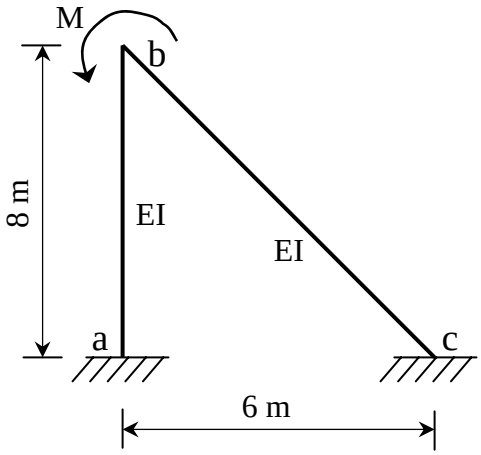
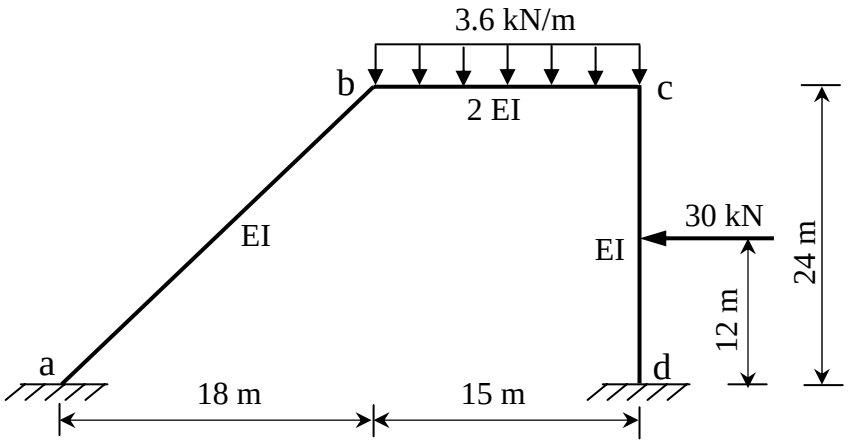
**Department of Civil Engineering, IIT Bombay**

**CE317 Structural Mechanics –II Tutorial 1**

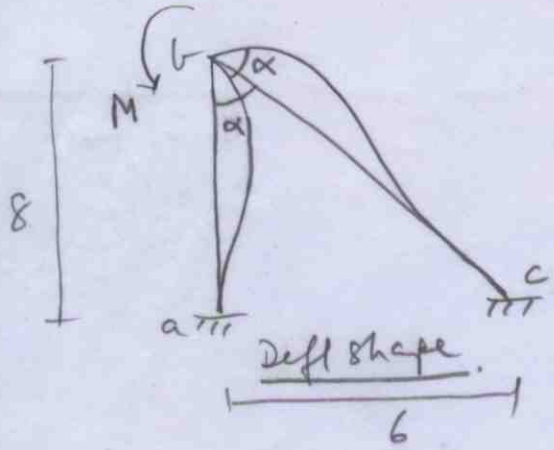
Use slope deflection method. Draw BMD, SFD, and deflected shape.




P.1




P.2




No sway.

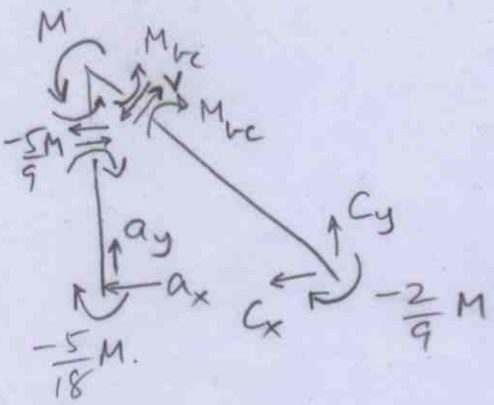
1-dof, $\theta_b$

$M_{ba} + M_{bc} = -M$.

