DEPARTMENT OF CIVIL ENGINEERING
CE-317 STRUCTURAL MECHANICS II
Midsem 11/9/13

PAPER CODE: A

Note: Write your name and roll no. on the answerbook and on the summary answer sheet provided on the reverse.
You must submit the question-paper-cum-summary-answer-sheet along with the answerbook.
Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.
All **three questions** carry equal marks

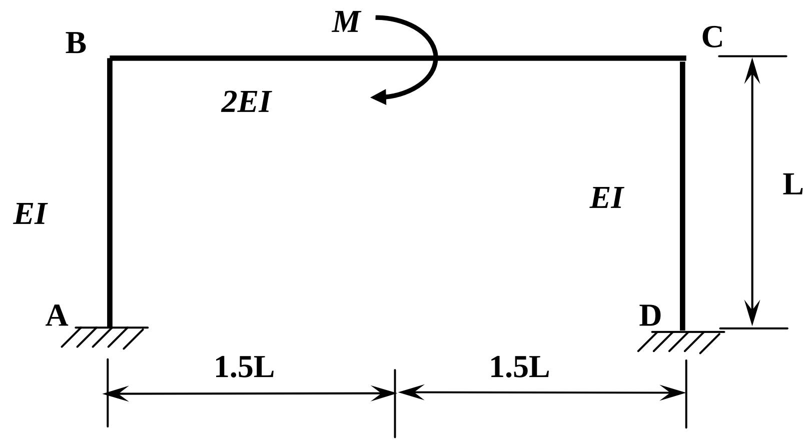
Problem 1

You must use only Slope Deflection Method in this question.
Consider the rigid-jointed frame in **Fig. 1.** A point moment ***M*** is applied at the midpoint of member ***BC*** as shown. **Find**:

(i) **Rotation at *B*.**
(ii) **All reactions at *A*.**

(Fixed end moment due to point moment *M* applied at mid-span is $M/4$)

Fig. 1

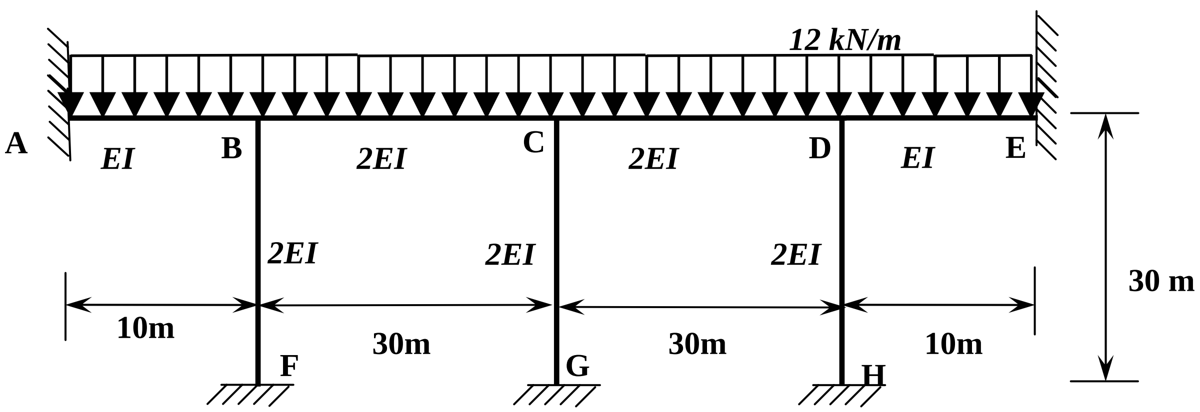


Problem 2

You must use only Moment Distribution Method in this question.
For the rigid-jointed frame in **Fig. 2**, **find all reactions at *F***

(Fixed end moment due to uniformly distributed load is $wL^2/12$)

