DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

**CE-317 STRUCTURAL MECHANICS II**

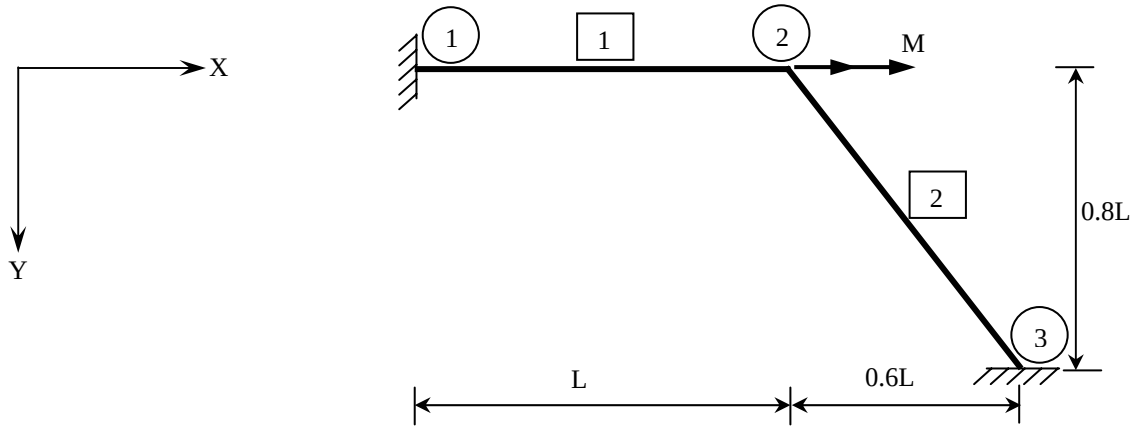
**Quiz-2 19/10/11**

**Read these instructions applicable to both problems**

1. For both problems you only have to find the numerical values of the stiffness matrix $\mathbf{K}_{II}$ (ie., roman one roman one) and the load vector $\mathbf{P}_I$ (ie., roman one) that are required to solve for the displacements.
2. You do not need to invert $\mathbf{K}_{II}$ and do not need to solve for displacements using $\mathbf{K}_{II}^{-1}\mathbf{P}_I$
3. Settlement must be handled by including it in load vector $\mathbf{P}_I$ .
4. Numerical answers must be accurate to 2 or more places after decimal.
5. Must use member end (local) coordinate system as done in class. Must use convention for all forces and displacements (linear and angular) as in class. Must use numbering sequence of structure's nodal forces and displacements as in class.
6. All data provided in units of N, m.
7. In each problem, all members have same geometric and physical properties.

**Problem 1**

Data: $EI_y = EI_z = 6,\quad A/I_y = 10,\quad L = 6,\quad GJ/EI_y = 0.25,\quad M = 1$ . $M$ is a torque applied.



