DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE-317 STRUCTURAL MECHANICS II

Quiz-2 29/10/12

Read these instructions applicable to both problems

1. For both problems you have to solve for the displacements.
2. Settlement must be handled by including it in load vector $\mathbf{P_I}$.
3. Must use member end (local) coordinate system as done in class. Must use convention for all forces and displacements (linear and angular) as in class. Must use numbering sequence of structure's nodal forces and displacements as in class. Must use **global coordinate system provided with the problem.**
4. All data provided in units of N, m, radian.
5. In each problem, all members have same geometric and physical properties.

Problem 1

Data: $EI_y = EI_z = 9$, $A/I_y = 2/3$, $L = 3$, $GJ/EI_y = 0.5$, $M = 2$. M is a torque applied.

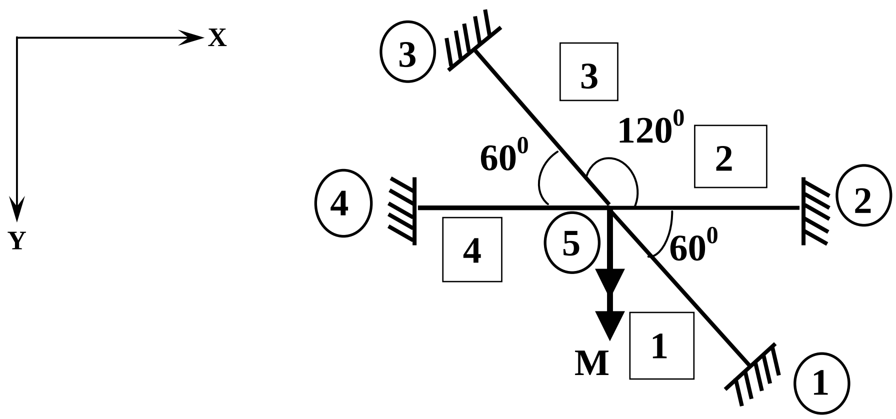


Fig. P1

Problem 2

Data: $EI_y = EI_z = 9$, $A/I_y = 2/3$, $L = 3$, $GJ/EI_y = 0.5$, $P = 2$.

Rotational spring constant = 6

Settlement of support node 2: 0.2m rightward.

Misfit of member 1: 0.2 radians counterclockwise.

Member 3 is heated 20 ^oC above ambient temperature, coefficient of linear expansion is 0.01/^oC.

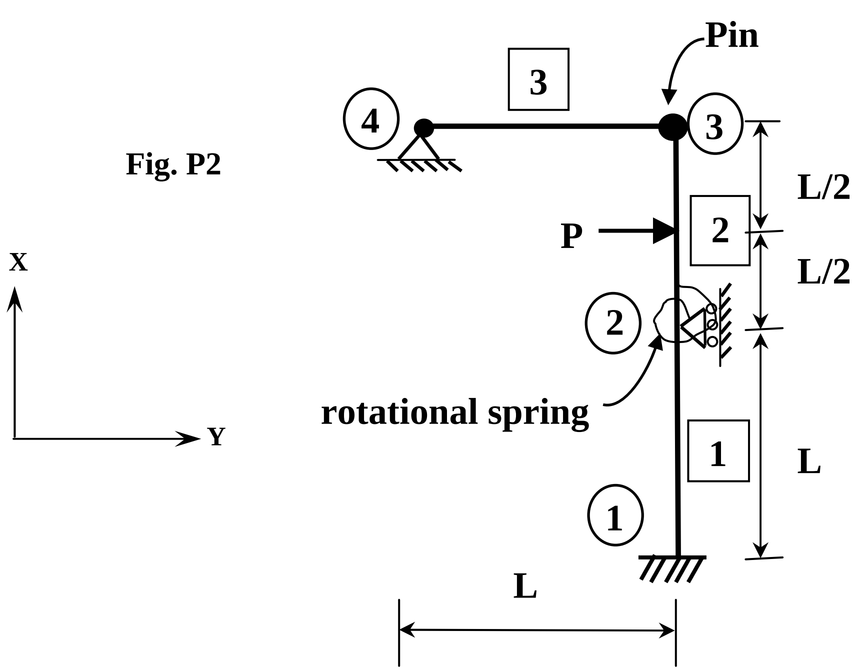


Fig. P2

P1 Grid problem

$$K_{II} = K_{55}$$

$$a_{51} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & -1/2 \end{bmatrix}, \quad a_{52} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$a_{53} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \boxed{-ve} & \\ 0 & & \end{bmatrix}, \quad a_{54} = I$$

