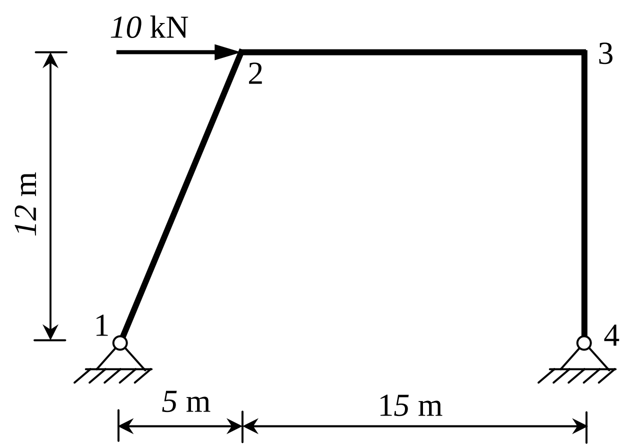
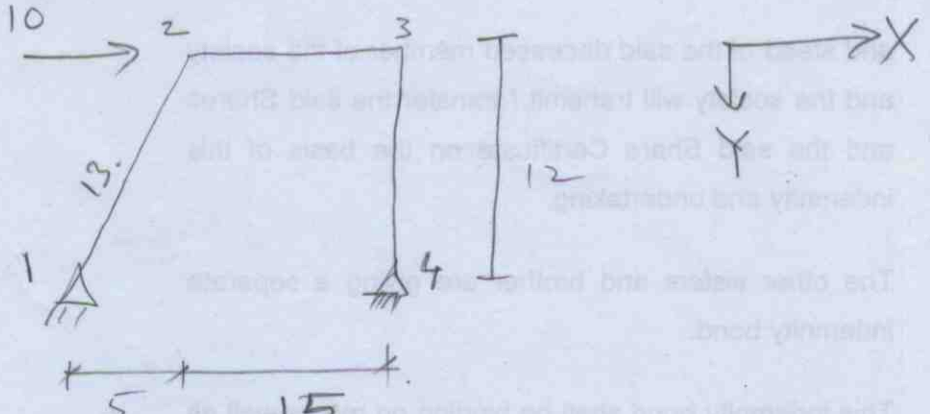
Department of Civil Engineering, IIT Bombay

CE317 **Structural Mechanics –II** **Tutorial 10**

Solve by Flexibility Matrix method. Select horizontal reaction at node 4 as the redundant. Take $\mathbf{F} = \mathbf{F_I} = \{\mathbf{F}_{21}^T\ \mathbf{F}_{32}^T\ \mathbf{F}_{43}^T\}^T$. Take $I/A = 3000\,\text{mm}^2$, and *E, I, A,* same for all members. Determine redundants, members end forces, nodal displacements.