Fig. 2



Problem 3

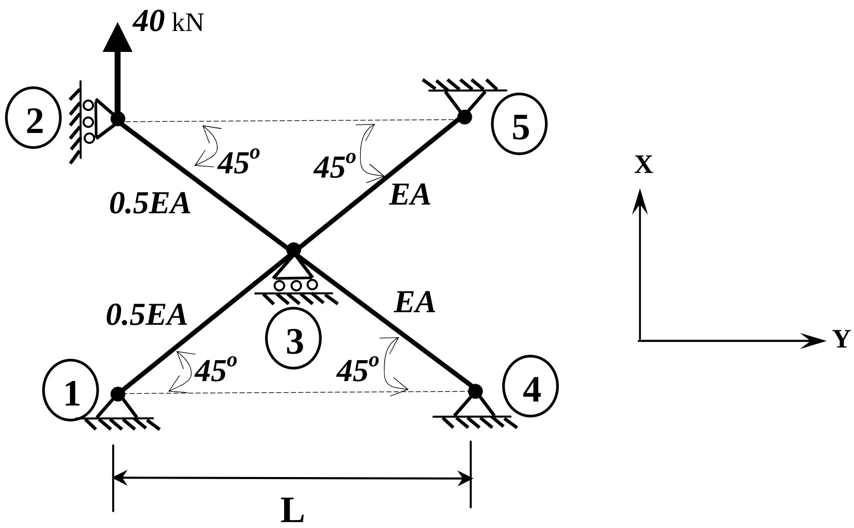
<u>You must use only Stiffness Method in this question</u>. (You must use joint numbering and global coordinate system as shown in **Fig. 3**. Assume unit of ***EA*** is kN.)

For the 4-member pin-jointed truss with load as shown in **Fig. 3**, **find:**

(i) The $\mathbf{K}_{\mathrm{II}}$ (i.e., the reduced stiffness matrix K-roman-one-roman-one).

(ii) Displacement of joints 2 and 3.

Fig. 3



SUMMARY ANSWER SHEET

PAPER CODE: A

Name:

Roll no:

Problem 1

<u>(i)</u> **Rotation at *B*** =

<u>(ii)</u> **Reactions at *A*** are A_x = ; A_y = ; M_A =

Problem 2

Reactions at *F* are F_x = ; F_y = ; M_F =

Problem 3

(i) **Reduced stiffness matrix** $\mathbf{K}_{\mathrm{II}}$ =

(ii) **Displacement of joint 2 is =**

Displacement of joint 3 is =

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PAPER CODE: B

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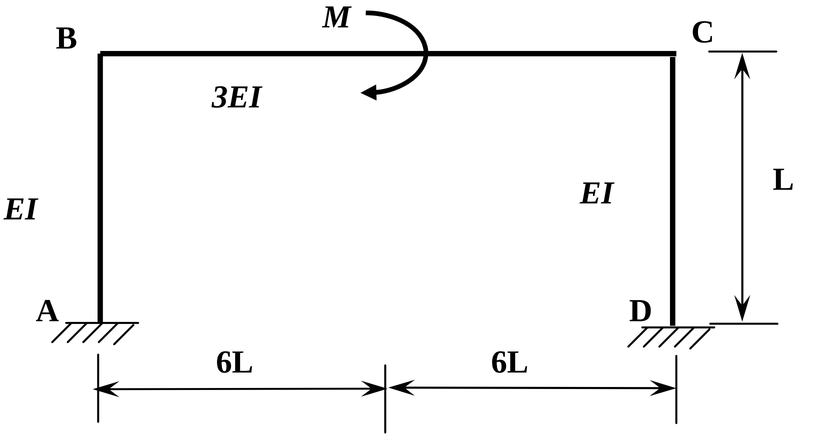
Problem 1

You must use only Slope Deflection Method in this question.
Consider the rigid-jointed frame in **Fig. 1.** A point moment ***M*** is applied at the midpoint of member ***BC*** as shown. **Find**:

(iii) **Rotation at *B*.**
(iv) **All reactions at *A*.**

(Fixed end moment due to point moment *M* applied at mid-span is $M/4$)

Fig. 1

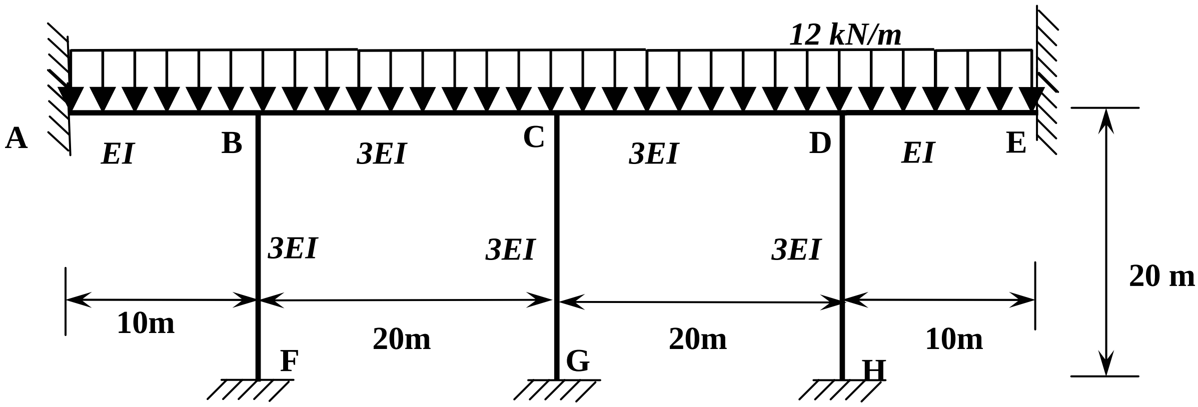


Problem 2

You must use only Moment Distribution Method in this question.
For the rigid-jointed frame in **Fig. 2**, **find all reactions at *F***