$EI\left(\frac{4}{8}+\frac{4}{10}\right)\theta_b = -M \Rightarrow \theta_b = -\frac{10}{9}\frac{M}{EI}$

$M_{ab} = \frac{EI}{8}\cdot 2\left(-\frac{10}{9}\frac{M}{EI}\right) = -\frac{5}{18}M$, $M_{cb} = \frac{EI}{10}\,2\left(-\frac{10}{9}\frac{M}{EI}\right) = -\frac{2}{9}M$

$M_{ba} = \frac{EI}{8}\,4\left(-\frac{10}{9}\frac{M}{EI}\right) = -\frac{5}{9}M$




$\sum M_a = 0 = -\frac{5}{18}M - M - \frac{2}{9}M - C_y \cdot 6 = 0$

$\Rightarrow C_y = -\frac{1.5}{6} = -0.25$ kN (if M in kN.m)

$a_y = 0.25$

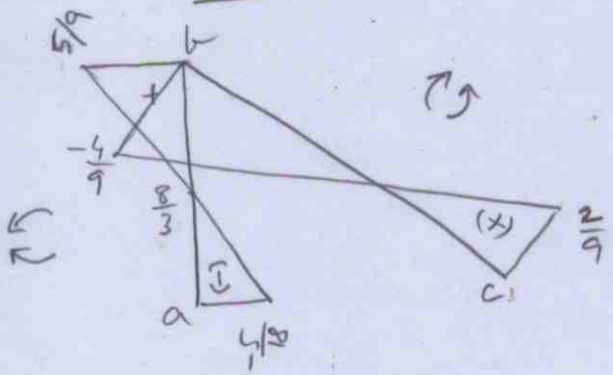
$\sum M_b = 0 = -\frac{5}{9}M - \frac{5}{18}M + 8a_x$

$\Rightarrow a_x = \frac{5}{48}$, $V = \frac{(M_{cb}-M_{bc})}{L_{bc}}$

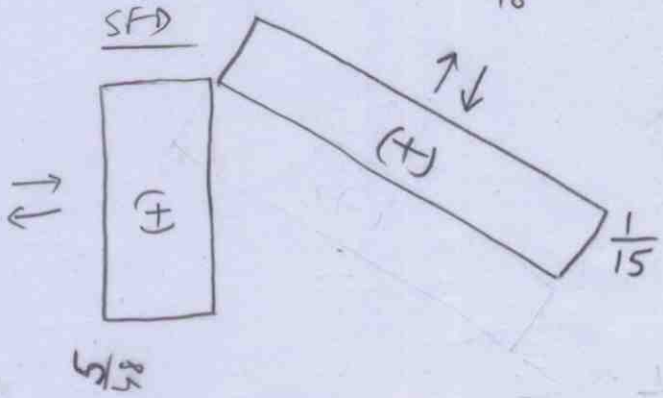
$= \frac{6}{9}\cdot\frac{1}{10}$

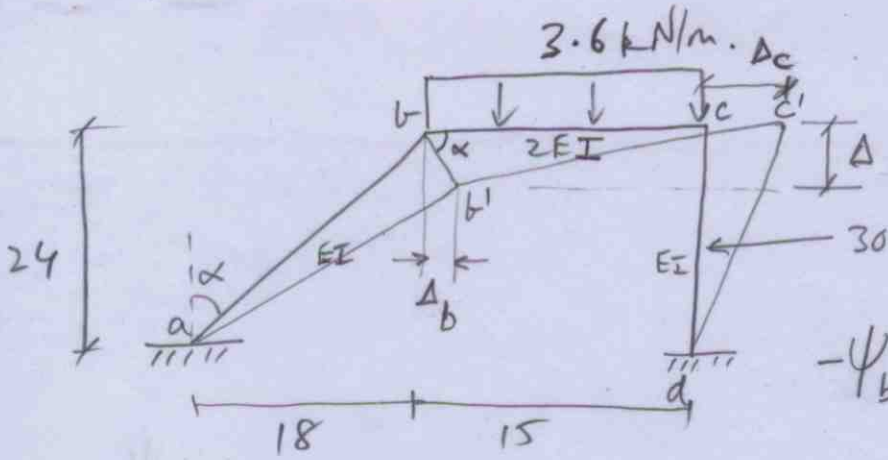
$= \frac{1}{15}$

BMD




SFD






3-dof, $\theta_b, \theta_c, \Delta_b$

$$\psi_{ab} = \frac{\Delta_b}{\cos\alpha}\cdot\frac{1}{L_{ab}} = \frac{5}{4}\cdot\frac{1}{30}\Delta_b = \frac{\Delta_b}{24}$$

$$-\psi_{bc} = \frac{\Delta}{L_{BC}} = \frac{\Delta_b \tan\alpha}{15} = \frac{3}{4}\cdot\frac{1}{15}\Delta_b = \frac{\Delta_b}{20}$$

$$\Delta_c = \Delta_b + \sqrt{(b'c')^2 - \Delta^2} - bc = \Delta_b + bc\sqrt{1-\left(\frac{\Delta}{bc}\right)^2} - bc = \Delta_b + bc - \frac{1}{2}\frac{\Delta^2}{bc} - bc$$