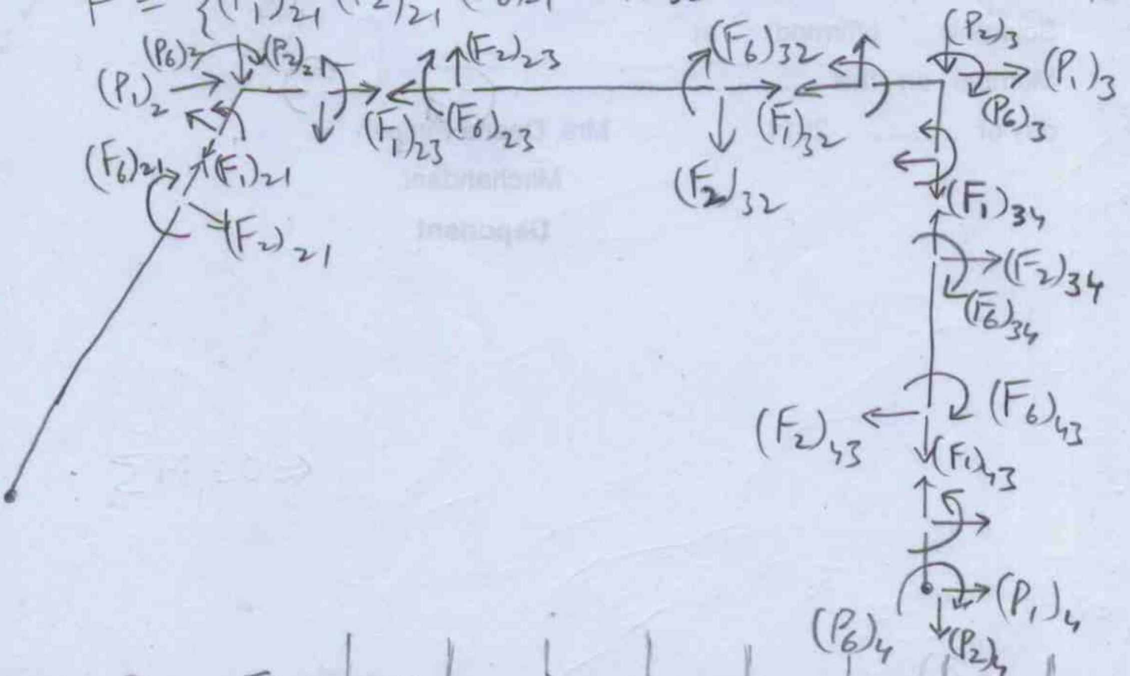
CE317 Tutorial 10.



$$P_{II} = \{(P_1)_4\}^T$$

$$P_I = \{(P_6)_1 \; (P_1)_2 \; (P_2)_2 \; (P_6)_2 \; (P_1)_3 \; (P_2)_3 \; (P_6)_3 \; (P_6)_4\}^T$$

$$F = \{(F_1)_{21} \; (F_2)_{21} \; (F_6)_{21} \; (F_1)_{32} \; (F_2)_{32} \; (F_6)_{32} \; (F_1)_{43} \; (F_2)_{43} \; (F_6)_{43}\}^T$$



$$
\begin{Bmatrix} (F_1)_{21} \\ (F_2)_{21} \\ (F_6)_{21} \\ (F_1)_{32} \\ (F_2)_{32} \\ (F_6)_{32} \\ (F_1)_{43} \\ (F_2)_{43} \\ (F_6)_{43} \end{Bmatrix}
=
\left[\begin{array}{cccccccc|c}
\frac{12}{13}\cdot\frac{1}{20} & \frac{5}{13}+\frac{12}{13}\cdot\frac{12}{20} & -\frac{12}{13}+\frac{12}{13}\cdot\frac{5}{20} & \frac{12}{13}\cdot\frac{1}{20} & \frac{5}{13}+\frac{12}{13}\cdot\frac{12}{20} & 0 & \frac{12}{13}\cdot\frac{1}{20} & \frac{12}{13}\cdot\frac{1}{20} & \frac{5}{13} \\
-\frac{5}{13}\cdot\frac{1}{20} & \frac{12}{13}-\frac{5}{13}\cdot\frac{12}{20} & \frac{5}{13}-\frac{5}{13}\cdot\frac{5}{20} & -\frac{5}{13}\cdot\frac{1}{20} & \frac{12}{13}-\frac{5}{13}\cdot\frac{12}{20} & 0 & -\frac{5}{13}\cdot\frac{1}{20} & -\frac{5}{13}\cdot\frac{1}{20} & \frac{12}{13} \\
-\frac{1}{20}\cdot 15 & -\frac{12}{20}\cdot 15 & -\frac{5}{20}\cdot 15 & 1-\frac{1}{20}\cdot 15 & -\frac{12}{20}\cdot 15 & 0 & 1-\frac{1}{20}\cdot 15 & 1-\frac{1}{20}\cdot 15 & -12 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
-1/20 & -12/20 & -5/20 & -1/20 & -12/20 & 1-1 & -1/20 & -1/20 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -12 \\
-1/20 & -12/20 & -5/20 & -1/20 & -12/20 & -1 & -1/20 & -1/20 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\begin{Bmatrix} (P_6)_1 \\ (P_1)_2 \\ (P_2)_2 \\ (P_6)_2 \\ (P_1)_3 \\ (P_2)_3 \\ (P_6)_3 \\ (P_6)_4 \\ \hline (P_1)_4 \end{Bmatrix}
$$

