CE-317 STRUCTURAL ANALYSIS I
DEPARTMENT OF CIVIL ENGINEERING
Midsem; September 16, 2019, 11am-1pm

<u>Problems carry equal weightage</u>

Problem 1 YOU MUST USE ONLY SLOPE DEFLECTION METHOD

Refer Fig. 1. For the frame having an internal hinge at ***C***, and loaded as shown, **determine the rotation at *B* (i.e., θ_B) and reactions (i.e., forces and moment) at the support *D*.** Use $EI = 4\times10^{13}$ N.mm^2 for all members, and $w = 6$ kN/m.

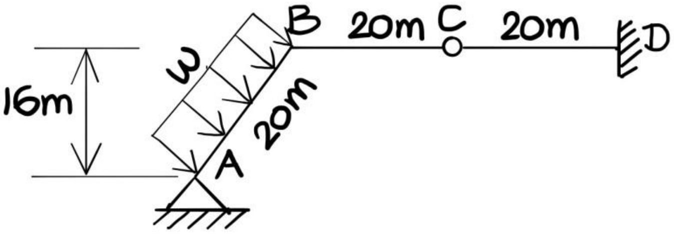


Fig.1

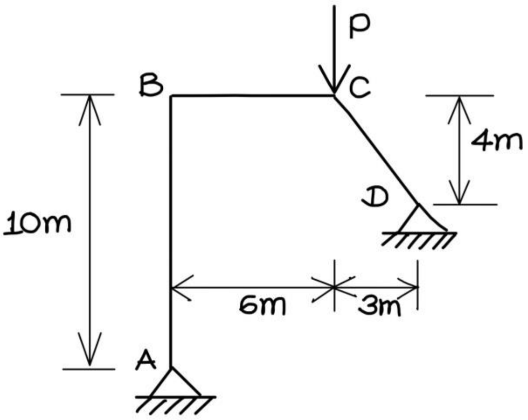


Fig.2

Problem 2 YOU MUST USE ONLY MOMENT DISTRIBUTION METHOD

Refer Fig. 2. For the frame loaded as shown, **determine all support reactions (i.e., forces and moments)** based on two iterations of moment distribution. Use $EI = 4\times10^{13}$ N.mm^2 for all members, and $P = 50$ kN.

Problem 3 YOU MUST USE ONLY STIFFNESS MATRIX METHOD

Refer Fig. 3. Determine all member forces for the truss loaded as shown. Use the joint and member notation as shown. Use $EA = 2\times10^{8}$ N for all members, and $P = 50$ kN.

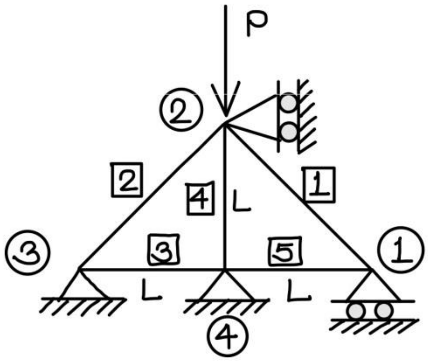
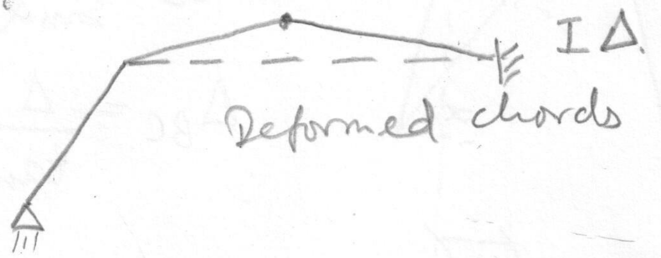


Fig.3

P1 D.o. sway = 1.



$\psi_{AB} = 0$

$\psi_{BC} = -\psi_{CD} = -\frac{\Delta}{L}$

L = 20m

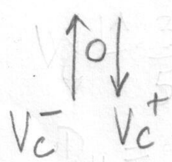
$$M_{BA} = \frac{3EI}{L}\theta_B + \frac{WL^2}{8}$$

$$M_{BC} = \frac{3EI}{L}(\theta_B - \psi_{BC}) = \frac{3EI}{L}\left(\theta_B + \frac{\Delta}{L}\right)$$

$$M_{DC} = \frac{3EI}{L}(-\psi_{DC}) = -\frac{3EI}{L}\frac{\Delta}{L}$$

Equilibrium

$$M_{BA} + M_{BC} = 0 = \frac{6EI}{L}\theta_B + \frac{3EI}{L}\frac{\Delta}{L} + \frac{WL^2}{8} \longrightarrow ①$$