(Fixed end moment due to uniformly distributed load is $wL^2/12$)

Fig. 2



Problem 3

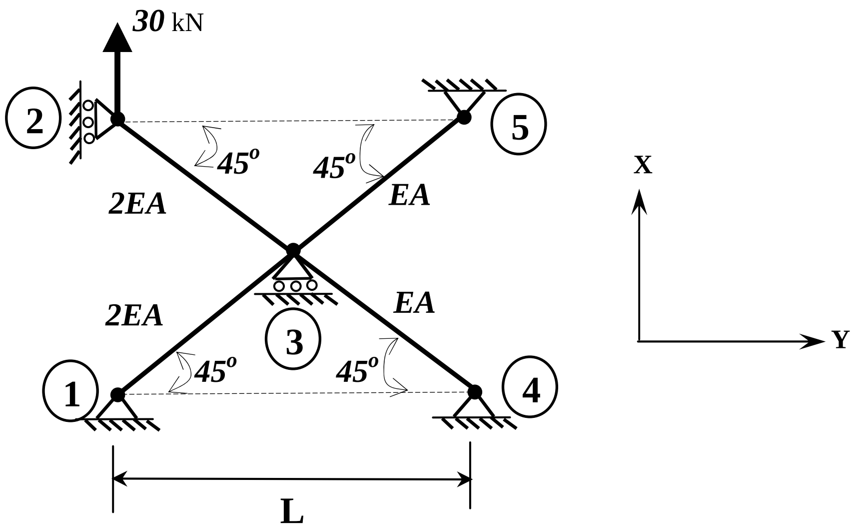
You must use only Stiffness Method in this question. (You must use joint numbering and global coordinate system as shown in **Fig. 3**. Assume unit of ***EA*** is kN.)

For the 4-member pin-jointed truss with load as shown in **Fig. 3**, **find:**

(ii) The $\mathbf{K}_{II}$ (i.e., the reduced stiffness matrix K-roman-one-roman-one).

(ii) Displacement of joints 2 and 3.

Fig. 3



SUMMARY ANSWER SHEET

PAPER CODE: B

Name:

Roll no:

Problem 1

(iii) **Rotation at *B*** =

(iv) **Reactions at *A*** are $A_x =$; $A_y =$; $M_A =$

Problem 2

Reactions at *F* are $F_x =$; $F_y =$; $M_F =$

Problem 3

(iii) **Reduced stiffness matrix** $\mathbf{K}_{II} =$

(iv) **Displacement of joint 2 is =**

Displacement of joint 3 is =

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Midsem 11/9/13

PAPER CODE: C

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(ii) **All reactions at *A*.**

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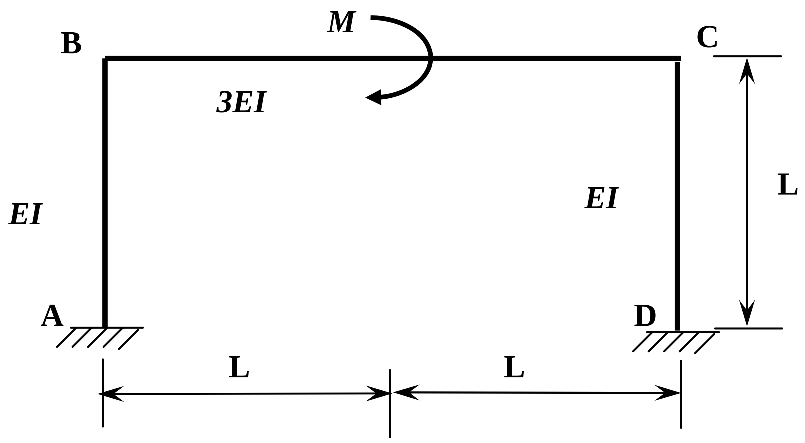
Fig. 1



