CE 317: MIDSEM
Department of Civil Engineering, IIT Bombay

Date: 9th September 2018 **Maximum Marks: 60** **Duration: 120 Minutes**

Instructions:

1. Attempt all questions.
2. All questions carry equal weight.
3. Make suitable assumptions, if necessary, and state the same clearly.

Q. 1. For the frame loaded as shown in **Fig. 1,** the deflections were found as $\theta_B = 0.2041 \text{ x } 10^{-2}$ rad (clockwise), $\theta_C = 0.5174 \text{ x } 10^{-3}$ rad (counter-clockwise), $\theta_D = 0.2824 \text{ x } 10^{-4}$ rad (clockwise), $\Delta_B = 0.8633 \text{ x } 10^{-2}$ m (→), $\Delta_D = 0.1357 \text{ x } 10^{-1}$ m (→). Using ***Slope Deflection Method,*** find **ALL reactions** at joint ***A*****.** Joints ***B*** and ***D*** are at the same horizontal level. Member lengths are shown against each member**.** $EI = 2 \times 10^{14}$ N.mm^2 for all members.

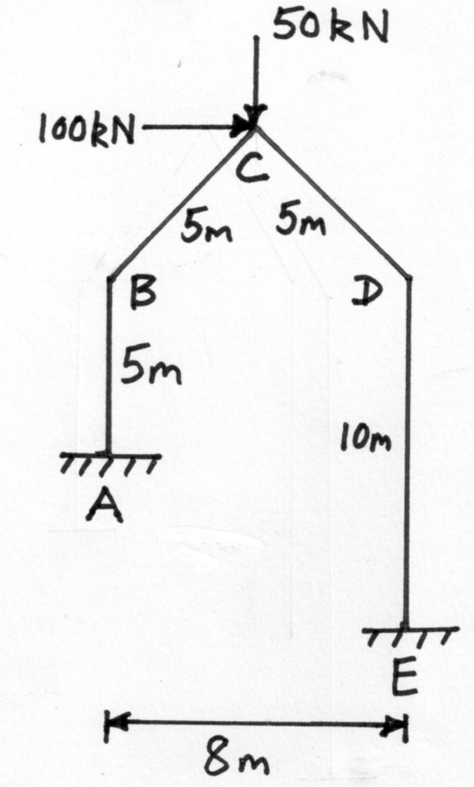


Fig. 1

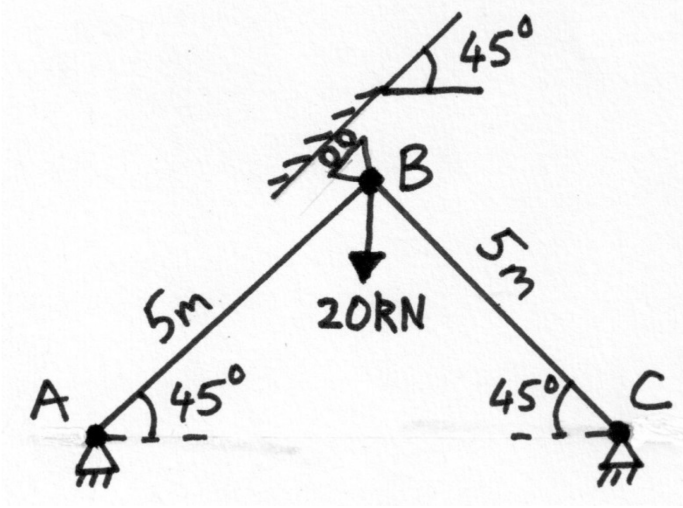


Fig.2

Q. 2. Using **Matrix Stiffness Method**, determine the **displacement of joint *B*** of the two-member plane truss shown in **Fig. 2**. Joint ***B*** is an inclined roller support. $EA = 8 \times 10^{6}$ N for all members.

Q. 3 Joint displacements due to loads ***V*** and ***Q*** applied to the simply-supported plane truss in **Fig. 3** are given in **Table 1**. **<u>Determine the loads *V* and *Q*</u>** that cause these displacements. $EA = 8\times10^6$ N for all members.

Table 1.