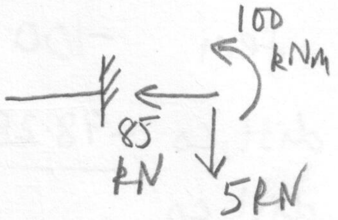


$$V_{C-} = V_{C+} \longrightarrow \frac{M_{BC}}{L} = \frac{M_{DC}}{L} \rightarrow \theta_B = -2\frac{\Delta}{L} \rightarrow ②$$

$$①,② \longrightarrow \frac{\Delta}{L} = \frac{WL^3}{EI}\frac{1}{72}, \quad \theta_B = -\frac{WL^3}{EI}\frac{1}{36} = 0.0333 \text{ rad.}$$

$= 1.91^\circ$

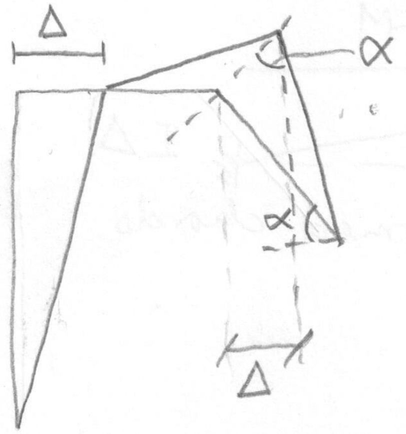
$$M_{DC} = -\frac{WL^2}{24} = -100 \text{ kN.m}$$



$$D_y = \frac{M_{DC}}{L} = \frac{-WL}{24} = -5 \text{ kN}$$

$$D_x = \frac{1}{16}\left(-\frac{WL^2}{2} - M_{DC} + D_y \cdot 52\right) = -85 \text{ kN}$$

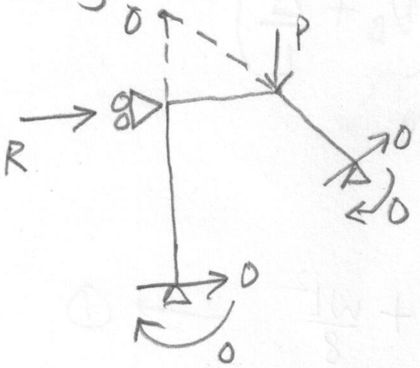
P2



$$\Delta_{CD} = \frac{\Delta}{\sin\alpha} = \frac{5}{4}\Delta$$

$$\Delta_{BC} = \frac{\Delta}{\tan\alpha} = \frac{3}{4}\Delta$$

No sway soln



No Fem's so no moment distribution.

$$\Sigma M_0 = 0 \rightarrow R = \frac{6P}{8} = 37.5$$

BM 1 = 0

Sway soln.

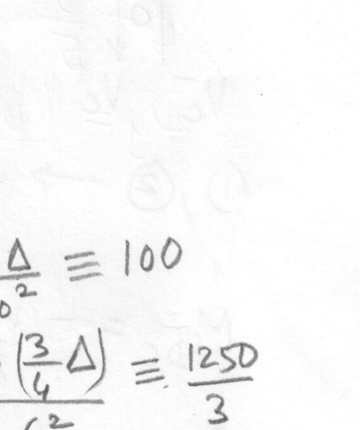
	BA	BC	CB	CD
k	3/10	4/6	4/6	3/5
df	9/29	20/29	10/19	9/19
fem	-100	$\frac{1250}{3}$	$\frac{1250}{3}$	-500
dist,co	-98.28	-218.39 →	-109.20	
dist,co		50.665 ←	101.33	91.20
dist,co	-15.72	-34.94 →	-17.47	
dist,co		4.6 ←	9.20	8.28
	-214	218.60	400.53	-400.52 — BM2

$$\frac{3\Delta}{10^2} \equiv 100$$

$$\frac{6(\frac{3}{4}\Delta)}{6^2} \equiv \frac{1250}{3}$$

$$\frac{3(\frac{5}{4}\Delta)}{5^2} \equiv 500$$

$$V_A = \frac{M_{BA}}{10} = -21.4,\quad V_D = \frac{M_{CD}}{5} = -80.104$$



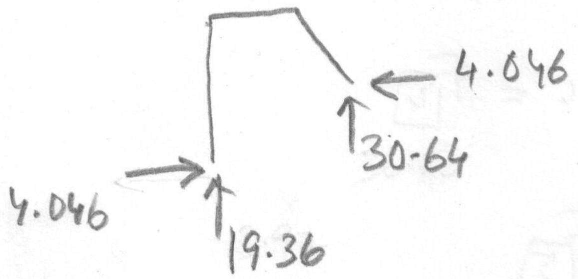
$$\Sigma M_0 = 0 \rightarrow R' = \frac{21.4*18 + 80.104*15}{8} = 198.35$$

