CE-317 STRUCTURAL ANALYSIS I
DEPARTMENT OF CIVIL ENGINEERING
Quiz 1; September 2, 2019, 9-10pm

<u>Problems carry equal weightage</u>

YOU MUST USE ONLY DIRECT STIFFNESS METHOD FOR BOTH PROBLEMS

Problem 1

Refer Fig. 1. Support at node 1 settles down 50mm. Member 1 is heated 10^0C above ambient temperature. Member 2 is short by 2mm. Use $\alpha = 2\times10^{-5}\ /\ {}^0\mathrm{C}$, $A = 5\times10^3\ \mathrm{mm}^2$, $I = 2\times10^8\ \mathrm{mm}^4$, $E = 2\times10^5\ \mathrm{N/mm}^2$. D**etermine all reactions at support 1.**

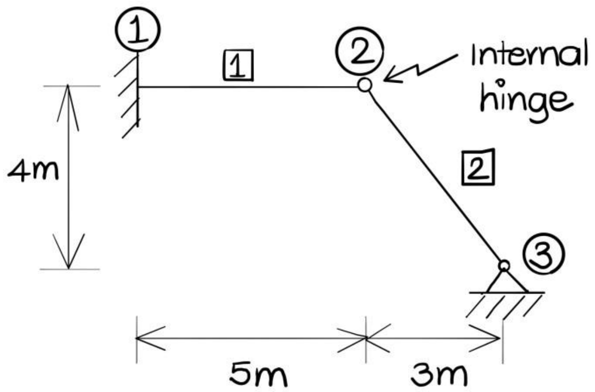


Figure 1

Problem 2

Refer Fig. 2. The support at the right end comprises a fixed roller attached to a spring. Use, $A = 5\times10^3\ \mathrm{mm}^2$, $I = 2\times10^8\ \mathrm{mm}^4$, $E = 2\times10^5\ \mathrm{N/mm}^2$, $M = 10\,\mathrm{kN.m}$, $P = 10\,\mathrm{kN}$, and spring constant $k = 8EI/L^3$. D**etermine the force in the spring.**

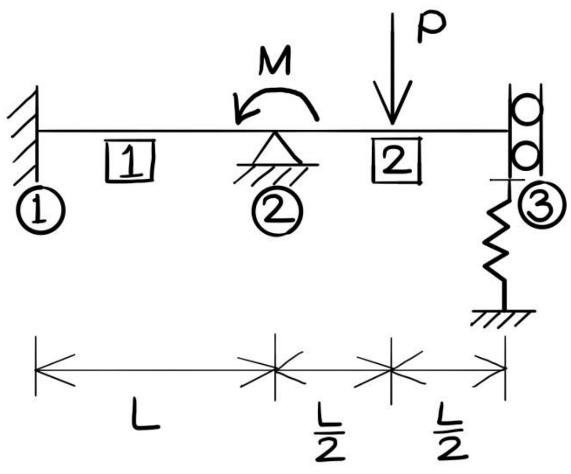
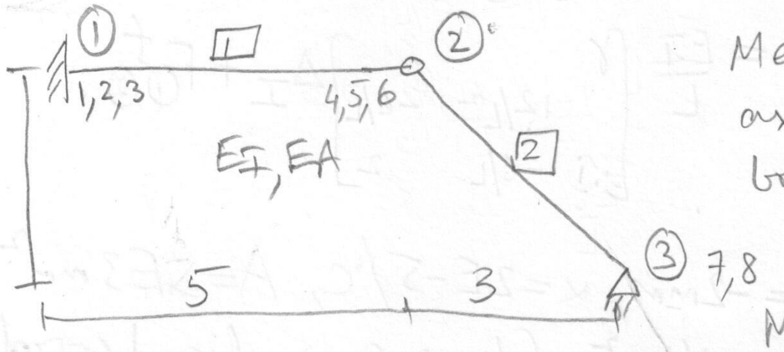


