4EI}{L} \equiv 1$

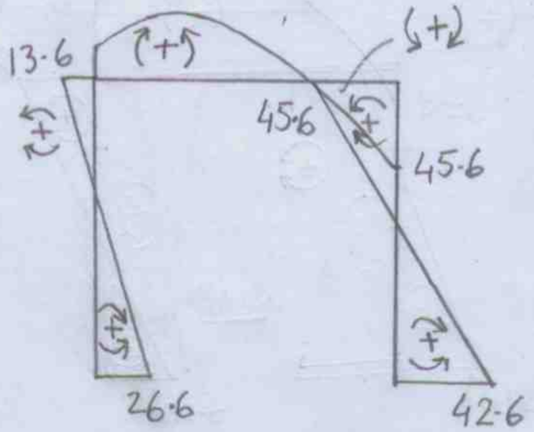
$\frac{6E2I}{L} \equiv 3$

$$\Sigma F_x : R_c' - k\Delta - 2\left\{-\frac{(-87.5 - 75)}{4}\right\} = 0 \; ; \quad \Delta = 100 \cdot \frac{4^2}{6EI}$$

$$\Rightarrow R_c' = 100 \cdot 100 \cdot \frac{4^2}{6 \cdot \frac{4000}{3}} + 2\left(\frac{162.5}{4}\right) = 101.25 \ (\rightarrow)$$

	AB	BA	BC	CB	CD	DC
BM	−2152/81 = −26.6	−368/27 = −13.6	368/27 = 13.6	1232/27 = 45.6	−1232/27 = −45.6	−3448/81 = −42.6

$$BM = BM1 + \left(\frac{R_c}{R_c'}\right) BM2$$



For BC, $V_B = -\left(45.6 + 13.6 - \frac{24 \cdot 4^2}{2}\right) \cdot \frac{1}{4}$
$= 896/27 = 33.19 \ (\uparrow)$
$V_c = -\frac{1696}{27} = 62.81 (\uparrow)$

Not reqd. Only to verify slope of BMD

P3 $\quad K_{II} = K_{77} = \sum_{j=1}^{6} k_{77}^{j} = \sum_{j=1}^{6} a_{7j}^{T} k_{77}^{j} a_{7j}$

$a_{71} = [0.5 \ \ -\sqrt{3}/2] = -a_{74}$; $\quad a_{72} = [-0.5 \ \ -\sqrt{3}/2] = -a_{75}$

$a_{73} = [-1 \ \ 0] = -a_{76}$

$$K_{II} = \frac{AE}{L}\left(\begin{bmatrix} 1/4 & -\sqrt{3}/4 \\ -\sqrt{3}/4 & 3/4 \end{bmatrix}\left(2+\frac{1}{2}\right) + \begin{bmatrix} 1/4 & \sqrt{3}/4 \\ \sqrt{3}/4 & 3/4 \end{bmatrix}\left(2+\frac{1}{2}\right) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}(1+1)\right)$$

$$= \frac{AE}{L}\begin{bmatrix} 26/8 & 0 \\ 0 & 30/8 \end{bmatrix}$$

$$\Delta_I = K_{II}^{-1} P_I = \frac{L}{AE}\begin{bmatrix} 8/26 & 0 \\ 0 & 8/30 \end{bmatrix}\begin{Bmatrix} 20 \\ 10 \end{Bmatrix} = \frac{L}{AE}\begin{Bmatrix} 160/26 \\ 80/30 \end{Bmatrix} = \frac{L}{AE}\begin{Bmatrix} 6.15 \\ 2.67 \end{Bmatrix}$$

$F_{ij} = k_{ii}^{j} a_{ij} \Delta_i + k_{ij} a_{ji} \cancel{\Delta_j} \rightarrow = 0$ for far end when $i = 7$

Tensile (+)

$$F_{71} = \frac{2AE}{L}\left(0.5 \cdot \frac{160}{26} - \frac{\sqrt{3}}{2} \cdot \frac{80}{30}\right)\frac{L}{AE} = 1.54 = -4F_{74} \Rightarrow F_{74} = -0.38$$

$$F_{72} = \frac{2AE}{L}\left(-0.5 \cdot \frac{160}{26} - \frac{\sqrt{3}}{2} \cdot \frac{80}{30}\right)\frac{L}{AE} = -10.77 = -4F_{75}$$

$\Rightarrow F_{75} = 2.69$

$$F_{73} = -F_{76} = \frac{AE}{L}\left(-\frac{160}{26}\right)\frac{L}{AE} = -6.15$$