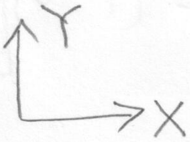
$$BM = BM1 - \frac{R}{R'}.BM2$$

$M_{BA} = 40.46$, and similarly for M_{BC}, M_{CB}, M_{CD}.

$$A_x = \frac{M_{BA}}{10} = 4.046 \Rightarrow D_x = -4.046$$

$$D_y = \frac{P.6 + D_x.6}{9} = 30.64 \Rightarrow A_y = P - D_y = 19.36$$





P.3 $a_{\textcircled{1}\textcircled{2}} = \begin{bmatrix}\frac{1}{\sqrt2} & -\frac{1}{\sqrt2}\end{bmatrix}$, $a_{\textcircled{2}\textcircled{3}} = \begin{bmatrix}\frac{1}{\sqrt2} & \frac{1}{\sqrt2}\end{bmatrix}$

$a_{\textcircled{3}\textcircled{4}} = \begin{bmatrix}-1 & 0\end{bmatrix}$, $a_{\textcircled{1}\textcircled{4}} = \begin{bmatrix}1 & 0\end{bmatrix}$, $a_{\textcircled{2}\textcircled{4}} = \begin{bmatrix}0 & 1\end{bmatrix}$

$$K_{\boxed{1}} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \frac{1}{2\sqrt2} & -\frac{1}{2\sqrt2} & -\frac{1}{2\sqrt2} & \frac{1}{2\sqrt2} \\ & \frac{1}{2\sqrt2} & \frac{1}{2\sqrt2} & -\frac{1}{2\sqrt2} \\ & & \frac{1}{2\sqrt2} & -\frac{1}{2\sqrt2} \\ & & & \frac{1}{2\sqrt2} \end{bmatrix} \end{array} ; \quad K_{\boxed{2}} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \frac{1}{2\sqrt2} & \frac{1}{2\sqrt2} \xrightarrow{} & -ve & \\ & \frac{1}{2\sqrt2} \searrow & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$K_{\boxed{4}} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ & & 0 & 0 \\ & & 0 & 1 \end{bmatrix} \end{array} ; \quad K_{\boxed{5}} = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ & & 1 & 0 \\ & & 0 & 0 \end{bmatrix} \end{array} ; \quad K_{\boxed{3}} \text{ not required}$$

$$K_{II} = \begin{array}{c|cc} & 1 & 4 \\ \hline 1 & \frac{1}{2\sqrt2}+1 & \frac{1}{2\sqrt2} \\ 4 & & \frac{1}{2\sqrt2}+\frac{1}{2\sqrt2}+1 \end{array} \frac{EA}{L.} ; \quad P_I = \begin{array}{c} 1 \\ 4 \end{array} \begin{bmatrix} 0 \\ -P \end{bmatrix} = \begin{bmatrix} 0 \\ -50 \end{bmatrix}$$

$$\Delta_I = K_{II}^{-1} P_I = \frac{L}{EA} \begin{bmatrix} 8.0880 \\ -30.9644 \end{bmatrix}$$

$$F_{①②} = \frac{EA}{L\sqrt{2}}\left\{\begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}8.0880\\0\end{bmatrix}\frac{L}{EA} + \begin{bmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}0\\-30.9644\end{bmatrix}\frac{L}{EA}\right\}$$

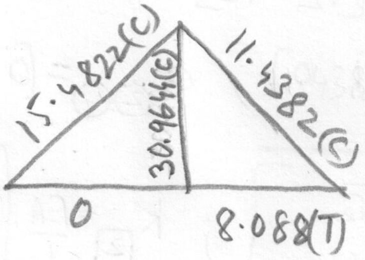
$$= -11.4382 = F_{\boxed{1}}$$

$$F_{②③} = \frac{1}{\sqrt{2}}\left\{\left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right)\begin{bmatrix}0\\-30.9644\end{bmatrix}\right\} = -15.4822 = F_{\boxed{2}}$$