$$k^1_{55} = k^2_{55} = k^3_{55} = k^4_{55} = \frac{EI_y}{L} \begin{bmatrix} 12/L^2 & 0 & 6/L \\ 0 & GJ_x/EI_y & 0 \\ 6/L & 0 & 4 \end{bmatrix}$$

$$= \frac{9}{3} \begin{bmatrix} 12/9 & 0 & 6/3 \\ 0 & 1/2 & 0 \\ 6/3 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 6 \\ 0 & 3/2 & 0 \\ 6 & 0 & 12 \end{bmatrix}$$

$$a^T_{51} k^1_{55} a_{51} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & -1/2 \end{bmatrix} \begin{bmatrix} 4 & 3\sqrt{3} & -3 \\ 0 & -3/4 & -3\sqrt{3}/4 \\ 6 & 6\sqrt{3} & -6 \end{bmatrix} = \begin{bmatrix} 4 & 3\sqrt{3} & -3 \\ 3\sqrt{3} & 75/8 & -\frac{21}{8}\sqrt{3} \\ -3 & -\frac{21}{8}\sqrt{3} & 33/8 \end{bmatrix}$$

$$a^T_{52} k^2_{55} a_{52} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 & -6 \\ 0 & -3/2 & 0 \\ 6 & 0 & -12 \end{bmatrix} = \begin{bmatrix} 4 & 0 & -6 \\ 0 & 3/2 & 0 \\ -6 & 0 & 12 \end{bmatrix}$$

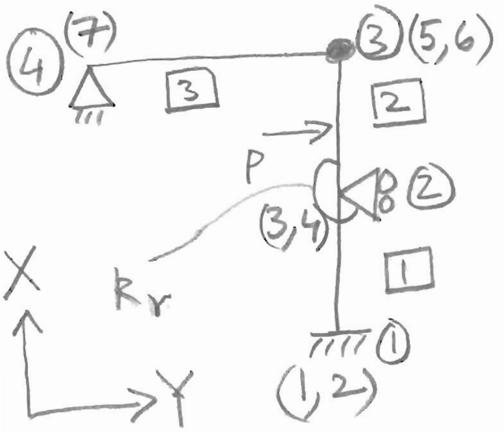
$$a^T_{53} k^3_{55} a_{53} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 4 & -3\sqrt{3} & 3 \\ 0 & 3/4 & 3\sqrt{3}/4 \\ 6 & -6\sqrt{3} & 6 \end{bmatrix} = \begin{bmatrix} 4 & -3\sqrt{3} & 3 \\ -3\sqrt{3} & 75/8 & -\frac{21}{8}\sqrt{3} \\ 3 & -\frac{21}{8}\sqrt{3} & \frac{33}{8} \end{bmatrix}$$

$$a^T_{54} k^3_{55} a_{54} = k^3_{55}$$

$$K_{55} = K_{II} = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 21.75 & -9.093 \\ 0 & -9.093 & 32.25 \end{bmatrix} ; \quad P_I = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}^T$$

$$\Delta_I = \left[(\Delta_5)_3 \;\; (\Delta_5)_4 \;\; (\Delta_5)_5 \right]^T = K^{-1}_{II} P_I = \begin{bmatrix} 0 & 0.0294 & 0.0703 \end{bmatrix}^T$$

P2 $\because$ temp increase is constant thru thickness of $\boxed{3}$, and $\boxed{3}$ has no mech loads, it acts as a truss element. Further, since $\boxed{3}$ is horizontal, it transfers only horz force at node 3. Then since $\boxed{1}$, $\boxed{2}$ have only transverse loads applied, they act as beam elements.



So we need rows/cols (4, 5, 6) for K_{II}.

$$K = \begin{array}{c|c} & \begin{matrix} 1,2 & 3,4 & 5,6 & 7 \end{matrix} \\ \begin{matrix} 1,2 \\ 3,4 \\ 5,6 \\ 7 \end{matrix} & \begin{bmatrix} K_{11} & K_{12} & 0 & 0 \\ K_{21} & K_{22} & K_{23} & 0 \\ 0 & K_{32} & K_{33} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} + \begin{array}{c|c} & \begin{matrix} 1,2 & 3,4 & 5,6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} & & & \\ & & & \\ & k_r & & \\ & \text{rest are zeros} & & \\ & & & \end{bmatrix} \end{array}$$

$K_{\boxed{1}}$, $K_{\boxed{2}}$

elements $\boxed{1}$, $\boxed{2}$ — Rot spring element

$$+ \begin{array}{c|c} & \begin{matrix} 1,2 & 3,4 & 5,6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} & \text{rest all zeros} & & \\ & & & \\ & & \times & \times \\ & & & \\ & & \times & \times \end{bmatrix} \end{array}$$

