DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Quiz-2 19/10/13 PAPER CODE: A

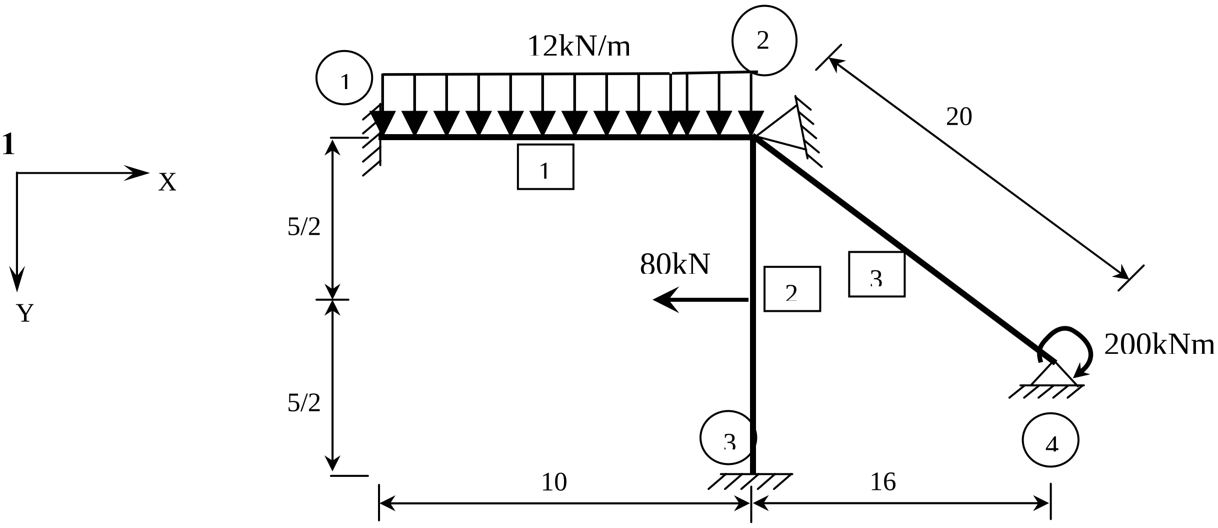
Note: Write your name & roll no. on answerbook and on summary answer sheet provided on the reverse.
You must submit the question-paper-cum-summary-answer-sheet along with the answerbook.
Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.
Both questions carry equal marks. All answers should be upto at least three significant digits.
Must use only Stiffness Matrix Method in both questions.
Must use global coordinate system provided with the problem.

Problem 1

For this question, settlement must be handled only through self straining by including it in the load vector $\mathbf{P}_{\mathbf{I}}$. Thus, settlement must not be handled through $\mathbf{\Delta}_{\mathbf{II}}$ term for this question.

Consider the 3-member **rigid-jointed** frame in **Fig. 1.** It is fixed supported at joint-1 and joint-3 and pin supported at joint-2 and joint-4. Note that all members are rigidly welded to each other at joint 2 which is then pin-supported.
The support at joint-4 settles 0.1m downward.
Member-3 is 0.05m too long (i.e., misfit). It is heated/cooled such that $T_L = 50^0\text{C}, T_U = -50^0\text{C}$, both measured with respect to the ambient temperature.
For all members use data:
$EI = 10^5\,\text{kNm}^2;\ GJ = 0.16*10^5\,\text{kNm}^2;\ A/I = 300\text{m}^{-2};\ h = 0.1\text{m} = \text{depth of member};\ \alpha = 0.000012\,/^0\text{C}$
Find: (i) $\mathbf{K}_{\mathbf{II}}$; (ii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector) ; (iii) $\Delta_{\mathbf{I}}$ (joint displacement vector)