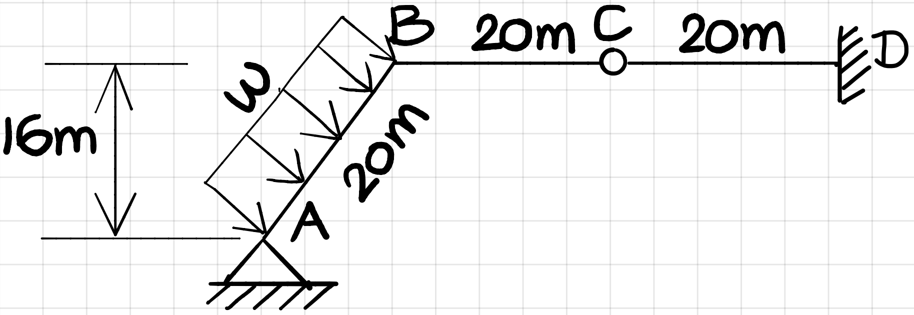
$$F_{③④} = 0 \text{ (no jt displacements)} = F_{\boxed{3}}$$

$$F_{②④} = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}0\\-30.9644\end{bmatrix} = -30.9644 = F_{\boxed{14}}$$

$$F_{①④} = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}8.088\\0\end{bmatrix} = 8.088 = F_{\boxed{5}}$$



①



$$L = 20m, \quad \psi_{AB} = 0, \quad \psi_{BC} = -\psi_{CD} = \psi$$

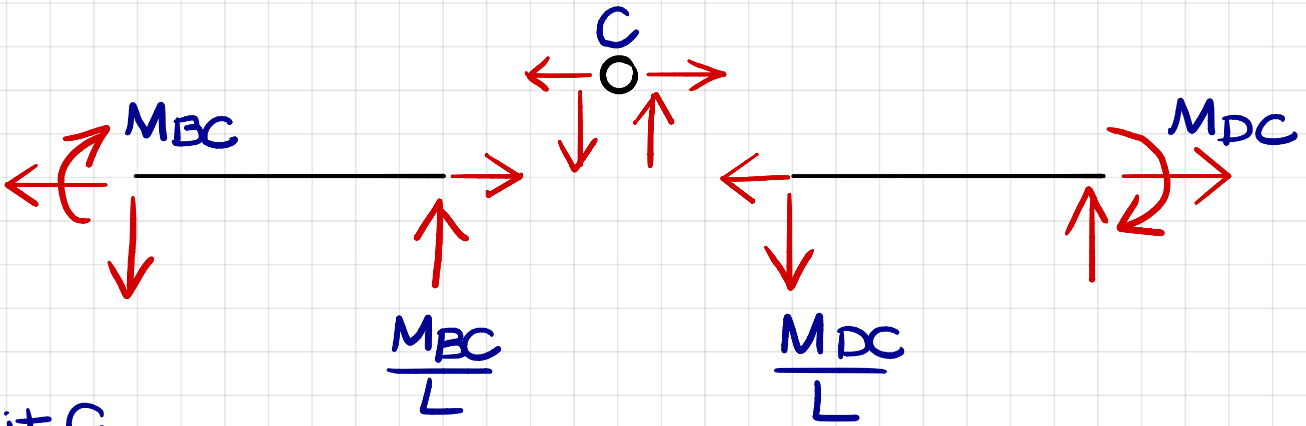
$$M_{BA} = \frac{3EI}{L}\left[\theta_B\right] + \frac{wL^2}{12} + \frac{wL^2}{24} = \frac{3EI}{L}\theta_B + \frac{wL^2}{8}$$

$$M_{BC} = \frac{3EI}{L}\left(\theta_B - \psi\right)$$

$$M_{DC} = \frac{3EI}{L}\psi$$

Jt B

$$M_{BA} + M_{BC} = \frac{3EI}{L}\left(2\theta_B - \psi\right) + \frac{wL^2}{8} = 0 \quad -①$$



jt C

$$\uparrow \Sigma F_y = \frac{M_{BC}}{L} - \frac{M_{DC}}{L} = 0 \quad - ②$$

$$\Rightarrow \theta_B - 2\psi = 0 \Rightarrow \theta_B = 2\psi$$

From ① and ②

$$\frac{3EI}{L}(3\psi) + \frac{wL^2}{8} = 0 \Rightarrow \psi = -\frac{wL^3}{72EI}, \quad \theta_B = -\frac{wL^3}{36EI}$$

@ support D

$$M_{DC} = \frac{3EI}{L}\psi = -\frac{3EI}{L} \times \frac{wL^3}{72EI} = -\frac{wL^2}{24} \circlearrowright$$