Joint	X - displacement (m)	Y - displacement (m)
1	0.00000	0.00000
2	0.07294	0.00000
3	0.01425	-0.10700
4	0.06919	-0.08835
5	0.03338	-0.16080
6	0.05494	-0.15450
7	0.05046	-0.16230
8	0.03452	-0.15460
9	0.06883	-0.10670
10	0.02252	-0.08540
11	0.08083	0.00000
12	0.02252	0.00000

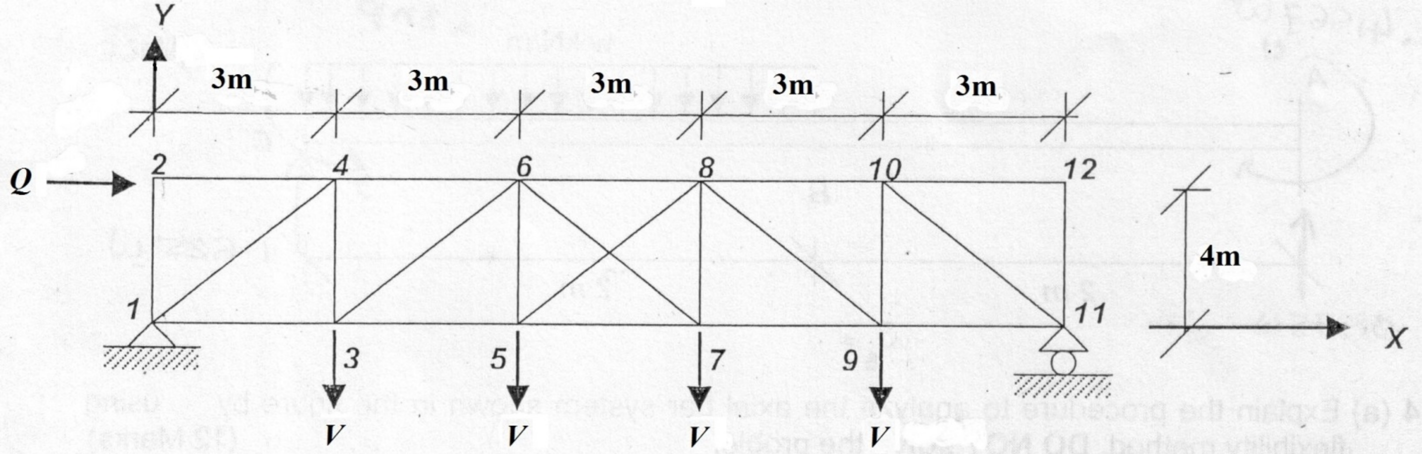


Fig. 3

Q. 4 The space truss in **Fig. 4** comprises three members ***AD***, ***BD***, ***CD***, each with length **1 m.** Coordinates (in **m**) are: $A=\left(0,0,\dfrac{1}{\sqrt{3}}\right)$, $B=\left(-\dfrac{1}{2},0,-\dfrac{1}{2\sqrt{3}}\right)$, $C=\left(\dfrac{1}{2},0,-\dfrac{1}{2\sqrt{3}}\right)$, $D=\left(0,\sqrt{\dfrac{2}{3}},0\right)$. Supports ***A***, ***B***, ***C***, are ball and sockets. Member ***CD*** is heated to 70 ^{0}C above ambient temperature. Determine displacements at joint ***D***. Use $EA = 8\times10^6$ N for all members, and $\alpha = 5\times10^{-6}\,/\,{}^0\mathrm{C}$ for ***CD***.

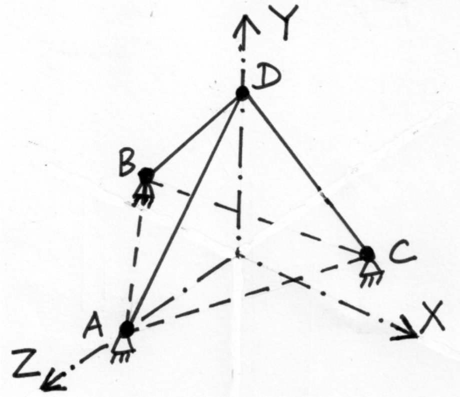


Fig. 4

CE317 MIDSEM 2018

①

Q1

$$M_{AB} = \frac{2*10^{14}}{5*10^3}\left(2*0.2041*10^{-2} - 6*\frac{0.8633*10^{-2}}{5}\right) = -2.51104*10^{8}\ \text{N.mm} = -2.511*10^{2}\ \text{kN.m}$$

$$M_{BA} = \frac{2*10^{14}}{5*10^3}\left(4*0.2041*10^{-2} - 6*\frac{0.8633*10^{-2}}{5}\right) = -8.7824*10^{7}\ \text{N.mm}$$