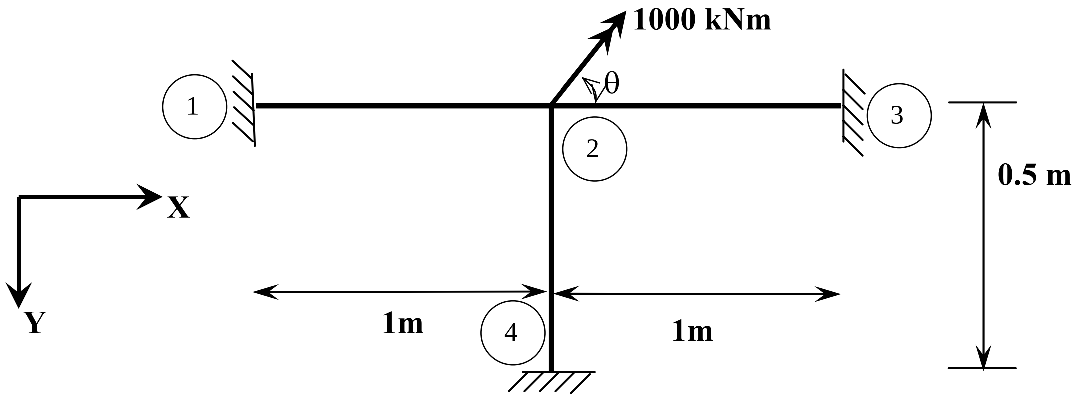
Fig. 1



Problem 2

The plane frame has a torque applied in the XY plane, at an angle $\theta = \cos^{-1}(0.8)$ as shown.
Use data: $EI = 10^5\,\text{kNm}^2;\ GJ = 0.16*10^5\,\text{kNm}^2;\ A/I = 300\text{m}^{-2}$,
Find: (i) $\mathbf{K}_{\mathbf{II}}$; (ii) $\mathbf{P}_{\mathbf{I}}$ (joint load vector) ; (iii) $\Delta_{\mathbf{I}}$ (joint displacement vector)

Fig. 2



SUMMARY ANSWER SHEET

PAPER CODE: A

Name:

Roll no:

Problem 1

<u>(i)</u> $\mathbf{K_{II}}$

(ii) $\mathbf{P_I} =$

(iii) $\Delta_{\mathbf{I}} =$

Problem 2

<u>(ii)</u> $\mathbf{K_{II}}$

(ii) $\mathbf{P_I} =$

(iii) $\Delta_{\mathbf{I}} =$

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Quiz-2 19/10/13 PAPER CODE: B

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Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.

Both questions carry equal marks. All answers should be upto at least three significant digits.

Must use only Stiffness Matrix Method in both questions.

Must use global coordinate system provided with the problem.

Problem 1

For this question, settlement must be handled only through self straining by including it in the load vector $\mathbf{P_I}$. Thus, settlement must not be handled through Δ_{II} term for this question.

Consider the 3-member **rigid-jointed** frame in **Fig. 1.** It is fixed supported at joint-1 and joint-3 and pin supported at joint-2 and joint-4. Note that all members are rigidly welded to each other at joint 2 which is then pin-supported.

The support at joint-4 settles 0.05m downward.

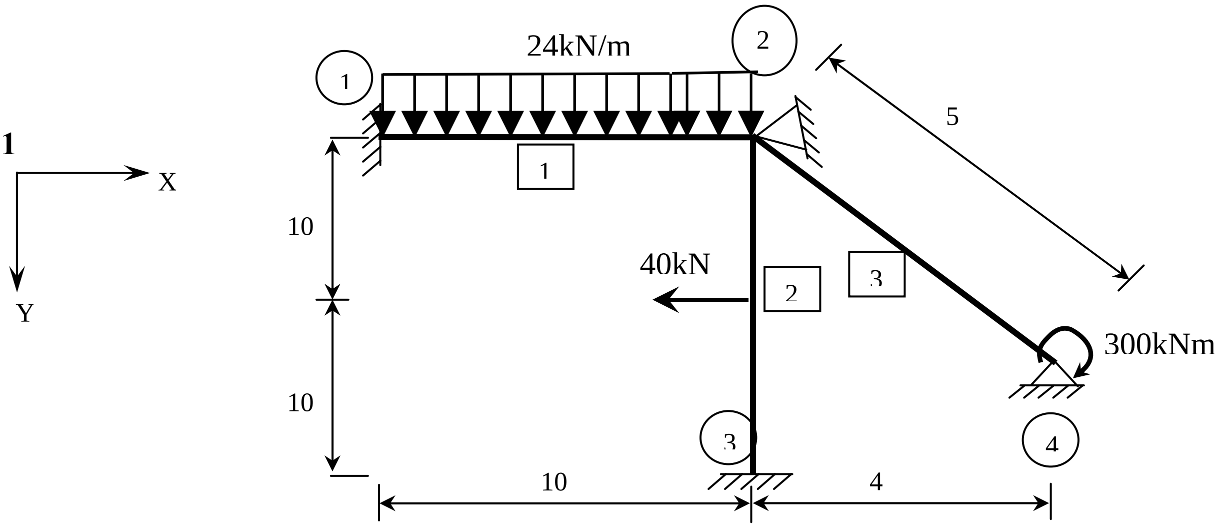
Member-3 is 0.1m too long (i.e., misfit). It is heated/cooled such that $T_L = 25^0\text{C}$, $T_U = -25^0\text{C}$, both measured with respect to the ambient temperature.