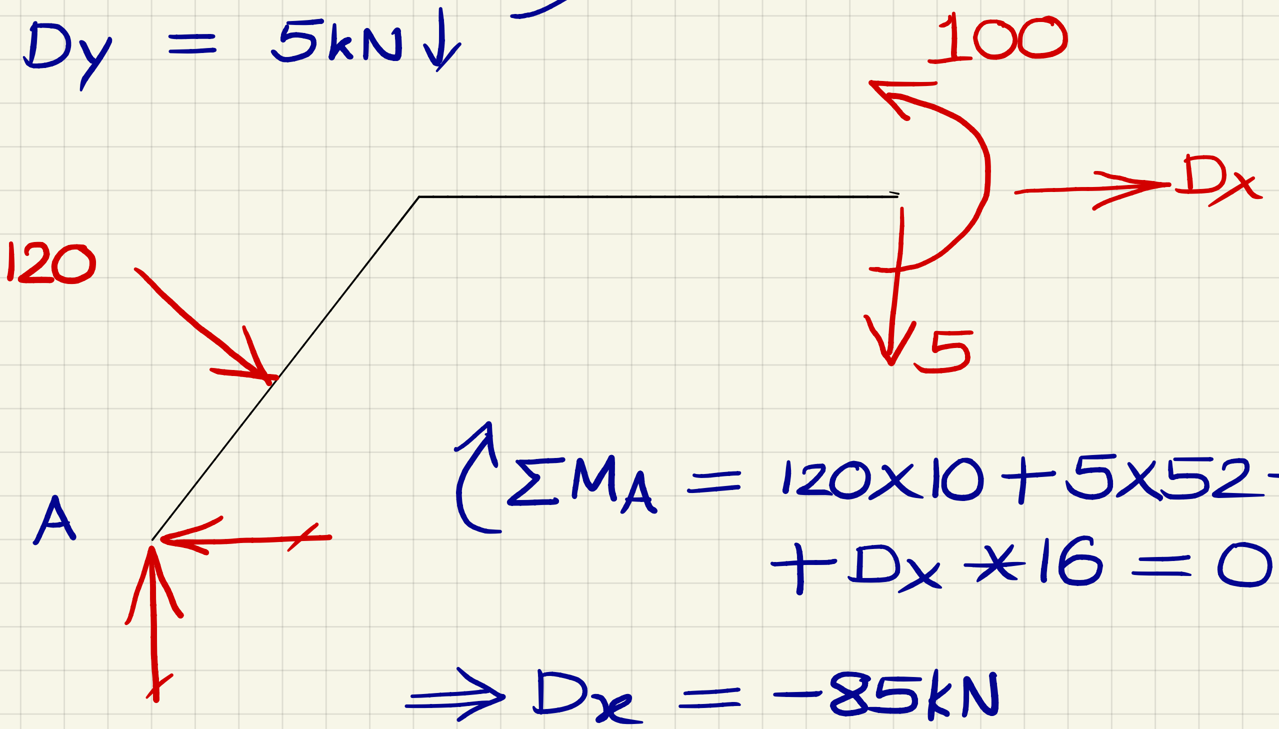
$$V_D = \frac{M_{DC}}{L} = -\frac{wL}{24} \uparrow = D_y$$

Numerical values

$\theta_B = 0.033$ rad

$M_D = 100$ kN·m $\circlearrowleft$

$D_y = 5$ kN ↓

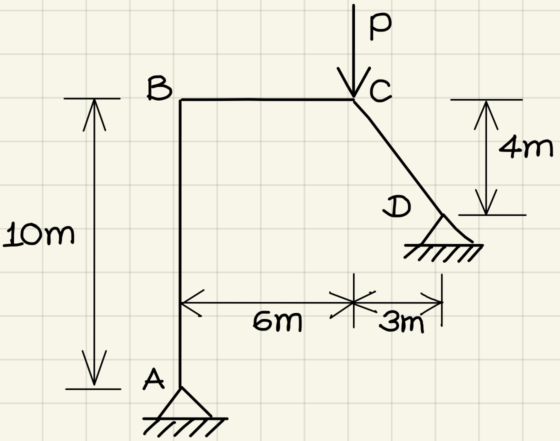


$$\circlearrowleft \Sigma M_A = 120 \times 10 + 5 \times 52 - 100 + D_x * 16 = 0$$

$$\Rightarrow D_x = -85 \text{kN}$$

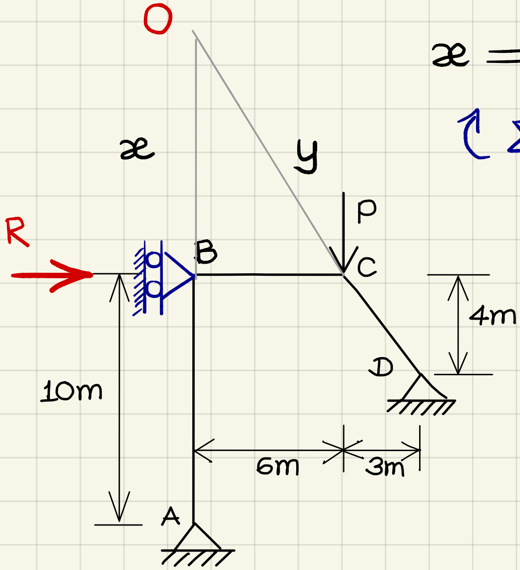
$D_x = 85$ kN ←

②



step-1, No sway

$$\frac{y}{y+5} = \frac{x}{4+x} = \frac{6}{9} = \frac{2}{3} \Rightarrow \frac{x}{4} = 2$$