b_I $\quad$ b_{II}

$$\Sigma M_1 = 0 \Rightarrow (P_1)_2 \cdot 12 + (P_1)_3 \cdot 12 + (P_2)_2 \cdot 5 + (P_2)_3 \cdot 20$$
$$(P_6)_1 + (P_6)_2 + (P_6)_3 + (P_6)_4 + (P_2)_4 \cdot 20 = 0.$$
$$(P_2)_4 = (F_1)_{43}$$

$$(F_1)_{21} - (P_1)_2 \cdot \frac{5}{13} + (P_2)_2 \cdot \frac{12}{13} - (F_1)_{23} \cdot \frac{5}{13} + (F_2)_{23} \cdot \frac{12}{13} = 0.$$
$$(F_1)_{23} \to (F_1)_{32}, \quad (F_2)_{23} \to (F_2)_{32}$$

$$(F_2)_{21} - (P_1)_2 \cdot \frac{12}{13} - (P_2)_2 \cdot \frac{5}{13} - (F_1)_{32} \frac{12}{13} - (F_2)_{32} \cdot \frac{5}{13} = 0.$$

$$(F_6)_{32} - (P_6)_3 + (F_6)_{34} = 0.$$
$$(F_6)_{34} \to -(F_6)_{43} - (F_2)_{43} \cdot 12$$

$$(F_6)_{21} - (P_6)_2 + (F_6)_{23} = 0$$
$$(F_6)_{23} \to -(F_6)_{32} - (F_2)_{32} \cdot 15$$

$$d_u = \begin{bmatrix} d^1_{22} & 0 & 0 \\ 0 & d^2_{33} & 0 \\ 0 & 0 & d^3_{44} \end{bmatrix}; \quad \frac{1}{EA} = \frac{1}{EI_z} \cdot \frac{I}{A} \qquad \alpha = \frac{3000}{10^6} = 3E\text{-}3.$$

$$d_u = \frac{1}{EI_z}\begin{bmatrix} 13\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{13^3}{3} & \frac{13^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{13^2}{2} & 13 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{15^3}{3} & \frac{15^2}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{15^2}{2} & 15 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{12^3}{3} & \frac{12^2}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{12^2}{2} & 12 \\ 0 & 0 & 0 & 0 & 0 & & & & \end{bmatrix}$$

$$D_{I\,I} = b_I^T d_u b_I \; ; \; D_{I\,II} = b_I^T d_u b_{II}$$

$$D_{II\,I} = b_{II}^T d_u b_I \; ; \; D_{II\,II} = b_{II}^T d_u b_{II}$$

For $\Delta_{II} = 0$,

$$P_{II} = -D_{II\,II}^{-1} D_{II\,I} P_I$$

$$P_I = \{0 \; 10 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0\}^T$$

$$\Delta_I = \left(D_{I\,I} - D_{I\,II} D_{II\,II}^{-1} D_{II\,I}\right) P_I$$

$$F = F_I = b_I P_I + b_{II} P_{II}$$

See MATLAB file.

%Tutorial 10 solution.
%Externally statically indeterminate

format long;

%redundant is taken as horizontal force at node 4
%member end forces in equilibrium conditions pertain to cut end at higher node number for respective members, i.e., F21, F32, F43.

%equilibrium matrix bI
bI=[12/13*1/20 5/13+12/13*12/20 -12/13+12/13*5/20 12/13*1/20 5/13+12/13*12/20 0 12/13*1/20 12/13*1/20; -5/13*1/20 12/13-5/13*12/20 5/13-5/13*5/20 -5/13*1/20 12/13-5/13*12/20 0 -5/13*1/20 -5/13*1/20; -1/20*15 -12/20*15 -5/20*15 1-1/20*15 -12/20*15 0 1-1/20*15 1-1/20*15; 0 0 0 0 1 0 0 0; -1/20 -12/20 -5/20 -1/20 -12/20 0 -1/20 -1/20; 0 0 0 0 0 0 1 1; -1/20 -12/20 -5/20 -1/20 -12/20 -1 -1/20 -1/20; 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 1];

%bI matrix (9X8), rows separated by semi-colon.