Figure 2

P1



Mem [2] has misfit along its axis, and it is hinged at both ends. Hence it will exhibit TRUSS action.
Mem [3] fixed at one end, attached to inclined mem [2]. Hence it exhibits FRAME action.

$$a_{①②} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad a_{②③} = \begin{bmatrix} -0.6 & -0.8 \end{bmatrix} \qquad \frac{A}{I} = \gamma$$

$$K_{II} = \frac{EI}{L} \begin{array}{c} \begin{array}{ccc} 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \gamma + 0.36\gamma & 0.48\gamma & 0 \\ & \frac{12}{L^2} + 0.64\gamma & -\frac{6}{L} \\ \text{symm} & & 4 \end{bmatrix} \end{array} \begin{array}{c} 4 \\ 5 \\ 6 \end{array}$$

$$K_{I\,II} = \frac{EI}{L} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 7 & 8 \end{array} \\ \begin{bmatrix} -\gamma & 0 & 0 & -0.36\gamma & -0.48\gamma \\ 0 & -\frac{12}{L^2} & -\frac{6}{L} & -0.48\gamma & -0.64\gamma \\ 0 & \frac{6}{L} & 2 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 4 \\ 5 \\ 6 \end{array}$$

not reqd if we deal with <u>settlement</u> thru self-straining, as <u>done below</u>.

Cuts: at node ① for mem [1], and node ② for mem [2]

$$F^f_{①②} = \frac{EI}{L} \begin{bmatrix} \gamma & & \\ & 12/L^2 & -6/L \\ & -6/L & 4 \end{bmatrix} \begin{Bmatrix} -\alpha TL \\ S \\ 0 \end{Bmatrix}; \quad F^f_{②①} = \frac{EI}{L} \begin{bmatrix} \gamma & & \\ & 12/L^2 & -6/L \\ & -6/L & 2 \end{bmatrix} \begin{Bmatrix} -\alpha TL \\ S \\ 0 \end{Bmatrix}$$

$$F_{②③} = \frac{EI}{L}\gamma(-m)$$

minus due to "fitting it back"

$$P_{②} = \frac{EI}{L} \begin{bmatrix} -\gamma\alpha TL \\ \frac{12}{L^2}S \\ -\frac{6}{L}S \end{bmatrix} + \begin{bmatrix} -0.6 \\ -0.8 \end{bmatrix} \frac{EI\gamma}{L}(-m) = \frac{EI}{L} \begin{bmatrix} -\gamma\alpha TL + 0.6\gamma m \\ \frac{12S}{L^2} + 0.8\gamma m \\ -\frac{6S}{L} \end{bmatrix} = -P_I$$

due to reversal of fixity loads.

$\Delta_I = K_{II}^{-1} P_I$; $F_{12} = k_{12}\,\delta_{21} + F^f_{12}$

$(\delta_{21} = \Delta_2 = \Delta_I)$ $= \dfrac{EI}{L}\begin{bmatrix}\gamma & & \\ & 12/L^2 & -6/L \\ & -6/L & 2\end{bmatrix}\Delta_I + F^f_{12}$

data: units N, mm

$s = -50\,mm$, $T = 10°C$, $m = -2\,mm$, $\alpha = 2E{-}5/°C$, $A = 5E3\,mm^2$

$I = 2E8\,mm^4$, $E = 2E5\,N/mm^2$. (here s in local coords).

$$K_{II} = \begin{bmatrix} 272000 & 96000 & \\ & 131840 & -96E5 \\ & & 3.2E10 \end{bmatrix}$$

$$P_I = \begin{bmatrix} 440E3 \\ 512E3 \\ -480E6 \end{bmatrix} \begin{matrix} N \rightarrow \\ N \downarrow \\ N.mm \circlearrowright \end{matrix}$$

$$\Delta_I = \begin{bmatrix} 0.8281 \\ 2.2372 \\ -0.01433 \end{bmatrix} \begin{matrix} mm \rightarrow \\ mm \downarrow \\ rad \circlearrowright \end{matrix}$$

+ve sign convention indicated throughout.