$$x = 8 \; , \; y = 10$$

$$\circlearrowleft \Sigma M_O = P \times 6 - R \times 8 = 0$$

$$\Rightarrow R = \frac{3}{4} P = \frac{75}{2} \text{ kN}$$

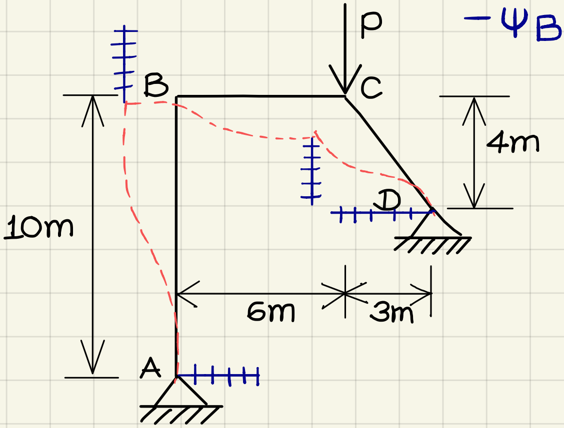
No internal forces

Step-2

$\psi_{CD} = \psi$

$\psi_{AB} \times 10 = \psi_{CD} \times 4$

$-\psi_{BC} \times 6 = \psi_{CD} \times 3$



$\psi_{AB} = \frac{2}{5}\psi$

$\psi_{BC} = -\frac{\psi}{2}$

FEM :

$$M_{CD} = M_{DC} = \frac{6EI}{5}\psi = 100$$

$$M_{BC} = -M_{CB} = -\frac{6EI}{6} \times \frac{\psi}{2} = -\frac{6EI}{5}\psi \times \frac{5}{12}$$

$$= -100 \times \frac{5}{12} = -\frac{125}{3}$$

$$M_{BA} = M_{AB} = \frac{6EI}{10} \times \frac{2}{5}\psi = \frac{6EI}{5}\psi \times \frac{1}{5} = 20$$

Distribution factors :

jt B: $K_{AB} = \frac{3}{4} \times \frac{1}{10}$

$K_{BC} = \frac{1}{6}$

$DF_{AB} = 0.310$

$DF_{BC} = 0.69$

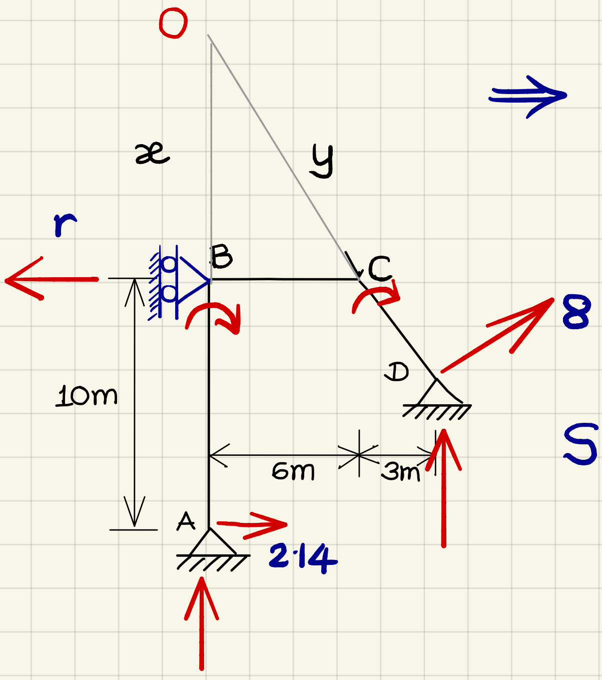
jt C : $K_{BC} = \frac{1}{6}$

$K_{CD} = \frac{3}{4} \times \frac{1}{5}$

$DF_{BC} = 0.526$

$DF_{CD} = 0.474$

A	B	B		C	C	D
1	0.31	0.69		0.526	0.474	1
20	20	-41.67		-41.67	100	100
-20	-10				-50	-100
	9.82	21.85	→	10.93		
		-5.07	←	-10.13	-9.13	
	1.57	3.5	→	1.75		
			←---	-0.92	-0.83	
0	21.39	-21.39		-40.04	40.04	0