For all members use data:

$EI = 10^5\,\text{kNm}^2$; $GJ = 0.16*10^5\,\text{kNm}^2$; $A/I = 300\text{m}^{-2}$; $h = 0.1\text{m} = \text{depth of member}$; $\alpha = 0.000012/^0\text{C}$

Find: (i) $\mathbf{K_{II}}$; (ii) $\mathbf{P_I}$ (joint load vector) ; (iii) $\Delta_{\mathbf{I}}$ (joint displacement vector)

Fig. 1



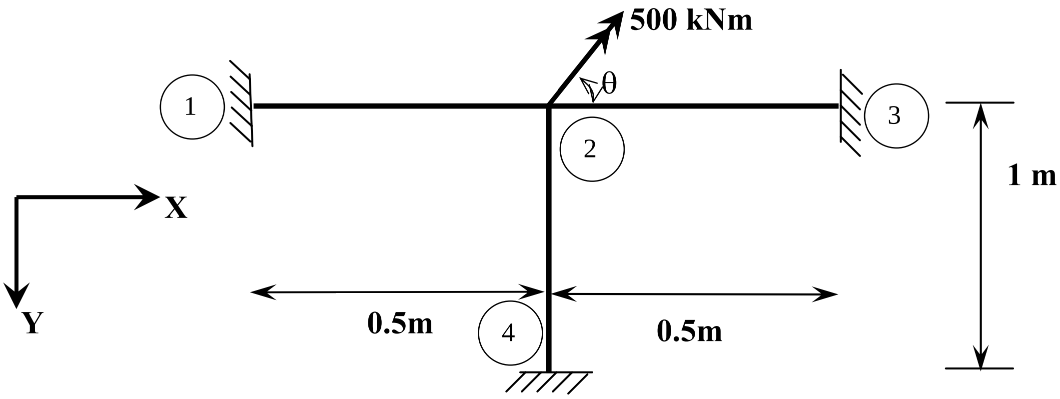
Problem 2

The plane frame has a torque applied in the XY plane, at an angle $\theta = \cos^{-1}(0.6)$ as shown.

Use data: $EI = 10^5\,\text{kNm}^2$; $GJ = 0.16*10^5\,\text{kNm}^2$; $A/I = 300\text{m}^{-2}$,

Find: (i) $\mathbf{K_{II}}$; (ii) $\mathbf{P_I}$ (joint load vector) ; (iii) $\Delta_{\mathbf{I}}$ (joint displacement vector)

Fig. 2



SUMMARY ANSWER SHEET

PAPER CODE: B

Name:

Roll no:

Problem 1

<u>(iii)</u> $\mathbf{K_{II}}$

(ii) $\mathbf{P_I} =$

(iii) $\Delta_I =$

Problem 2

<u>(iv)</u> $\mathbf{K_{II}}$

(ii) $\mathbf{P_I} =$

(iii) $\Delta_I =$

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Quiz-2   19/10/13   PAPER CODE: C

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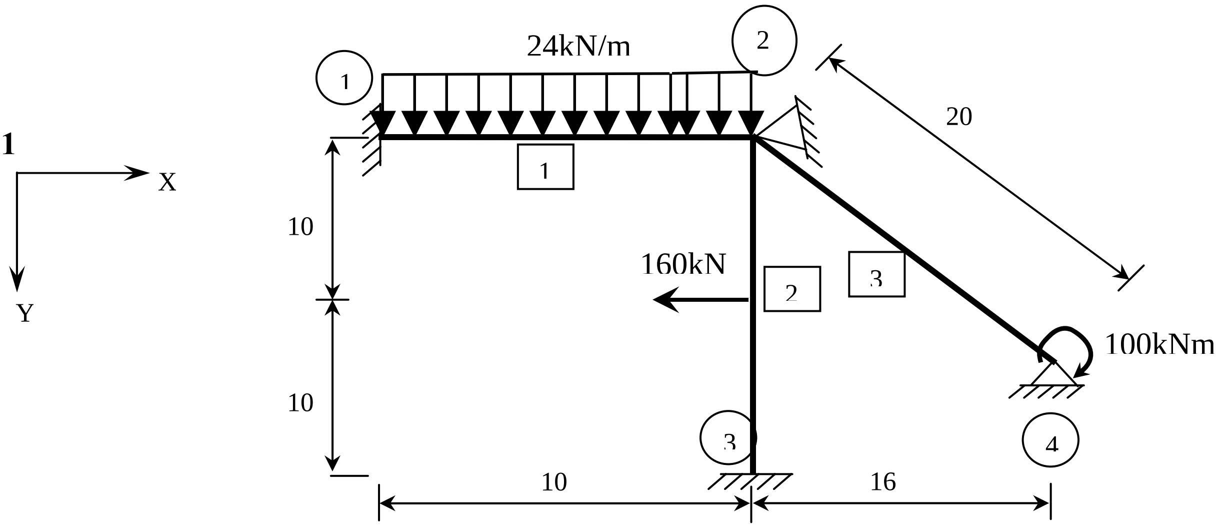
Both questions carry equal marks. All answers should be upto at least three significant digits.
Must use only Stiffness Matrix Method in both questions.
Must use global coordinate system provided with the problem.