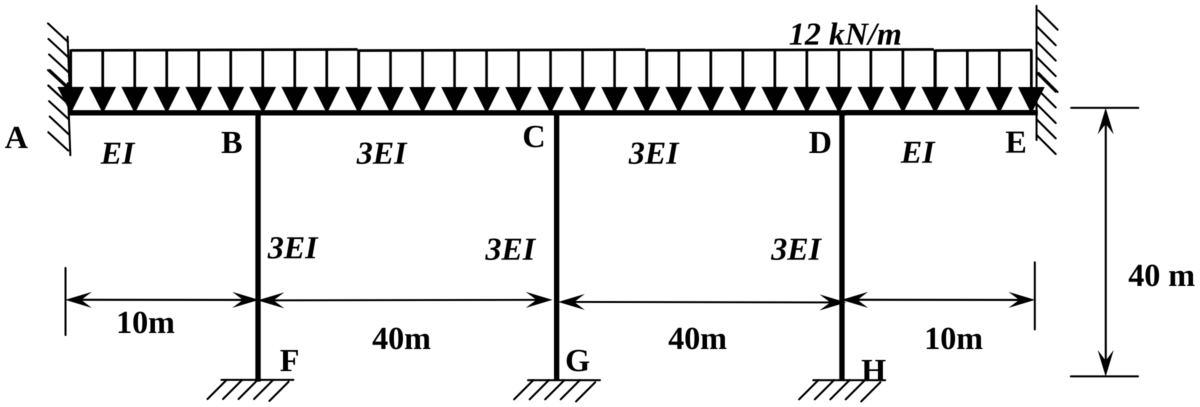
Problem 2

You must use only Moment Distribution Method in this question.

For the rigid-jointed frame in **Fig. 2**, **find all reactions at *F***

(Fixed end moment due to uniformly distributed load is $wL^2/12$)

Fig. 2



Problem 3

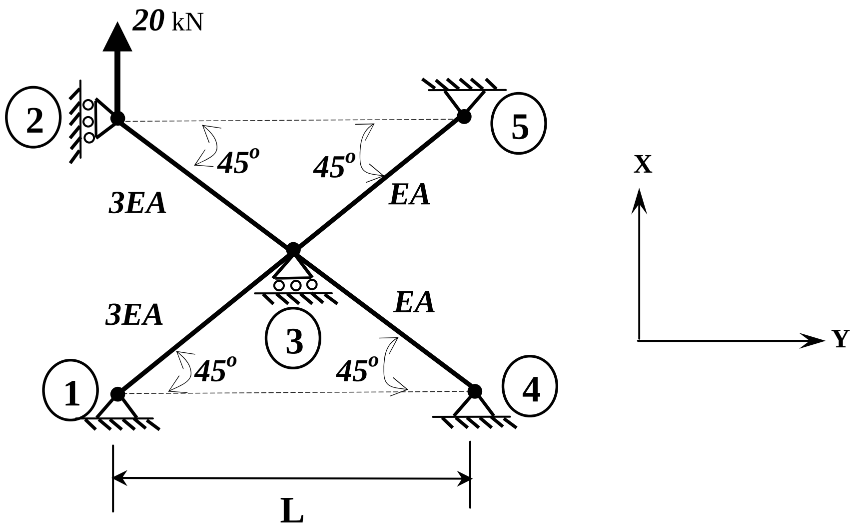
<u>You must use only Stiffness Method in this question</u>. (You must use joint numbering and global coordinate system as shown in **Fig. 3**. Assume unit of ***EA*** is kN.)

For the 4-member pin-jointed truss with load as shown in **Fig. 3**, **find:**

(i) The $\mathbf{K}_{II}$ (i.e., the reduced stiffness matrix K-roman-one-roman-one).

(ii) Displacement of joints 2 and 3.

Fig. 3



SUMMARY ANSWER SHEET

PAPER CODE: C

Name:

Roll no:

Problem 1

<u>(i)</u> **Rotation at *B*** =

<u>(ii)</u> **Reactions at *A*** are $A_x =$; $A_y =$; $M_A =$

Problem 2

Reactions at *F* are $F_x =$; $F_y =$; $M_F =$

Problem 3

(i) **Reduced stiffness matrix** $\mathbf{K}_{II} =$

(ii) **Displacement of joint 2 is =**

Displacement of joint 3 is =

DEPARTMENT OF CIVIL ENGINEERING
CE-317 STRUCTURAL MECHANICS II
Midsem 11/9/13

PAPER CODE: D

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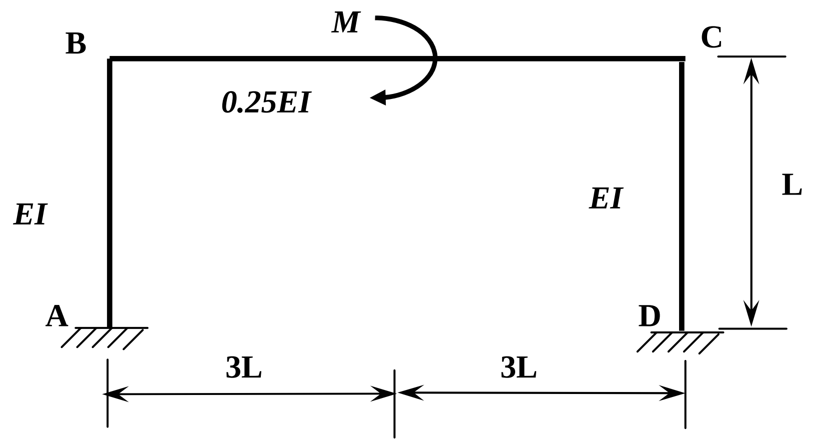
(iv) **All reactions at *A*.**

(Fixed end moment due to point moment *M* applied at mid-span is $M/4$)

Fig. 1

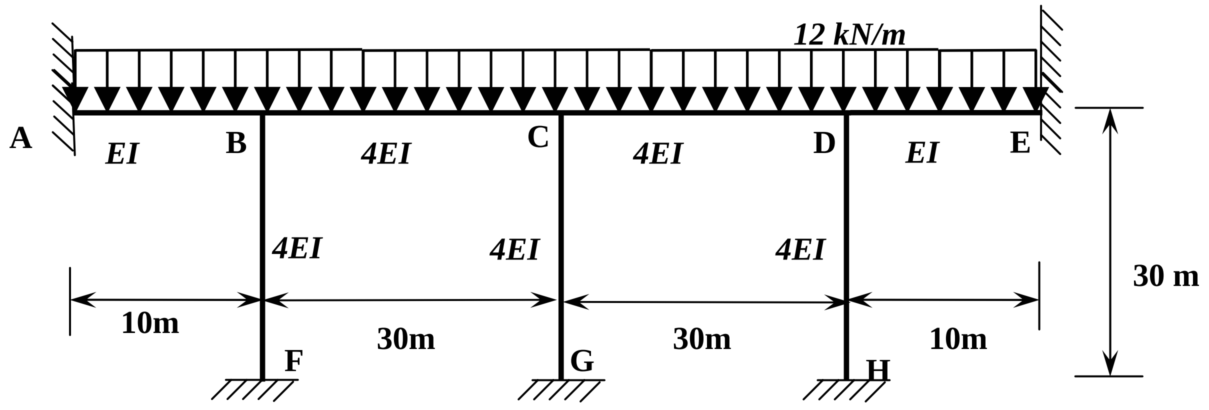


Problem 2

You must use only Moment Distribution Method in this question.
For the rigid-jointed frame in **Fig. 2**, **find all reactions at *F***

(Fixed end moment due to uniformly distributed load is $wL^2/12$)

Fig. 2



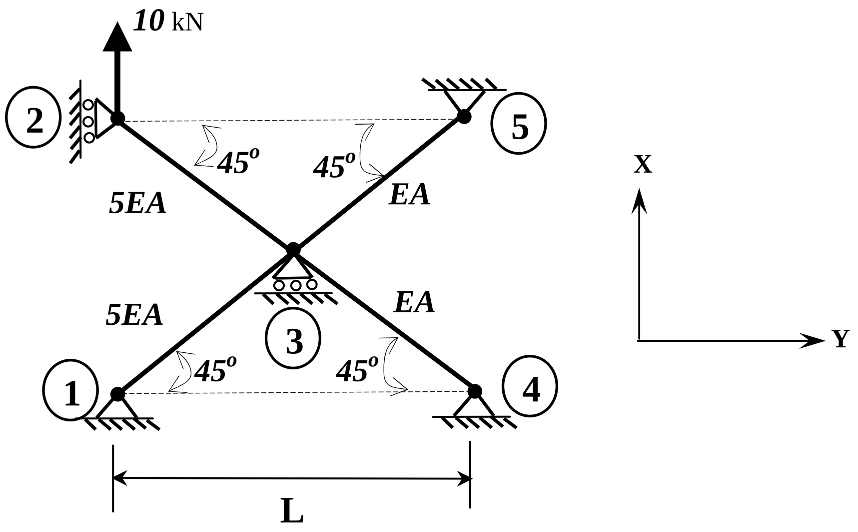
Problem 3

<u>You must use only Stiffness Method in this question</u>. (You must use joint numbering and global coordinate system as shown in **Fig. 3**. Assume unit of ***EA*** is kN.)
For the 4-member pin-jointed truss with load as shown in **Fig. 3**, **find:**

(ii) The $\mathbf{K}_{II}$ (i.e., the reduced stiffness matrix K-roman-one-roman-one).