$$\circlearrowleft \Sigma M_O = r \times 8 - 8 \times 15 - 2.14 \times 18$$

$$= 0$$

$$\Rightarrow r = 19.81$$

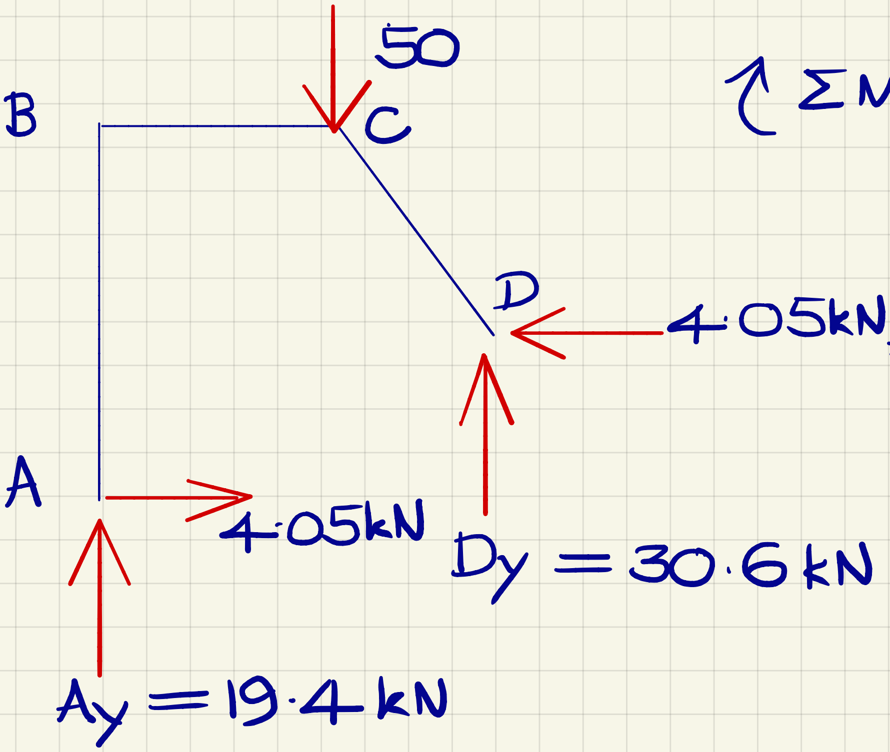
Scaling factor : $S = \frac{37.5}{19.81} = \frac{R}{r}$

$$= 1.89$$

Support reaction at A (shear)