(HOT)

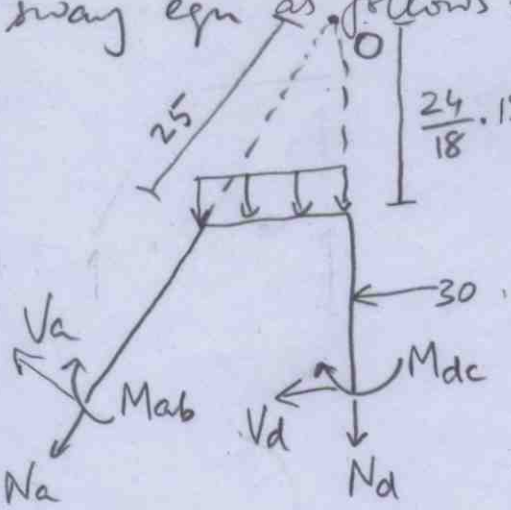
$$\Delta_c \simeq \Delta_b$$

$$\psi_{cd} = \frac{\Delta_b}{24}$$

$$EI\begin{bmatrix} 4\left(\frac{1}{30}+\frac{2}{15}\right) & 2\left(\frac{2}{15}\right) & -6\left(\frac{1}{24}\cdot\frac{1}{30}-\frac{1}{20}\cdot\frac{2}{15}\right) \\ 2\left(\frac{2}{15}\right) & 4\left(\frac{2}{15}+\frac{1}{24}\right) & -6\left(-\frac{1}{20}\cdot\frac{2}{15}+\frac{1}{24^2}\right) \\ \left(2\cdot\frac{1}{30}\cdot\left(-\frac{25}{30}\right)\right) & \left(4\cdot\frac{1}{24}\cdot\left(-\frac{44}{24}\right)\right) & -6\left(\frac{1}{24}\cdot\frac{1}{30}\cdot\left[-\frac{25}{30}-\frac{55}{30}\right]\right. \\ \left(+4\cdot\frac{1}{30}\cdot\left(-\frac{55}{30}\right)\right) & \left(+2\cdot\frac{1}{24}\cdot\left(-\frac{20}{24}\right)\right) & \left.+\frac{1}{24^2}\cdot\left[-\frac{44}{24}-\frac{20}{24}\right]\right) \end{bmatrix} \begin{Bmatrix} \theta_b \\ \theta_c \\ \Delta_b \end{Bmatrix} = \begin{Bmatrix} \frac{3.6(15)^2}{12} \\ -3.6\frac{(15)^2}{12}+\frac{30(24)}{8} \\ \left(\frac{30(24)}{8}\right)\left(-\frac{44}{24}+\frac{20}{24}\right) \\ +\frac{44}{24}.30.12+3.6\frac{(15)^2}{2}-30(32) \end{Bmatrix}$$

where we used $M_{ba}+M_{bc}=0 \rightarrow (i)$, $M_{cb}+M_{cd}=0 \rightarrow (ii)$

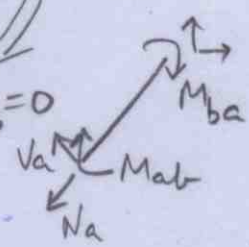
sway eqn as follows:



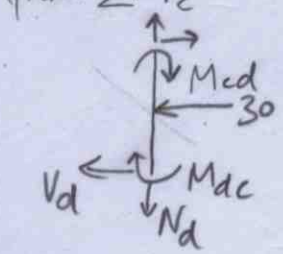

$$\sum_{ext} M_O = 0 = V_a(55) + V_d(44) + M_{ab} + M_{dc} - 3.6\frac{(15)^2}{2} + 30(32) \rightarrow (iii)$$

and $V_a = -\frac{(M_{ab}+M_{ba})}{30} \rightarrow (a)$, $V_d = -\frac{(M_{dc}+M_{cd}+30(12))}{24} \rightarrow (b)$

from $\sum M_b = 0$




from $\sum M_c = 0$




(iii), (a), (b) $\Rightarrow M_{ab}\left(1-\frac{55}{30}\right)+M_{ba}\left(-\frac{55}{30}\right)+M_{cd}\left(-\frac{44}{24}\right)+M_{dc}\left(1-\frac{44}{24}\right)$

$$-\frac{44}{24}.30.12-3.6\frac{(15)^2}{12}+30(32)=0.$$

$EI\{\theta_b \;\; \theta_c \;\; \Delta_b\}^T = \{82.3595, -24.9018, 607.3936\}^T$