Problem 1

For this question, settlement must be handled only through self straining by including it in the load vector $\mathbf{P}_\mathrm{I}$. Thus, settlement must not be handled through Δ_{II} term for this question.

Consider the 3-member **rigid-jointed** frame in **Fig. 1.** It is fixed supported at joint-1 and joint-3 and pin supported at joint-2 and joint-4. Note that all members are rigidly welded to each other at joint 2 which is then pin-supported.
The support at joint-4 settles 0.2 downward.
Member-3 is 0.05m too long (i.e., misfit). It is heated/cooled such that $T_L = 25^0\mathrm{C}$, $T_U = -25^0\mathrm{C}$, both measured with respect to the ambient temperature.
For all members use data:
$EI = 10^5\,\mathrm{kNm}^2$; $GJ = 0.16*10^5\,\mathrm{kNm}^2$; $A/I = 300\mathrm{m}^{-2}$; $h = 0.1\mathrm{m}$ = depth of member; $\alpha = 0.000012\,/^0\mathrm{C}$
Find: (i) $\mathbf{K}_{\mathrm{II}}$; (ii) $\mathbf{P}_\mathrm{I}$ (joint load vector) ; (iii) Δ_I (joint displacement vector)

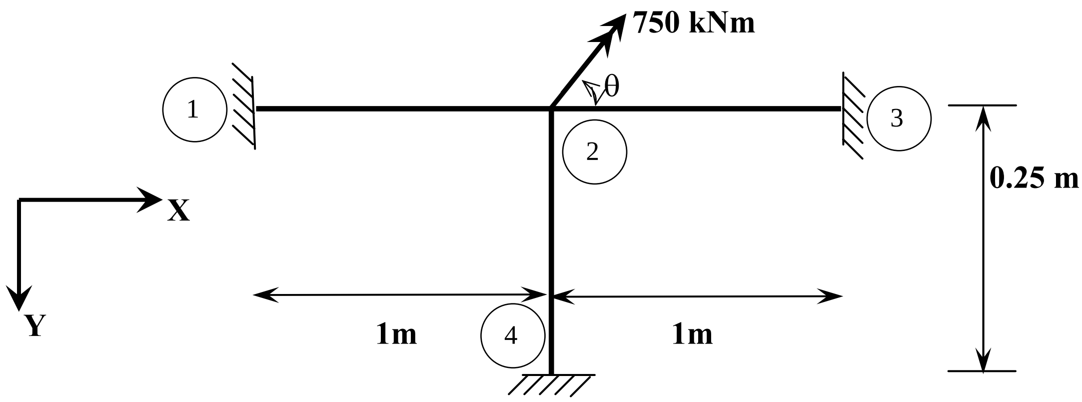
Fig. 1



Problem 2

The plane frame has a torque applied in the XY plane, at an angle $\theta = 45^0$ as shown.
Use data: $EI = 10^5\,\mathrm{kNm}^2$; $GJ = 0.16*10^5\,\mathrm{kNm}^2$; $A/I = 300\mathrm{m}^{-2}$,
Find: (i) $\mathbf{K}_{\mathrm{II}}$; (ii) $\mathbf{P}_\mathrm{I}$ (joint load vector) ; (iii) Δ_I (joint displacement vector)

Fig. 2



SUMMARY ANSWER SHEET

PAPER CODE: C

Name:

Roll no:

Problem 1

(v) K_{II}

(ii) $P_I =$

(iii) $\Delta_I =$

Problem 2

(vi) K_{II}

(ii) $P_I =$

(iii) $\Delta_I =$

DEPARTMENT OF CIVIL ENGINEERING

CE-317 STRUCTURAL MECHANICS II

Quiz-2 19/10/13 PAPER CODE: D

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Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.
Both questions carry equal marks. All answers should be upto at least three significant digits.
Must use only Stiffness Matrix Method in both questions.
Must use global coordinate system provided with the problem.