(ii) Displacement of joints 2 and 3.

Fig. 3



SUMMARY ANSWER SHEET

PAPER CODE: D

Name:

Roll no:

Problem 1

<u>(iii)</u> **Rotation at *B*** =

<u>(iv)</u> **Reactions at *A*** are A_x = ; A_y = ; M_A =

Problem 2

Reactions at *F* are F_x = ; F_y = ; M_F =

Problem 3

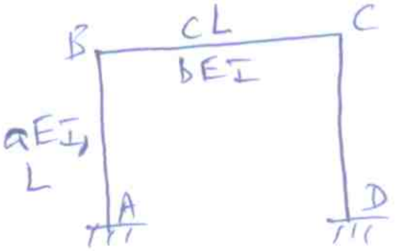
(iii) **Reduced stiffness matrix** $\mathbf{K}_{II}$ =

(iv) **Displacement of joint 2 is =**

Displacement of joint 3 is =

CE317 Midsem 2013

P(1)



Antisymm $\Rightarrow \theta_B = \theta_C$
↳ sway.

$M_{BA} + M_{BC} = 0 \rightarrow$

$M_{CB} + M_{CD} = 0 \rightarrow$ (Not required ∵ using antisymmetry.)

$V_A + V_D = 0 \rightarrow$

$$\frac{EI}{L}\begin{bmatrix} \frac{a}{1}\cdot 4 + \frac{b}{c}\cdot 4 & \frac{b}{c}\cdot 2 & -\frac{6}{L}\cdot\frac{a}{1} \\ & & \\ \frac{a}{1}\cdot 2 + \frac{a}{1}\cdot 4 \Big/ L & & \frac{2\left(-\frac{6}{L}\cdot\frac{a}{1} - \frac{6}{L}\cdot\frac{a}{1}\right)}{L} \end{bmatrix}\begin{Bmatrix}\theta_B \\ \theta_C \\ \Delta\end{Bmatrix} = \begin{Bmatrix} -M/4 \\ -M/4 \\ 0 \end{Bmatrix}$$

use $\theta_B = \theta_C \rightarrow$

$$\left(4a + 6\frac{b}{c}\right)\theta_B - \frac{6}{L}a\Delta = -\frac{M}{4}\cdot\frac{L}{EI}$$

$$12a\theta_B - \frac{24}{L}a\Delta = 0$$

$$\Rightarrow \left(4a + 6\frac{b}{c} - \frac{6}{2}a\right)\theta_B = -\frac{M}{4}\frac{L}{EI}$$

$$\theta_B = -\frac{M}{4}\cdot\frac{1}{(a + 6b/c)}\cdot\frac{L}{EI}$$

$$\Delta = -\frac{M}{4}\frac{1}{(a+6b/c)}\frac{L}{EI}\cdot\frac{L}{2}$$

$$M_{AB} = a\frac{EI}{L}\left(2\left(-\frac{M}{4}\right)\frac{1}{(a+6b/c)} - 6\left(-\frac{M}{4}\right)\frac{1}{(a+6b/c)}\cdot\frac{L}{2}\cdot\frac{1}{L}\right)\cdot\frac{L}{EI}$$

$$A_y = \frac{(2M_{AB} + M)}{cL} \quad ; \quad \because M_{AB} = -M_{BA},\ A_x = 0$$

Paper code	a	b	c	θ_B	M_{AB}	A_y	A_x
A	1	2	3	$-\frac{ML}{EI}\cdot\frac{1}{20}$	$M\cdot\frac{1}{20}$	$\frac{M}{L}\cdot\frac{11}{30}$	0
B	1	3	12	$-\frac{ML}{EI}\cdot\frac{1}{10}$	$M\cdot\frac{1}{10}$	$\frac{M}{L}\cdot\frac{1}{10}$	0
C	1	3	2	$-\frac{ML}{EI}\cdot\frac{1}{40}$	$M\cdot\frac{1}{40}$	$\frac{M}{L}\cdot\frac{21}{40}$	0
D	1	0.25	6	$-\frac{ML}{EI}\cdot\frac{1}{5}$	$M\cdot\frac{1}{5}$	$\frac{M}{L}\cdot\frac{7}{30}$	0