$M_{ab} = EI\left(\frac{2}{30}\theta_B - 6\frac{\Delta_b}{24}\cdot\frac{1}{30}\right) = 0.4290$

$M_{ba} = EI\left(\frac{4}{30}\theta_B - 6\frac{\Delta_b}{24}\cdot\frac{1}{30}\right) = 5.9197$

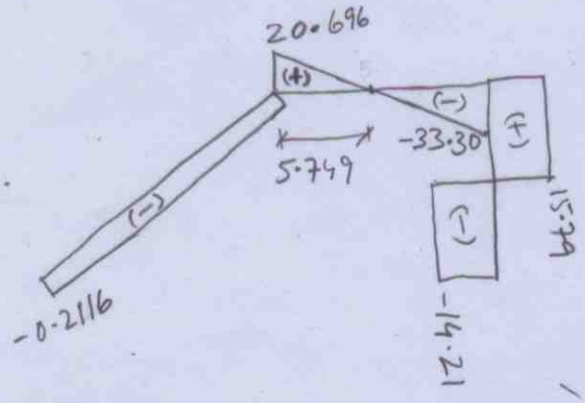
$M_{bc} = EI\left(\frac{4}{15}\cdot 2\cdot\theta_B + \frac{2}{15}\cdot 2\cdot\theta_c + 6\frac{\Delta_b}{20}\cdot\frac{2}{15}\right) - 3\cdot 6\frac{(15)^2}{12} = -5.9197$

$M_{cb} = EI\left(\frac{2}{15}\cdot 2\cdot\theta_B + \frac{4}{15}\cdot 2\cdot\theta_c + 6\frac{\Delta_b}{20}\cdot\frac{2}{15}\right) + 3\cdot 6\frac{(15)^2}{12} = 100.4773$

$M_{cd} = EI\left(\frac{4}{24}\cdot\theta_c - 6\cdot\frac{\Delta_b}{24}\cdot\frac{1}{24}\right) - 30\frac{(24)}{8} = -100.4773$

$M_{dc} = EI\left(\frac{2}{24}\theta_c - 6\frac{\Delta_b}{24}\cdot\frac{1}{24}\right) + 30\frac{(24)}{8} = 81.5978$

<Result matche with MDM — see Tute 4>




use (a), (b) on previous pg,

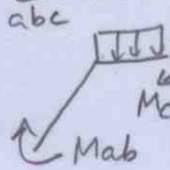
$V_d = -\frac{(-100.4773 + 81.5978 + 30*12)}{24}$

$= -14.2134$

$V_a = -(0.4290 + 5.9197)/30$

$= -0.2116$

$\sum_{abc} M_a = 0 \Rightarrow V_{cb} = -\frac{1}{33}(M_{ab} + M_{cb} + 3.6*15*25.5 + (-15.79).24)$



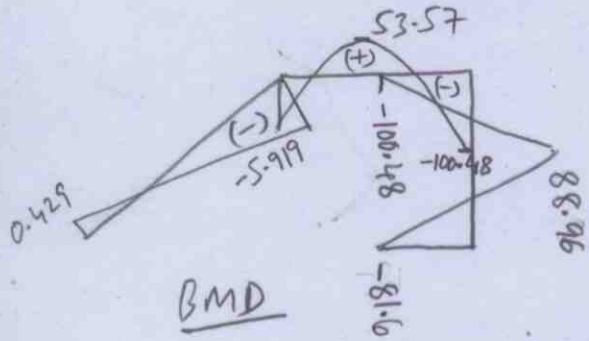

$\Rightarrow V_{cb} = -33.3039$

check: from $\Sigma M_a = 0$

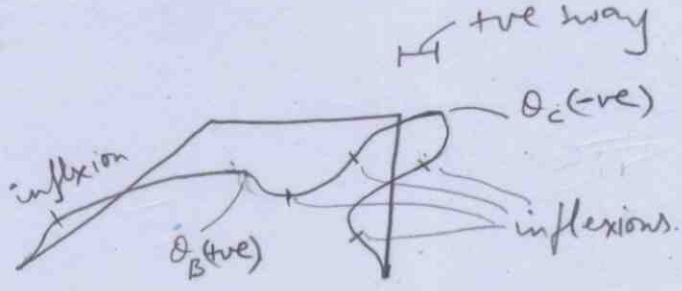
$N_d = \frac{-1}{33}(M_{dc} + M_{ab} - 30*12 + 3.6*15*25.5)$

$= -33.3038 = V_{cb}$

Can draw AFD if reqd.




BMD




Defl shape