Problem 1

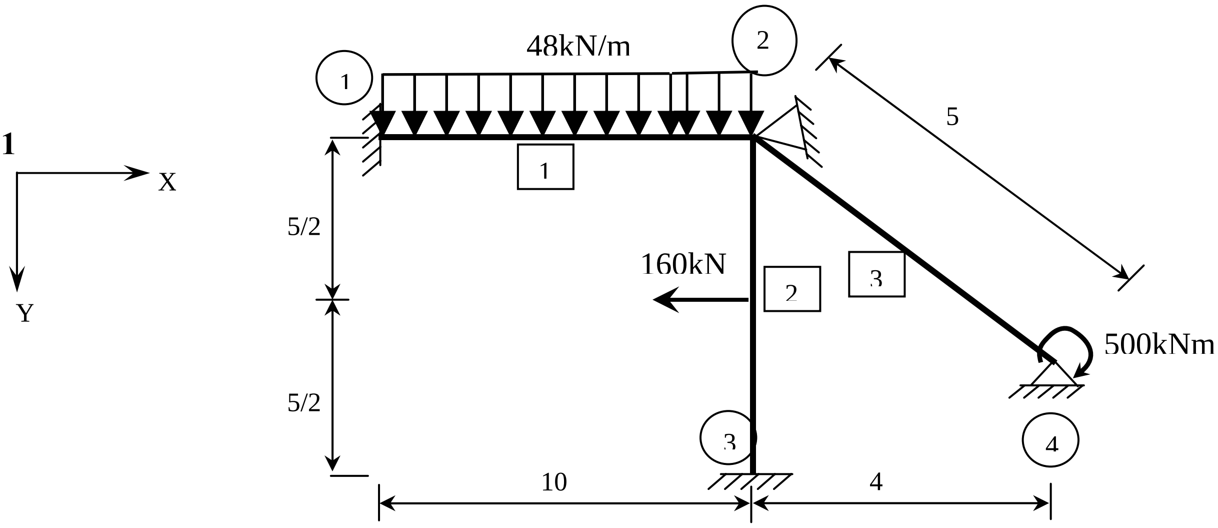
For this question, settlement must be handled only through self straining by including it in the load vector $\mathbf{P_I}$. Thus, settlement must not be handled through $\mathbf{\Delta_{II}}$ term for this question.

Consider the 3-member **rigid-jointed** frame in **Fig. 1.** It is fixed supported at joint-1 and joint-3 and pin supported at joint-2 and joint-4. Note that all members are rigidly welded to each other at joint 2 which is then pin-supported.
The support at joint-4 settles 0.2m downward.
Member-3 is 0.1m too long (i.e., misfit). It is heated/cooled such that $T_L = 50^0\text{C}, T_U = -50^0\text{C}$, both measured with respect to the ambient temperature.
For all members use data:

$EI = 10^5\,\text{kNm}^2;\ GJ = 0.16*10^5\,\text{kNm}^2;\ A/I = 300\text{m}^{-2};\ h = 0.1\text{m} = \text{depth of member};\ \alpha = 0.000012\,/^0\text{C}$

Find: (i) $\mathbf{K_{II}}$; (ii) $\mathbf{P_I}$ (joint load vector) ; (iii) $\Delta_\mathbf{I}$ (joint displacement vector)

Fig. 1



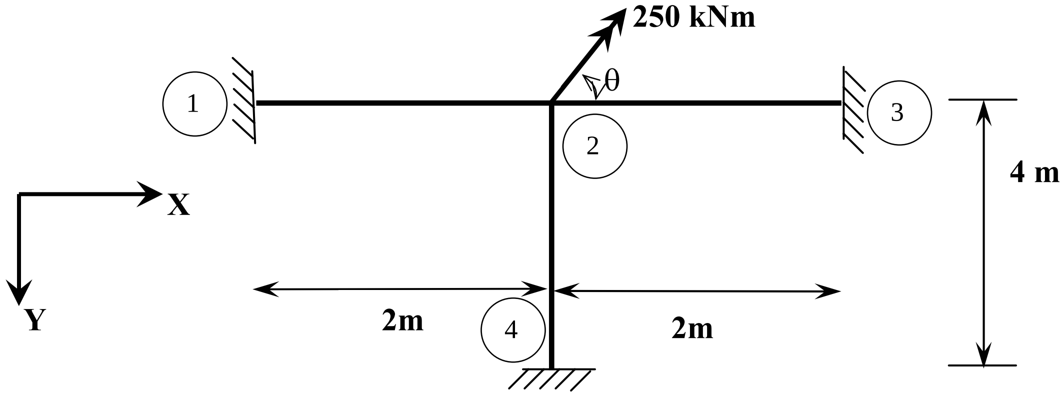
Problem 2

The plane frame has a torque applied in the XY plane, at an angle $\theta = \cos^{-1}(0.5)$ as shown.