P(2) Symmetry ⇒

mem end	AB	BA	BF	BC	CB	FB
rel stiff	1	1	a/b	c/d	c/d	a/b
df	—	$\frac{1}{1+\frac{a}{b}+\frac{c}{d}}$	$\frac{a/b}{1+\frac{a}{b}+\frac{c}{d}}$	$\frac{c/d}{1+\frac{a}{b}+\frac{c}{d}}$	—	—
fem	$-\frac{wL^2}{12}$	$\frac{wL^2}{12}$	0	$-\frac{wd^2L^2}{12}$	$\frac{wd^2L^2}{12}$	0
df, co	$q/2$	q	r	s	$s/2$	$r/2$
BM	$\frac{q}{2}-\frac{wL^2}{12}$	$q+\frac{wL^2}{12}$	r	$s-\frac{wd^2L^2}{12}$	$\frac{s}{2}+\frac{wd^2L^2}{12}$	$r/2$

where,

$$q = \frac{wL^2}{12}(d^2-1)\cdot\frac{bd}{bd+ad+bc} = \frac{24}{7}\cdot 100$$

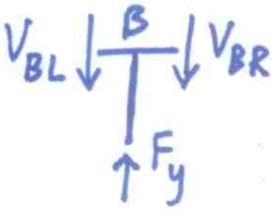
$$r = \frac{wL^2}{12}(d^2-1)\cdot\frac{ad}{bd+ad+bc} = \frac{16}{7}\cdot 100$$

$$s = \frac{wL^2}{12}(d^2-1)\cdot\frac{bc}{bd+ad+bc} = \frac{16}{7}\cdot 100$$

} for Paper Code A.

	AB	BA	BF	BC	CB	FB
BM	71.43	442.86	228.57	-671.43	1014.29	114.29

$M_F = M_{FB} = 114.29$; $F_x = \frac{M_{FB}+M_{BF}}{bL} = \frac{114.29+228.57}{30} = 11.43$



$$F_y = V_{BL}+V_{BR} = \frac{M_{AB}+M_{BA}+wL^2/2}{L} + \frac{\left(-M_{BC}-M_{CB}+\frac{wd^2L^2}{2}\right)}{dL}$$

$$= (71.43+442.86+600)/10 + \frac{1}{30}(+671.43-1014.29+5400)$$

$$= 280$$

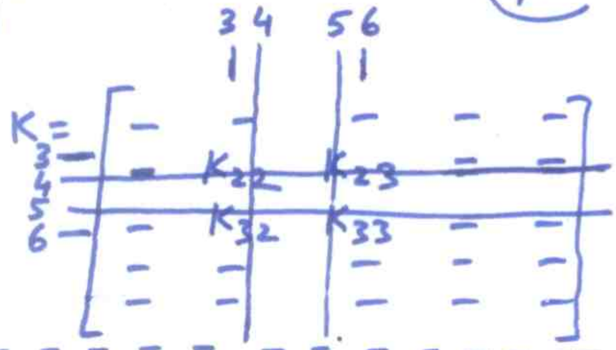
Paper Code	M_F	F_x	F_y
A	114.29	11.43	280
B	56.25	8.44	182.81
C	225	16.88	373.13
D	145.45	14.54	258.18

(for MDM tables)
(See xls file)

Problem2	Paper A											
	ab	ba	bf	bc	cb	fb		aEI	bL	cEI	dL
k	1	1	0.666667	0.666667	0.666667	0.666667		2	3	2	3
df		0.428571	0.285714	0.285714				L	w		
fem	-100	100	0	-900	900	0		10	12		
dist, co	171.4286	342.8571	228.5714	228.5714	114.2857	114.2857		Mf	Fx	VbL	VbR
BM	71.42857	442.8571	228.5714	-671.429	1014.286	114.2857		114.2857	11.42857	111.4286	168.5714
									Fy		
									280		
Problem2	Paper B										
	ab	ba	bf	bc	cb	fb		aEI	bL	cEI	dL
k	1	1	1.5	1.5	1.5	1.5		3	2	3	2
df		0.25	0.375	0.375				L	w		
fem	-100	100	0	-400	400	0		10	12		
dist, co	37.5	75	112.5	112.5	56.25	56.25		Mf	Fx	VbL	VbR
BM	-62.5	175	112.5	-287.5	456.25	56.25		56.25	8.4375	71.25	111.5625
									Fy		
									182.8125		
Problem2	Paper C										
	ab	ba	bf	bc	cb	fb		aEI	bL	cEI	dL
k	1	1	0.75	0.75	0.75	0.75		3	4	3	4
df		0.4	0.3	0.3				L	w		
fem	-100	100	0	-1600	1600	0		10	12		
dist, co	300	600	450	450	225	225		Mf	Fx	VbL	VbR
BM	200	700	450	-1150	1825	225		225	16.875	150	223.125
									Fy		
									373.125		
Problem2	Paper D										
	ab	ba	bf	bc	cb	fb		aEI	bL	cEI	dL
k	1	1	1.333333	1.333333	1.333333	1.333333		4	3	4	3
df		0.272727	0.363636	0.363636				L	w		
fem	-100	100	0	-900	900	0		10	12		
dist, co	109.0909	218.1818	290.9091	290.9091	145.4545	145.4545		Mf	Fx	VbL	VbR
BM	9.090909	318.1818	290.9091	-609.091	1045.455	145.4545		145.4545	14.54545	92.72727	165.4545
									Fy		
									258.1818		