$$F^f_{12} = \begin{bmatrix} -200E3 \\ -192E3 \\ 480E6 \end{bmatrix} \begin{matrix} N \leftarrow \\ N \uparrow \\ N.mm \circlearrowright \end{matrix}$$

$$F_{12} = \text{reaction at } 1 = \begin{bmatrix} -34.39E3 \\ -45.85E3 \\ 2.293E8 \end{bmatrix} \begin{matrix} N \rightarrow \\ N \downarrow \\ N.mm \circlearrowright \end{matrix}$$

<u>If settlement handled thru $K_{I\,II}$</u>, P_I as before w/o 's' terms, $\Delta_I = K_{II}^{-1}(P_I - K_{I\,II}\Delta_{II})$, $\Delta_{II} = \begin{bmatrix} 0 \\ s \\ 0 \\ 0 \\ 0 \end{bmatrix}$,

here s in global coords (ie $s = 50\,mm$),

$\Rightarrow K_{I\,II}\Delta_{II} = \begin{bmatrix} 0 \\ -\frac{12}{L^2}s \\ \frac{6s}{L} \end{bmatrix}$, $P_I = \begin{bmatrix} 44E4 \\ 32E4 \\ 0 \end{bmatrix}$, so this equals P_I on p. 1, hence Δ_I same as before.

$$F_{①②} = k_{11}^2 \delta_{①②} + \underbrace{k_{12}\delta_{②①}}_{\text{same as before}} + F^f_{①②}$$

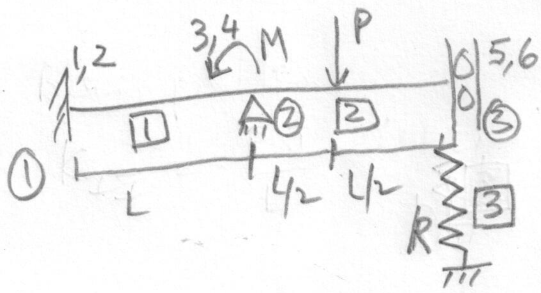
where, $\delta_{①②} = \begin{bmatrix} 0 \\ \ominus S \\ 0 \end{bmatrix}$ and $F^f_{①②} = k_{11}^2 \begin{bmatrix} -\alpha TL \\ 0 \\ 0 \end{bmatrix}$

$\because \delta_{①②}$ is local
and S now in global.

Thus $k_{11}^2 \delta_{①②} + F^f_{①②}$ equals $F^f_{①②}$ of before, hence
$F_{①②}$ unchanged, ie reaction same as before.

$$F^f_{12} = \begin{bmatrix} -200E3 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} N \leftarrow \\ N \uparrow \\ N.mm \circlearrowright \end{matrix}$$

P.2



Beam.
N, mm.

$$K_{II} = \begin{array}{cc} 4 & 5 \end{array}$$
$$\frac{EI}{L}\begin{bmatrix} 4+4 & -6/L \\ & \frac{12}{L^2} + \frac{8}{L^2} \end{bmatrix}$$

$$= \begin{bmatrix} 8E10 & -15E6 \\ & 125E2 \end{bmatrix}$$

; $P_I = \begin{bmatrix} -10 \\ 0 \end{bmatrix} - \begin{bmatrix} -10 * 4/8 \\ -10/2 \end{bmatrix}$

$= \begin{bmatrix} -5 \\ 5 \end{bmatrix} \begin{matrix} kN.m \circlearrowright \\ kN \downarrow \end{matrix}$

$= \begin{pmatrix} -5E6 \\ 5E3 \end{pmatrix} \begin{matrix} N.mm \circlearrowright \\ N \downarrow \end{matrix}$

$$\Delta_I = K_{II}^{-1} P_I = \begin{bmatrix} 0.00001613 \\ 0.4194 \end{bmatrix} \begin{matrix} rad \circlearrowright \\ mm \downarrow \end{matrix}$$

$$F_{spring} = k(0.4194) = 2097 N.$$