Use data: $EI = 10^5\,\text{kNm}^2;\ GJ = 0.16*10^5\,\text{kNm}^2;\ A/I = 300\text{m}^{-2}$,

Find: (i) $\mathbf{K_{II}}$; (ii) $\mathbf{P_I}$ (joint load vector) ; (iii) $\Delta_\mathbf{I}$ (joint displacement vector)

Fig. 2



SUMMARY ANSWER SHEET

PAPER CODE: D

Name:

Roll no:

Problem 1

(vii) K_{II}

(ii) $P_I =$

(iii) $\Delta_I =$

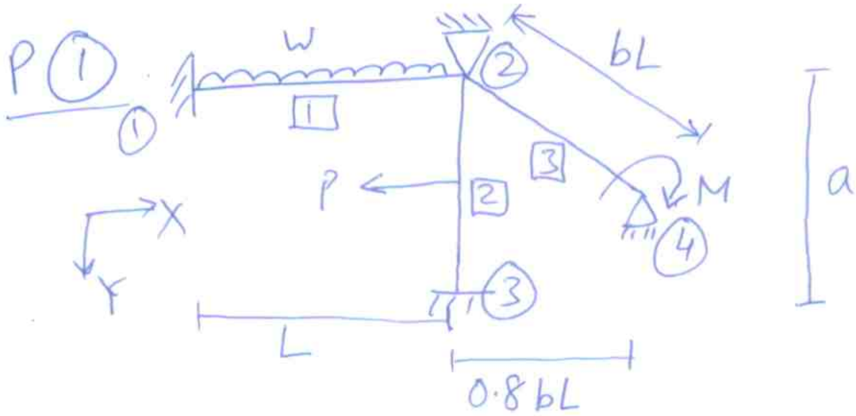
Problem 2

(viii) K_{II}

(ii) $P_I =$

(iii) $\Delta_I =$

P ①



$$a_{21} = I, \quad a_{23} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a_{24} = \begin{bmatrix} -0.8 & -0.6 & 0 \\ 0.6 & -0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$K_{II} = \begin{bmatrix} K_{22}(3,3) & K_{24}(3,3) \\ K_{42}(3,3) & K_{44}(3,3) \end{bmatrix}$$

Self straining:
misfit → m too long.
settlement → s ↓
temp → $\Delta T = T_L - T_u$, $T_{avg} = \frac{T_L + T_u}{2}$

$$K_{22} = \frac{EI}{L}\left(\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 4 \end{bmatrix} + \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & -\frac{6}{a^2L} & \frac{4}{a} \end{bmatrix} a_{23} + \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & -\frac{6}{b^2L} & \frac{4}{b} \end{bmatrix} a_{24} \right)$$

$$= \frac{EI}{L}\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 4\left(1+\frac{1}{a}+\frac{1}{b}\right) \end{bmatrix}$$

$$K_{24} = \frac{EI}{L}\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & -\frac{6}{b^2L} & \frac{2}{b} \end{bmatrix} a_{42} = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \frac{2}{b} \end{bmatrix}\frac{EI}{L} = K_{42}^T$$

$$K_{44} = \frac{EI}{L}\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & -\frac{6}{b^2L} & \frac{4}{b} \end{bmatrix} a_{42} = \frac{EI}{L}\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \frac{4}{b} \end{bmatrix} \Rightarrow K_{II} = \frac{EI}{L}\begin{bmatrix} 4*\left(1+\frac{1}{a}+\frac{1}{b}\right) & \frac{2}{b} \\ \frac{2}{b} & \frac{4}{b} \end{bmatrix}$$

$$P^e_{②} = \begin{Bmatrix} \times \\ \times \\ \frac{wL^2}{12} \end{Bmatrix} + \begin{Bmatrix} \times \\ \times \\ -\frac{PaL}{8} \end{Bmatrix} + a_{24}^T \underbrace{\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & -\frac{6}{b^2L} & \frac{2}{b} \end{bmatrix}}_{k_{②④}} \frac{EI}{L}\left(\left\{-\begin{pmatrix} m \\ 0 \\ 0 \end{pmatrix}\right\} + \left\{-\begin{pmatrix} \alpha T_{av} bL \\ -\alpha \frac{\Delta T}{2h} b^2L^2 \\ -\alpha \frac{\Delta T}{h} bL \end{pmatrix}\right\} + \underbrace{\begin{bmatrix} 0.8 & 0.6 & 0 \\ -0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{a_{42}} \left\{+\begin{pmatrix} 0 \\ s \\ 0 \end{pmatrix}\right\} \right)$$