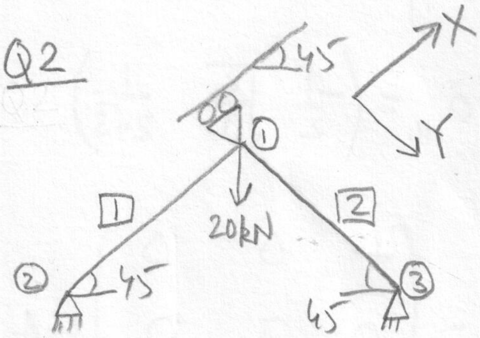
$$A_x = \frac{M_{AB}+M_{BA}}{5*10^3} = -6.77856*10^{4}\ \text{N} = -67.79\ \text{kN}.(\leftarrow)$$

$$M_{ED} = \frac{2*10^{14}}{10*10^3}\left(2*0.2824*10^{-4} - 6*\frac{0.1357*10^{-1}}{10}\right) = -1.617104*10^{8}$$

$$A_y = \frac{(+M_{AB} - M_{ED} - 100*10^3*13*10^3 + 50*10^3*4*10^3 - A_x*5*10^3)}{8*10^3}$$

$$A_y = -43532.2\text{N} = -43.53\text{kN}(\downarrow)$$

Q2



$$a_{①②} = [1\ \ 0],\quad a_{①③} = [0\ \ -1]$$

$$K_{\boxed{1}} = \frac{EA}{L}\begin{array}{c|cc|cc|}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 1 & 0 & & \\
2 & 0 & 0 & & \\
\hline
3 & & & & \\
4 & & & &
\end{array}
\qquad
K_{\boxed{2}} = \frac{EA}{L}\begin{array}{c|cc|cc|}
 & 1 & 2 & 5 & 6 \\
\hline
1 & 0 & 0 & & \\
2 & 0 & 1 & & \\
\hline
5 & & & & \\
6 & & & &
\end{array}$$

$$K_{II} = \begin{array}{cc} \left[\frac{EA}{L}\right] & 1 \\ 1 & \end{array};\quad P_I = \left[-\frac{20*10^3}{\sqrt{2}}\right]1;\quad \Delta_I = K_{II}^{-1}P_I$$

$$= -8.8388\ \text{mm}.$$

Q3

$$F_{24} = k^{4}_{22}a_{24}\Delta_2 + k_{24}a_{42}\Delta_4 = \frac{EA}{3}\left([-1\ \ 0]\begin{bmatrix}0.07294\\0\end{bmatrix} + [1\ \ 0]\begin{bmatrix}0.06919\\-0.08835\end{bmatrix}\right)$$

$$= -10000\ \text{N}$$

Joint equil (method of joints) at joint ② $\Rightarrow Q + F_{24} = 0 \Rightarrow Q = 10000\text{N}.$

$$F_{34} = \frac{EA}{4}\left([0\ \ -1]\begin{bmatrix}0.01425\\-0.10700\end{bmatrix} + [0\ \ 1]\begin{bmatrix}0.06919\\-0.08835\end{bmatrix}\right) = 37300\text{N}$$

$$F_{36} = \frac{EA}{5}\left([-0.6\ \ -0.8]\begin{bmatrix}0.01425\\-0.10700\end{bmatrix} + [0.6\ \ 0.8]\begin{bmatrix}0.05494\\-0.15450\end{bmatrix}\right) = -21737.6\text{N}$$

Joint equil at jt. ③ $\Rightarrow V = F_{34} + F_{36}*0.8 = \underline{19909.92\text{N}}$

$$F_{56} = \frac{EA}{4}\left([0\ \ -1]\begin{bmatrix}0.03338\\-0.16080\end{bmatrix} + [0\ \ 1]\begin{bmatrix}0.05494\\-0.15450\end{bmatrix}\right) = 12600\text{N}$$

(or)

$$F_{58} = \frac{EA}{5}\left([-0.6\ \ -0.8]\begin{bmatrix}0.03338\\-0.16080\end{bmatrix} + [0.6\ \ 0.8]\begin{bmatrix}0.03452\\-0.15460\end{bmatrix}\right) = 9030.4$$