$$A_x = 2.14 \times 1.89 = 4.05 \text{ kN}$$

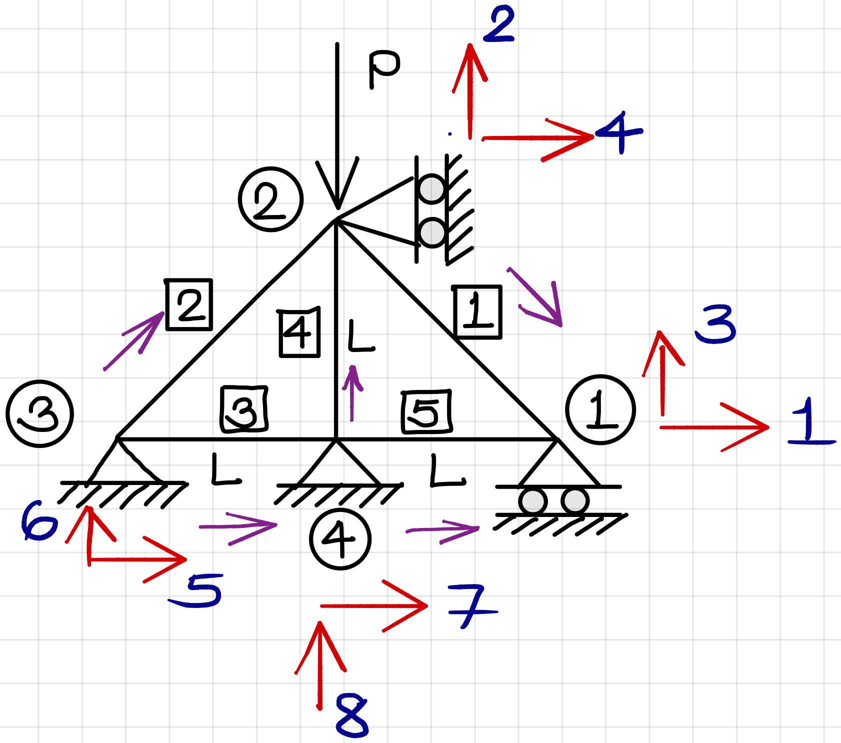
$$D_x = 4.05$$



$$\circlearrowleft \Sigma M_A = 50\times6 - 405\times6 - D_y\times9 = 0$$

$$\Rightarrow D_y = 30.6\,\text{kN}$$

③



Member-1, $\lambda_x = \frac{1}{\sqrt{2}}$, $\lambda_y = \frac{-1}{\sqrt{2}}$

$$k_1 = \frac{AE}{2\sqrt{2}L}\begin{array}{c} \begin{array}{cccc} \underline{1} & 3 & 4 & \underline{2} \end{array} \\ \left[\begin{array}{cccc} \textcircled{1} & -1 & -1 & \textcircled{1} \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ \textcircled{1} & -1 & -1 & \textcircled{1} \end{array}\right] \end{array}\begin{array}{c} \\ \underline{1} \\ 3 \\ 4 \\ \underline{2} \end{array}$$

Member 2 : $\lambda_x = 1/\sqrt{2}$, $\lambda_y = 1/\sqrt{2}$

$$k_2 = \frac{AE}{2\sqrt{2}L}\begin{array}{c} \begin{array}{cccc} 4 & \underline{2} & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} & & & \\ & \textcircled{1} & & \\ & & & \\ & & & \end{array}\right] \end{array}\begin{array}{c} \\ 4 \\ \underline{2} \\ 5 \\ 6 \end{array}$$

Member-5, $\lambda_x = 1$, $\lambda_y = 0$

$$k_5 = \frac{AE}{L}\begin{array}{c} \begin{array}{cccc} \underline{1} & 3 & 7 & 8 \end{array} \\ \left[\begin{array}{cccc} \textcircled{1} & & & \\ & & & \\ & & & \\ & & & \end{array}\right] \end{array}\begin{array}{c} \\ \underline{1} \\ 3 \\ 7 \\ 8 \end{array}$$

Member-4

$\lambda_x = 0$, $\lambda_y = 1$

$$k_4 = \frac{AE}{L}\begin{array}{c} \begin{array}{cccc} 4 & \underline{2} & 7 & 8 \end{array} \\ \left[\begin{array}{cccc} & & & \\ & \textcircled{1} & & \\ & & & \\ & & & \end{array}\right] \end{array}\begin{array}{c} \\ 4 \\ \underline{2} \\ 7 \\ 8 \end{array}$$

$$[k_{11}] = \frac{AE}{L}\begin{array}{cc} \begin{array}{cc} \quad 1 \quad & \quad 2 \quad \end{array} & \\ \begin{bmatrix} \frac{1}{2\sqrt{2}}+1 & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 2\times\frac{1}{2\sqrt{2}}+1 \end{bmatrix} & \begin{array}{c} 1 \\ \\ 2 \end{array} \end{array}$$

$$= \frac{AE}{L}\begin{array}{cc} \begin{array}{cc} \quad 1 \quad & \quad 2 \quad \end{array} & \\ \begin{bmatrix} 1.354 & 0.354 \\ 0.354 & 1.707 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

$$\{Q_k\} = \begin{Bmatrix} 0 \\ -50 \end{Bmatrix} , \quad \{D_u\} = \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix}$$

$$\{Q_k\} = [k_{11}]\{D_u\}$$

$$\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \frac{L}{AE}\begin{Bmatrix} 8.088 \\ -30.96 \end{Bmatrix}$$

Force in members :

$$① \quad q^1 = \frac{AE}{2L}\langle -1 \quad 1 \quad 1 \quad -1 \rangle \begin{array}{cc} \begin{Bmatrix} D_4 \\ D_2 \\ D_1 \\ D_3 \end{Bmatrix} & \begin{array}{c} N_x \\ N_y \\ F_x \\ F_y \end{array} \end{array}$$

$$= \frac{1}{2}(-30.96 + 8.088) = -11.44\,\text{kN} \quad (C)$$

$$② \quad q^2 = \frac{AE}{2L}\langle -1 \quad -1 \quad 1 \quad 1 \rangle \begin{Bmatrix} 0 \\ 0 \\ 0 \\ D_2 \end{Bmatrix} = -\frac{30.96}{2} = -15.48\,\text{kN} \; (C)$$

$$q^3 = 0$$

$$q^4 = \frac{AE}{L}\langle 0 \quad -1 \quad 0 \quad 1 \rangle \begin{Bmatrix} 0 \\ 0 \\ 0 \\ D_2 \end{Bmatrix} = -30.96\,kN(C)$$

$$q^5 = \frac{AE}{L}\langle -1 \quad 0 \quad 1 \quad 0 \rangle \begin{Bmatrix} 0 \\ 0 \\ D_1 \\ 0 \end{Bmatrix} = 8.09\,kN\,(T)$$