$$= \begin{Bmatrix} \times \\ \times \\ \frac{wL^2}{12} - \frac{PaL}{8} + \frac{EI}{L}\left[\left(-\frac{6}{b^2L}\right)\left(\alpha\frac{\Delta T}{2h}b^2L^2 + 0.8s\right) + \left(\frac{2}{b}\right)\left(\alpha\frac{\Delta T}{h}bL\right)\right] \end{Bmatrix} = \{\times \;\; \times \;\; A\}^T$$

$$P^e_{④} = \begin{Bmatrix} \times \\ \times \\ \frac{EI}{L}\left[\left(-\frac{6}{b^2L}\right)\left(\alpha\frac{\Delta T}{2h}b^2L^2 + 0.8s\right) + \left(\frac{4}{b}\right)\left(\alpha\frac{\Delta T}{h}bL\right)\right] \end{Bmatrix} = \begin{Bmatrix} \times \\ \times \\ B \end{Bmatrix}$$

$$P_I = \begin{Bmatrix} 0 \\ M \end{Bmatrix} - \begin{Bmatrix} P^e_{②}(3) \\ P^e_{④}(3) \end{Bmatrix} = \begin{Bmatrix} -A \\ M-B \end{Bmatrix}$$

$$\Delta_I = K_{II}^{-1} P_I = \frac{L}{EI} \cdot \frac{1}{\left(4\left(1+\frac{1}{a}+\frac{1}{b}\right)\frac{4}{b} - \frac{4}{b^2}\right)} \begin{bmatrix} 4/b & -2/b \\ -2/b & 4\left(1+\frac{1}{a}+\frac{1}{b}\right) \end{bmatrix} \begin{Bmatrix} A \\ M-B \end{Bmatrix} = \begin{Bmatrix} \Delta_{6②} \\ \Delta_{6④} \end{Bmatrix}$$

<u>Code A</u> : $M = 200$, $w = 12$, $P = 80$, $a = 0.5$, $b = 2$, $T_L = 50$, $T_U = -50$, $s = 0.1$, $m = 0.05$

$$K_{II} = 10^4 \begin{bmatrix} 14 & 1 \\ 1 & 2 \end{bmatrix} ; \quad P_I = \begin{Bmatrix} 1270 \\ -880 \end{Bmatrix}, \quad \Delta_I = \begin{Bmatrix} 0.0127 \\ -0.0503 \end{Bmatrix}$$

<u>Code B</u> : $M = 300$, $w = 24$, $P = 40$, $T_L = 25$, $T_U = -25$, $s = 0.05$, $m = 0.1$, $a = 2$, $b = 0.5$

$$K_{II} = 10^4 \begin{bmatrix} 14 & 4 \\ 4 & 8 \end{bmatrix} ; \quad P_I = \begin{Bmatrix} 1460 \\ 660 \end{Bmatrix} ; \quad \Delta_I = \begin{Bmatrix} 0.0094 \\ 0.0035 \end{Bmatrix}$$

<u>Code C</u> : $M = 100$; $w = 24$, $P = 160$, $T_L = 25$, $T_U = -25$, $s = 0.2$, $m = 0.05$, $a = 2$, $b = 2$

$$K_{II} = 10^4 \begin{bmatrix} 8 & 1 \\ 1 & 2 \end{bmatrix} ; \quad P_I = \begin{Bmatrix} 1040 \\ -260 \end{Bmatrix} ; \quad \Delta_I = \begin{Bmatrix} 0.0156 \\ -0.0208 \end{Bmatrix}$$

<u>Code D</u> : $M = 500$, $w = 48$, $P = 160$, $T_L = 50$, $T_U = -50$, $s = 0.2$, $m = 0.1$, $a = 0.5$, $b = 0.5$