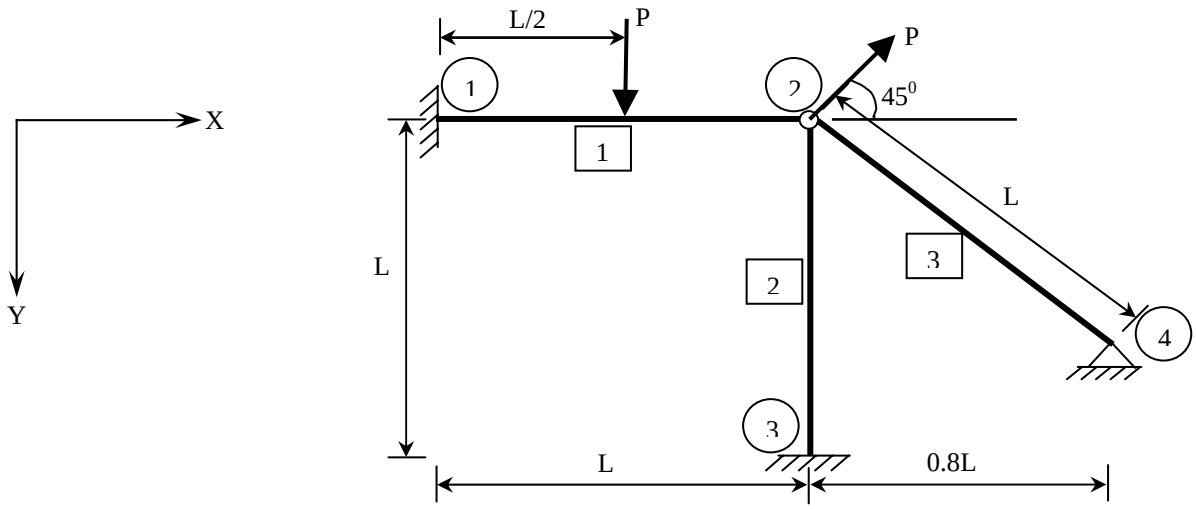

**Problem 2**

Data: $EI_y = EI_z = 6,\quad A/I_y = 10,\quad L = 6,\quad GJ/EI_y = 0.25,\quad P = 2.$

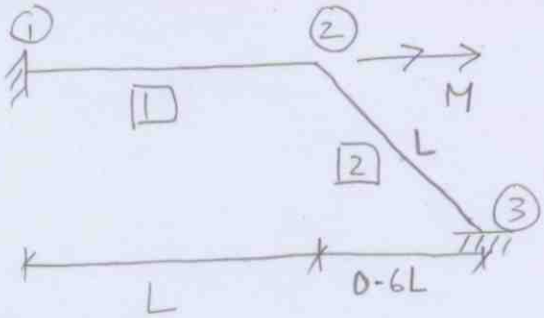
Settlement of support node 4 is 0.1m downward.

Misfit of member 3 is 0.04072m too short.

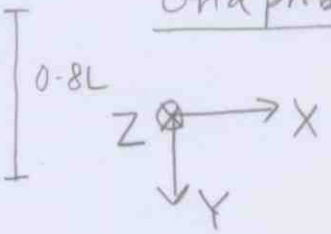
Member 3 is heated 10 $^{o}$C above ambient temperature, coefficient of linear expansion is 0.000012/ $^{o}$C.



(1)




Grid problem




$$a_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad a_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.6 & -0.8 \\ 0 & 0.8 & -0.6 \end{bmatrix}$$

For displacements need only $K_{22}$

$$K'_{22} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \frac{EI}{L} \begin{bmatrix} 12/L^2 & 0 & 6/L \\ 0 & 0.25 & 0 \\ 6/L & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} 12/L^2 & 0 & 6/L \\ 0 & 0.25 & 0 \\ 6/L & 0 & 4 \end{bmatrix}$$

$$K^3_{22} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.6 & 0.8 \\ 0 & -0.8 & -0.6 \end{bmatrix} \frac{EI}{L} \begin{bmatrix} 12/L^2 & 0 & 6/L \\ 0 & 0.25 & 0 \\ 6/L & 0 & 4 \end{bmatrix} a_{23} = \frac{EI}{L} \begin{bmatrix} 12/L^2 & 0 & 6/L \\ 4.8/L & -0.15 & 3.2 \\ -3.6/L & -0.2 & -2.4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.6 & -0.8 \\ 0 & 0.8 & -0.6 \end{bmatrix}$$

$$= \frac{EI}{L} \begin{bmatrix} 12/L^2 & 4.8/L & -3.6/L \\ 4.8/L & 2.65 & -1.8 \\ -3.6/L & -1.8 & 1.6 \end{bmatrix}$$

$$K_{II} = K_{22} = K'_{22} + K^3_{22} = \frac{EI}{L} \begin{bmatrix} 24/L^2 & 4.8/L & 2.4/L \\ 4.8/L & 2.9 & -1.8 \\ 2.4/L & -1.8 & 5.6 \end{bmatrix} \quad (EI/L = 1) = \begin{bmatrix} 2/3 & 0.8 & 0.4 \\ 0.8 & 2.9 & -1.8 \\ 0.4 & -1.8 & 5.6 \end{bmatrix}$$ ◀

$$P_I = \{0 \;\; M \;\; 0\}^T = \{0 \;\; 1 \;\; 0\}^T$$ ◀

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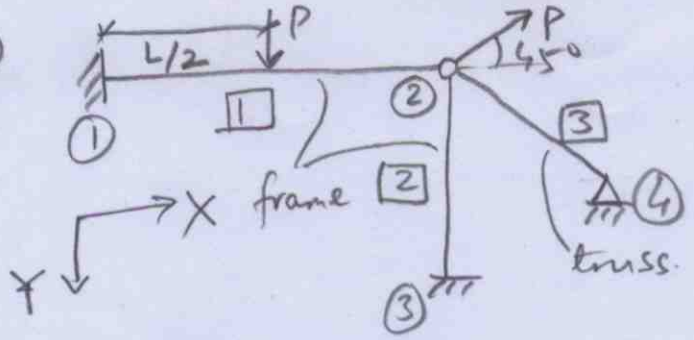
(Problem #2 contd from pg. 2) ×