0.046153846153846   0.938461538461538 -0.692307692307692
0.046153846153846   0.938461538461538   0
0.046153846153846   0.046153846153846;
  -0.019230769230769   0.692307692307692
0.288461538461538 -0.019230769230769   0.692307692307692
0 -0.019230769230769 -0.019230769230769;
  -0.750000000000000 -9.000000000000000 -
3.750000000000000   0.250000000000000 -9.000000000000000
0   0.250000000000000   0.250000000000000;
                   0   0   0   0
1.000000000000000   0   0   0;
  -0.050000000000000 -0.600000000000000 -
0.250000000000000 -0.050000000000000 -0.600000000000000
0 -0.050000000000000 -0.050000000000000;

```
                 0                 0                 0                 0                 0
0   1.000000000000000   1.000000000000000;
  -0.050000000000000  -0.600000000000000  -
0.250000000000000  -0.050000000000000  -0.600000000000000
-1.000000000000000  -0.050000000000000  -
0.050000000000000;
                 0                 0                 0                 0                 0
0                 0                 0;
                 0                 0                 0                 0                 0
0                 0   1.000000000000000;

%equilibrium matrix bII
bII=[5/13; 12/13; -12; 1; 0; -12; 0; -1; 0];

%A/I ratio
alpha=3e-3;

%member flexibility sub-matrices
d221=[13*alpha 0 0; 0 13^3/3 13^2/2; 0 13^2/2 13];
d332=[15*alpha 0 0; 0 15^3/3 15^2/2; 0 15^2/2 15];
d443=[12*alpha 0 0; 0 12^3/3 12^2/2; 0 12^2/2 12];

%unassembled flexibility matrix
du=[d221 zeros(3,3) zeros(3,3); zeros(3,3) d332 zeros(3,3);
zeros(3,3) zeros(3,3) d443];

du =
  1.0e+003 *

  Columns 1 through 8, rows separated by semi-colon, 9X8 matrix

   0.000039000000000                 0                 0                 0
0                 0                 0                 0;
                 0   0.732333333333333   0.084500000000000
0                 0                 0                 0                 0;
```

```
               0   0.084500000000000   0.013000000000000
0                  0                  0                  0                  0;
               0                  0                  0   0.000045000000000
0                  0                  0                  0;
               0                  0                  0                  0
1.125000000000000   0.112500000000000                  0
0;
               0                  0                  0                  0
0.112500000000000   0.015000000000000                  0
0;
               0                  0                  0                  0                  0
0   0.000036000000000                  0;
               0                  0                  0                  0                  0
0                  0   0.576000000000000;
               0                  0                  0                  0                  0
0                  0   0.072000000000000;

  Column 9

               0
               0
               0
               0
               0
               0
               0
   0.072000000000000
   0.012000000000000

%assembled system (global) flexibility matrices
DI_I=bI'*du*bI; DI_II=bI'*du*bII; DII_I=bII'*du*bI;
DII_II=bII'*du*bII;

%free joint loads
PI=[0 10 0 0 0 0 0 0]';
```

```
%redundants
PII=-inv(DII_II)*DII_I*PI

%free displacements, global
DeltaI=(DI_I-DI_II*inv(DII_II)*DII_I)*PI

%member end forces of cut ends (cut at higher node number of
respective member in the present solution)
FI=bI*PI+bII*PII

PII =
  -3.803555852864861

DeltaI =
  1.0e+003 *

   0.321053908572123
   2.699475154598626
   1.124446622198703
   0.032731265095582
   2.699303994585248
   0.000216000000000
   0.042371318611257
   0.316227340017527

FI =
   7.921709287359669
   3.412102289663205
 -44.357329765621671
  -3.803555852864861
  -6.000000000000000
  45.642670234378329
  -6.000000000000000
   3.803555852864861
                   0
```