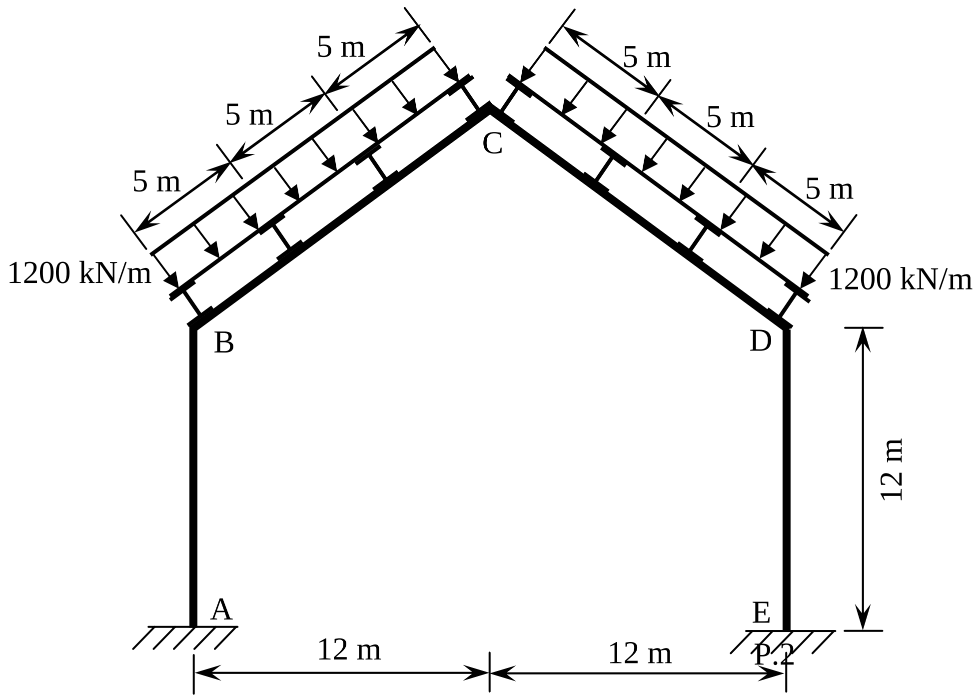
Department of Civil Engineering, IIT Bombay

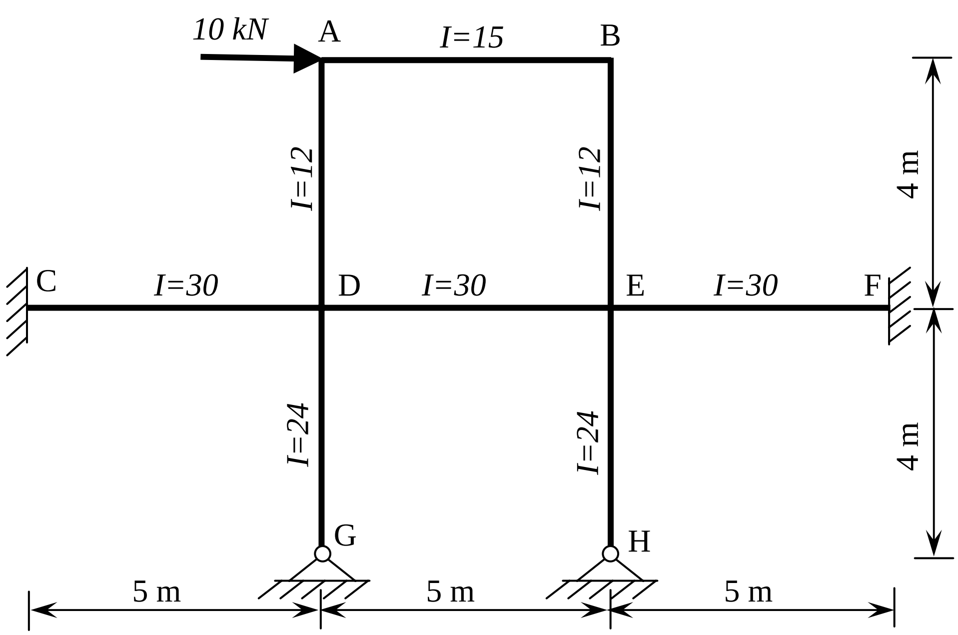
CE317 **Structural Mechanics –II** **Tutorial 2**

Use slope deflection method. Draw BMD, SFD, and deflected shape.