(P3) $\Delta_I = \{(\Delta_2)_2 \ (\Delta_1)_3\}^T = \{\Delta_4 \ \Delta_5\}^T$

K_{II} = keep cols/rows 4,5 of K →



$$\Rightarrow K_{II} = \begin{bmatrix} K_{22}(2,2) & K_{23}(2,1) \\ K_{32}(1,2) & K_{33}(1,1) \end{bmatrix}$$

$k_{23} = k_{32} = k^3_{22} = k^2_{33} = k_{13} = k_{31}$
$= k^3_{11} = k^1_{33} = a.EA/(L/\sqrt{2})$
$k_{34} = k_{43} = k_{35} = k_{53} = k^5_{33} = k^3_{55} = k^4_{33}$
$= k^3_{44} = EA/(L/\sqrt{2})$

$$a_{31} = \left(\tfrac{1}{\sqrt{2}} \ \tfrac{1}{\sqrt{2}}\right) = -a_{35}$$

$$a_{32} = \left(\tfrac{1}{\sqrt{2}} \ -\tfrac{1}{\sqrt{2}}\right) = -a_{23} = -a_{34}$$

$$K_{22} = a^T_{23} \, k^3_{22} \, a_{23} = \frac{EA}{L} \cdot \frac{a}{1/\sqrt{2}} \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \left(-1/\sqrt{2} \ \ 1/\sqrt{2}\right) = \frac{EA}{L} \begin{bmatrix} x & x \\ x & \frac{a\sqrt{2}}{2} \end{bmatrix}$$

where axial stiffness of members 1–3, 2–3 is a.EA

$$K_{33} = a^T_{32} k^2_{33} a_{32} + a^T_{31} k^1_{33} a_{31} + a^T_{34} k^4_{33} a_{34} + a^T_{35} k^5_{33} a_{35}$$

$$= \frac{EA}{L} \left\{ (\sqrt{2} + a\sqrt{2}) \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \left(1/\sqrt{2} \ \ 1/\sqrt{2}\right) + (\sqrt{2} + a\sqrt{2}) \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \left(1/\sqrt{2} \ \ -1/\sqrt{2}\right) \right\}$$

$$= \frac{EA}{L} \begin{bmatrix} \frac{\sqrt{2}}{2}(a+1).2 & x \\ x & x \end{bmatrix}$$

$$K_{23} = \frac{EA}{L} a\sqrt{2} \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \left(1/\sqrt{2} \ \ -1/\sqrt{2}\right) = \frac{EA}{L} a\sqrt{2} \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} = K_{32}$$

$$= a^T_{23} k_{23} a_{32}$$

$$K_{II} = \frac{EA}{L} \begin{bmatrix} a/\sqrt{2} & a/\sqrt{2} \\ a/\sqrt{2} & (a+1)\sqrt{2} \end{bmatrix} ; \ K^{-1}_{II} = \left(a^2 + a - \frac{a^2}{2}\right)^{-1} \frac{L}{EA} \begin{bmatrix} (a+1)\sqrt{2} & -a/\sqrt{2} \\ -a/\sqrt{2} & a/\sqrt{2} \end{bmatrix}$$

$$\Delta_I = K^{-1}_{II} \begin{Bmatrix} P_4 \\ P_5 \end{Bmatrix} = \left(a + \frac{a^2}{2}\right)^{-1} \frac{L}{EA} \begin{Bmatrix} \sqrt{2}(a+1) P_4 - \frac{a}{\sqrt{2}} P_5 \\ -\frac{a}{\sqrt{2}} P_4 + \frac{a}{\sqrt{2}} P_5 \end{Bmatrix} = \begin{Bmatrix} \Delta_4 \\ \Delta_5 \end{Bmatrix}$$

P_4 = as given, $P_5 = 0$, a = as given

Paper code	P_4	P_5	a	$\Delta_4 = (\Delta_2)_2$	$\Delta_5 = (\Delta_1)_3$
A	40	0	0.5	135.76 L/EA	−22.63 L/EA
B	30	0	2	31.82 L/EA	−10.61 L/EA
C	20	0	3	15.08 L/EA	−5.66 L/EA
D	10	0	5	4.85 L/EA	−2.02 L/EA.