$\times = AE/L$.

$$\Rightarrow K_{II} = \begin{array}{c|c} & \begin{matrix} 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} \left(K_{22}(2,2) + k_r\right) & K_{23}(2,1) & K_{23}(2,2) \\ K_{32}(1,2) & \left(K_{33}(1,1) + \frac{AE}{L}\right) & K_{33}(1,2) \\ K_{32}(2,2) & K_{33}(2,1) & K_{33}(2,2) \end{bmatrix} \end{array}$$

$$a_{21} = a_{32} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad a_{23} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$k^{1}_{22} = k^{3}_{22} = k^{2}_{33} = \frac{EI_z}{L}\begin{bmatrix} 12/L^2 & -6/L \\ -6/L & 4 \end{bmatrix} = \frac{9}{3}\begin{bmatrix} 12/9 & -6/3 \\ -6/3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix}$$

$$k_{21} = k_{23} = k_{32} = \begin{bmatrix} 4 & -6 \\ -6 & 6 \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix} + \begin{bmatrix} +4 & 6 \\ +6 & 12 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 24 \end{bmatrix}, \quad K_{33} = \begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix}$$

$$K_{23} = \begin{bmatrix} -4 & 6 \\ -6 & 6 \end{bmatrix}, \quad K_{32} = \begin{bmatrix} -4 & -6 \\ 6 & 6 \end{bmatrix}$$

$$K_{II} = \begin{bmatrix} (24+6) & -6 & 6 \\ -6 & \left(4 + \frac{2}{3}\cdot\frac{9}{3}\right) & -6 \\ 6 & -6 & 12 \end{bmatrix} = \begin{bmatrix} 30 & -6 & 6 \\ -6 & 6 & -6 \\ 6 & -6 & 12 \end{bmatrix} \blacktriangleleft$$

$$(P_I)_{mech} = \begin{Bmatrix} PL/8 \\ P/2 \\ -PL/8 \end{Bmatrix} = \begin{Bmatrix} 0.75 \\ 1 \\ -0.75 \end{Bmatrix} \qquad (P_a = 0)$$

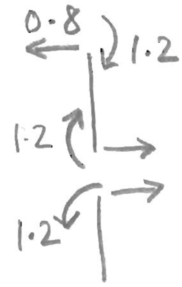
$$(P_I)_{temp} = \begin{Bmatrix} 0 \\ \alpha T L \frac{AE}{L} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ (0.01)\cdot 20 \cdot \frac{2}{3}\cdot 9 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1.2 \\ 0 \end{Bmatrix}$$

$$(F_{32})_{settle} = \begin{bmatrix} 4 & -6 \\ -6 & 6 \end{bmatrix}\begin{Bmatrix} -0.2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0.8 \\ 1.2 \end{Bmatrix}$$

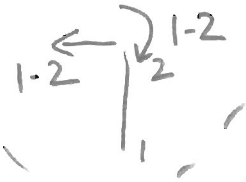
$$(F_{23})_{settle} = \begin{Bmatrix} 4 & -6 \\ -6 & 12 \end{Bmatrix}\begin{Bmatrix} -0.2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0.8 \\ 1.2 \end{Bmatrix}$$

$$(F_{21})_{settle} = \begin{Bmatrix} 4 & -6 \\ -6 & 12 \end{Bmatrix}\begin{Bmatrix} 0.2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.8 \\ -1.2 \end{Bmatrix}$$

$$\Rightarrow (P_I)_{settle} = \begin{Bmatrix} 0 \\ 0.8 \\ -1.2 \end{Bmatrix}$$



$$(F_{21})_{misfit} = \begin{bmatrix} 4 & -6 \\ -6 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ -(-0.2) \end{Bmatrix} = \begin{Bmatrix} -1.2 \\ 1.2 \end{Bmatrix} \Rightarrow (P_I)_{misfit} = \begin{Bmatrix} -1.2 \\ 0 \\ 0 \end{Bmatrix}$$



$$\Rightarrow P_I = (P_I)_{mech} + (P_I)_{temp} + (P_I)_{settle} + (P_I)_{misfit}$$

$$P_I = \begin{Bmatrix} -0.45 \\ 3 \\ -1.95 \end{Bmatrix} \blacktriangleleft$$

$$\Delta_I = K_{II}^{-1} P_I = \begin{Bmatrix} 0.1062 \\ 0.7813 \\ 0.1750 \end{Bmatrix}$$