$F_{fs24} = k^4_{22}\,\delta_{s/24} + k_{24}\,\delta_{s42} = 10(-0.00012 * 10 * 6 + 0.6 * 0.1 + 0.04072)$ ($\delta_{s/24} = 0$)

$= 1 \rightarrow$ self straining member end fef's.

$$P_{es2} = \begin{Bmatrix} -a^T_{24} F_{fs24} \\ 0 \\ 0 \end{Bmatrix} = \{0.8 \;\; 0.6 \;\; 0 \;\; 0\}^T \rightarrow \text{self straining equivalent nodal loads.}$$

$$P_I = P_a + P_{eM} + P_{es} = \{2.214 \;\; 0.186 \;\; -1.5 \;\; 0\}^T$$ ◀




(2)

Frame + truss elements, with load between nodes, temp, misfit, settlement.

$$a_{21}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad a_{23}=\begin{bmatrix}0&-1&0\\1&0&0\\0&0&1\end{bmatrix},\quad a_{24}=\begin{bmatrix}-0.8&-0.6\end{bmatrix}.$$

$$k^1_{22}=k^3_{22}=\frac{EI'}{L}\begin{bmatrix}A/I&0&0\\0&12/L^2&-6/L\\0&-6/L&4\end{bmatrix}=1.\begin{bmatrix}10&0&0\\0&1/3&-1\\0&-1&4\end{bmatrix}$$

$1\ 2\ 3 \rightarrow$ structure's dof's (global).

$$k^4_{22}=\left[\frac{EA}{L}\right]=[10]=k_{24}$$

$$K^1_{22}=a_{21}^T\,k^1_{22}\,a_{21}=k^1_{22}=\begin{bmatrix}10&0&0\\0&1/3&-1\\0&-1&4\end{bmatrix}$$

$$K^2_{22}=a_{23}^T\,k^3_{22}\,a_{23}=\begin{bmatrix}0&1/3&-1\\-10&0&0\\0&-1&4\end{bmatrix}\begin{bmatrix}0&-1&0\\1&0&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1/3&0&-1\\0&10&0\\-1&0&4\end{bmatrix}$$

$1\ \ 2\ \ 4$

$$K^4_{22}=\begin{bmatrix}-0.8\\-0.6\end{bmatrix}\begin{bmatrix}-0.8&-0.6\end{bmatrix}*10=\begin{bmatrix}6.4&4.8\\4.8&3.6\end{bmatrix}$$

$1\ \ 2$

$$K_{II}=K_{22}=\begin{bmatrix}10+\frac{1}{3}+6.4&0+4.8&0&-1\\0+4.8&\frac{1}{3}+10+3.6&-1&0\\0&-1&4&0\\-1&0&0&4\end{bmatrix}=\begin{bmatrix}16.733&4.8&0&-1\\4.8&13.933&-1&0\\0&-1&4&0\\-1&0&0&4\end{bmatrix}$$

**Loads:**

$P_a=[P/\sqrt{2}\quad -P/\sqrt{2}\quad 0\quad 0]^T$ → applied at nodes

$P_{EM}=[0\quad P/2\quad -PL/8\quad 0]^T$ → due to mechanical loads between nodes.

$\delta_{S4}=\delta_{ST4}+\delta_{SSt4}+\delta_{SM4}$ → self straining member end displs.

- $\delta_{S4}$ → local displ of node 4
- $\delta_{ST4}$ → due to temp → $-\alpha\Delta TL$ → $-0.000012*10*6$
- $\delta_{SSt4}$ → settlement → $+a_{42}\begin{Bmatrix}0\\0.1\end{Bmatrix}$ → $+0.6*0.1$
- $\delta_{SM4}$ → misfit → $0.04072$

(contd on pg. 1)