Joint equid. at jt. ⑤ $\Rightarrow V = F_{56} + F_{58}*0.8 = \underline{19824.32\text{N}}$

$$F_{78} = \frac{EA}{4}\left(\begin{bmatrix}0 & -1\end{bmatrix}\begin{bmatrix}0.05046\\-0.16230\end{bmatrix} + \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}0.03452\\-0.15460\end{bmatrix}\right) = 15400\,N$$

or

$$F_{76} = \frac{EA}{5}\left(\begin{bmatrix}0.6 & -0.8\end{bmatrix}\begin{bmatrix}0.05046\\-0.16230\end{bmatrix} + \begin{bmatrix}-0.6 & 0.8\end{bmatrix}\begin{bmatrix}0.05494\\-0.15450\end{bmatrix}\right) = 5683.2\,N$$

Joint equil at jt ⑦ $\Rightarrow V = F_{78} + F_{76} * 0.8 = \underline{19946.56\,N}$

$$F_{9,10} = \frac{EA}{4}\left(\begin{bmatrix}0 & -1\end{bmatrix}\begin{bmatrix}0.06883\\-0.10670\end{bmatrix} + \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}0.02252\\-0.08540\end{bmatrix}\right) = 42600\,N$$

or

$$F_{98} = \frac{EA}{5}\left(\begin{bmatrix}0.6 & -0.8\end{bmatrix}\begin{bmatrix}0.06883\\-0.10670\end{bmatrix} + \begin{bmatrix}-0.6 & 0.8\end{bmatrix}\begin{bmatrix}0.03452\\-0.15460\end{bmatrix}\right) = -28374\,N$$

Joint equil at jt ⑨ $\Rightarrow V = F_{9,10} + F_{98} * 0.8 = \underline{19900.8\,N}$

<u>P4</u> D → ①, A → ②, B → ③, C → ④ is jt. numbering.

$$K_{II} = \sum_{j=2,3,4} [K]^j_{11} = \sum a^T_{1j}[R]^j_{11} a_{1j} = \frac{EA}{1}\sum a^T_{1j}\, a_{1j}$$

$$a_{12} = \left(0, \sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right);\quad a_{13} = \left(\frac{1}{2}, \sqrt{\frac{2}{3}}, \frac{1}{2\sqrt{3}}\right);\quad a_{14} = \left(-\frac{1}{2}\ \sqrt{\frac{2}{3}}\ \ \frac{1}{2\sqrt{3}}\right)$$

$$K_{II} = \underset{8E6}{\underbrace{EA}}\begin{bmatrix} 0+\frac{1}{4}+\frac{1}{4} & 0+\frac{1}{\sqrt{6}}-\frac{1}{\sqrt{6}} & 0+\frac{1}{4\sqrt{3}}-\frac{1}{4\sqrt{3}} \\ & \frac{2}{3}+\frac{2}{3}+\frac{2}{3} & -\frac{\sqrt{2}}{3}+\frac{1}{\sqrt{18}}+\frac{1}{\sqrt{18}} \\ \text{symm} & & \frac{1}{3}+\frac{1}{12}+\frac{1}{12}\end{bmatrix} = \begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & \frac{1}{2}\end{bmatrix} EA$$

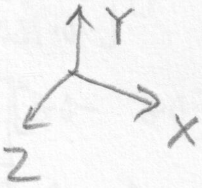
$$P_I = \overset{0}{\cancel{P_a}} - P^{se}_{①} = -a^T_{14}\, F^{sf}_{14} = -\begin{bmatrix}-\frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}}\end{bmatrix}^T \left(\underset{EA/\cancel{L}}{\underbrace{R^4_{11}}}\left(-\underset{\underset{5E-6}{\alpha}\,\underset{70}{\Delta T}\,\cdot\,\cancel{L}}{\underbrace{\delta^s_{14}}}\right) + R_{14}\left(-\underset{0}{\underbrace{\delta^s_{41}}}\right)\right)$$

$$= -\begin{bmatrix}-\frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}}\end{bmatrix}^T \cdot (-35*10^{-5}\cdot EA)$$

$$P_I = EA*10^{-5}\begin{bmatrix}-\frac{35}{2} & 35\sqrt{\frac{2}{3}} & \frac{35}{2\sqrt{3}}\end{bmatrix}^T$$

$$\Delta_I = 10^{-5}\begin{bmatrix}-35 & \frac{35}{\sqrt{6}} & \frac{35}{\sqrt{3}}\end{bmatrix}^T m = \begin{bmatrix}-0.35 & 0.1429 & 0.2021\end{bmatrix} mm$$

in



directions.