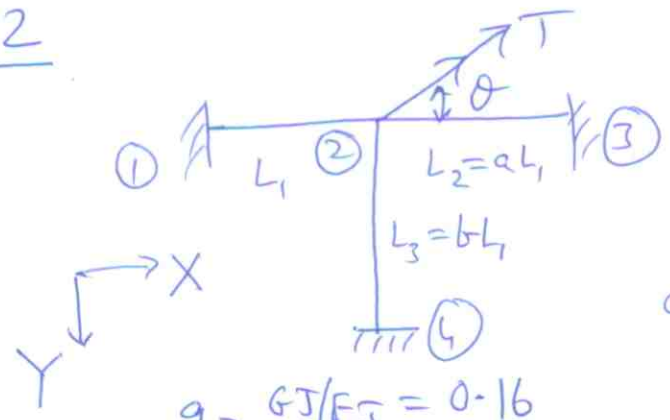
$$K_{II} = 10^4 \begin{bmatrix} 20 & 4 \\ 4 & 8 \end{bmatrix} ; \quad P_I = \begin{Bmatrix} 4740 \\ 3140 \end{Bmatrix} ; \quad \Delta_I = \begin{Bmatrix} 0.0176 \\ 0.0304 \end{Bmatrix}$$

Simplification of working for P_I :

$$P^e_{②} = \left\{ \times \quad \times \quad \frac{wL^2}{12} - \frac{PaL}{8} + EI\left(-\alpha\frac{\Delta T}{h} - \frac{4.8s}{b^2L^2}\right) \right\}^T$$

$$P^e_{④} = \left\{ \times \quad \times \quad EI\left(\alpha\frac{\Delta T}{h} - \frac{4.8s}{b^2L^2}\right) \right\}^T$$

P2



$g = GJ/EI = 0.16$

Grid

$$a_{12} = a_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}; \quad a_{21} = I = a_{32}$$

$$a_{24} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}; \quad a_{42} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$K_{II} = K_{22} = EI \begin{bmatrix} \frac{12}{L_1^3} + \frac{12}{L_2^3} + \frac{12}{L_3^3} & \frac{6}{L_3^2} & \frac{6}{L_1^2} - \frac{6}{L_2^2} \\ 6/L_3^2 & \frac{g}{L_1} + \frac{g}{L_2} + \frac{4}{L_3} & 0 \\ \frac{6}{L_1^2} - \frac{6}{L_2^2} & 0 & \frac{4}{L_1} + \frac{4}{L_2} + \frac{g}{L_3} \end{bmatrix}$$

$$P_I = \begin{Bmatrix} T\cos\theta \\ -T\sin\theta \end{Bmatrix}$$

Code A: $L_1 = 1$, $a = 1$, $b = 0.5$, $T = 1000$, $\cos\theta = 0.8$

$$K_{II} = 10^5 \begin{bmatrix} 120 & 24 & 0 \\ 24 & 0.32 + 8 & 0 \\ 0 & 0 & 8 + 0.32 \end{bmatrix}; \quad P_I = \begin{Bmatrix} 0 \\ 800 \\ -600 \end{Bmatrix}; \quad \Delta_I = 10^{-3} \begin{Bmatrix} -0.4545 \\ 2.2727 \\ -0.7212 \end{Bmatrix}$$

Code B: $L_1 = 0.5$, $a = 1$, $b = 2$, $T = 500$, $\cos\theta = 0.6$

$$K_{II} = 10^5 \begin{bmatrix} 204 & 6 & 0 \\ 6 & 0.64 + 4 & 0 \\ 0 & 0 & 16 + 0.16 \end{bmatrix}; \quad P_I = \begin{Bmatrix} 0 \\ 300 \\ -400 \end{Bmatrix}; \quad \Delta_I = 10^{-3} \begin{Bmatrix} -0.0198 \\ 0.6721 \\ -0.2475 \end{Bmatrix}$$

Code C: $L_1 = 1$, $a = 1$, $b = 0.25$, $T = 750$, $\theta = 45°$

$$K_{II} = 10^5 \begin{bmatrix} 792 & 96 & 0 \\ 96 & 0.32 + 16 & 0 \\ 0 & 0 & 8 + 0.64 \end{bmatrix}; \quad P_I = \begin{Bmatrix} 0 \\ 530.33 \\ -530.33 \end{Bmatrix}; \quad \Delta_I = 10^{-3} \begin{Bmatrix} -0.1372 \\ 1.1323 \\ -0.6138 \end{Bmatrix}$$

Code D: $L_1 = 2$, $a = 1$, $b = 2$, $T = 250$, $\cos\theta = 0.5$

$$K_{II} = 10^5 \begin{bmatrix} 3.1875 & 0.375 & 0 \\ 0.375 & 0.16 + 1 & 0 \\ 0 & 0 & 4 + 0.04 \end{bmatrix}; \quad P_I = \begin{Bmatrix} 0 \\ 125 \\ -216.51 \end{Bmatrix}; \quad \Delta_I = 10^{-3} \begin{Bmatrix} -0.1318 \\ 1.1202 \\ -0.5359 \end{Bmatrix}$$