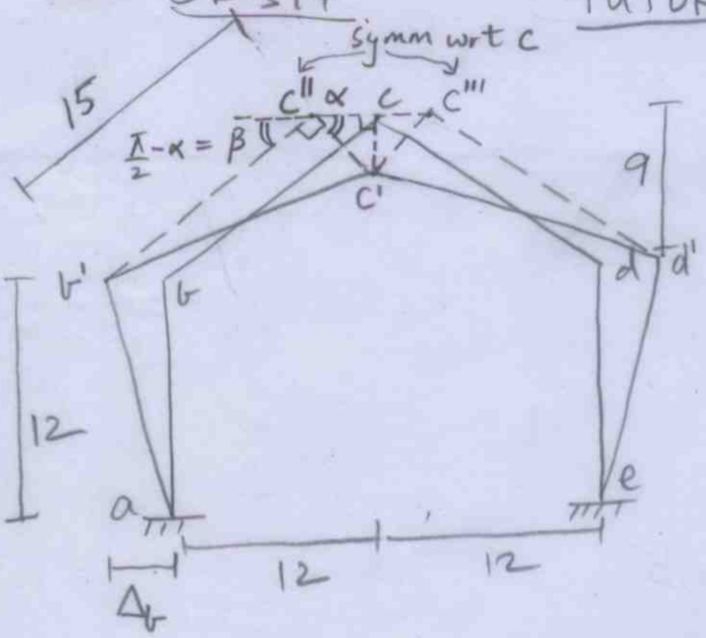
P1



P2



CE 317 TUTORIAL-2 ①



Structure & loading symmetric.
So only one sway dof
Dof's : (θ_b, Δ_b). Note $\theta_d=\theta_b, \Delta_d=\Delta_b$
$\theta_c=0$, from symm.

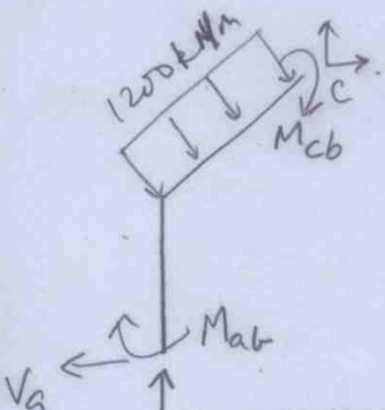
$$\psi_{ab} = -\frac{\Delta_b}{12}$$

$$\psi_{bc} = \frac{c''c'}{bc} = \frac{cc''/\cos\alpha}{bc} = \frac{cc''/\sin\beta}{bc}$$

$$= \Delta_b \frac{5}{3}\cdot\frac{1}{15} = \frac{\Delta_b}{9}$$

$$EI\begin{bmatrix} 4\left(\frac{1}{12}+\frac{1}{15}\right) & -6\left(-\frac{1}{12^2}+\frac{1}{9}\cdot\frac{1}{15}\right) \\ 2\left(\frac{1}{12}\right)\left(\frac{1}{12}-\frac{1}{21}\right) & -6\left(-\frac{1}{12^2}\left[\frac{1}{12}-\frac{1}{21}+\frac{1}{12}\right]\right. \\ +4\left(\frac{1}{12}\right)\left(\frac{1}{12}\right)+2\left(\frac{1}{15}\right)\left(-\frac{1}{21}\right) & \left.+\frac{1}{9}\cdot\frac{1}{15}\cdot\left[-\frac{1}{21}\right]\right) \end{bmatrix}\begin{Bmatrix}\theta_b \\ \Delta_b\end{Bmatrix} = \begin{Bmatrix} \frac{2\cdot1200\cdot5\cdot15}{9} \\ \left(-1200\frac{(15)^2}{2}+14400*12\right)\cdot\frac{1}{21} \\ -\frac{2\cdot1200\cdot5\cdot15}{9}\left(-\frac{1}{21}\right) \end{Bmatrix}$$

where we used $M_{ba}+M_{bc}=0 \rightarrow$ (i) and sway eqn as follows.



$$\sum_{abc} M_c = 0 \Rightarrow V_a = -\frac{\left(M_{ab}+M_{cb}+N_a\cdot12-1200\frac{(15)^2}{2}\right)}{21} \rightarrow \text{(ii)}$$

$$\text{Also } \quad V_a = -\frac{(M_{ab}+M_{ba})}{12} \rightarrow \text{(a)}$$

$N_a = 1200\times15\cos\beta$ (4/5)
$= 14400$, from symmetry.

$$\text{(ii),(a)} \Rightarrow M_{ab}\left(\frac{1}{12}-\frac{1}{21}\right)+M_{ba}\left(\frac{1}{12}\right) + M_{cb}\left(-\frac{1}{21}\right) = \frac{\left(-1200\frac{(15)^2}{2}+14400\times12\right)}{21}$$

Soln is, $EI\theta_B = 3.4516*10^4$ $\quad EI\Delta_B = 2.5539*10^5$

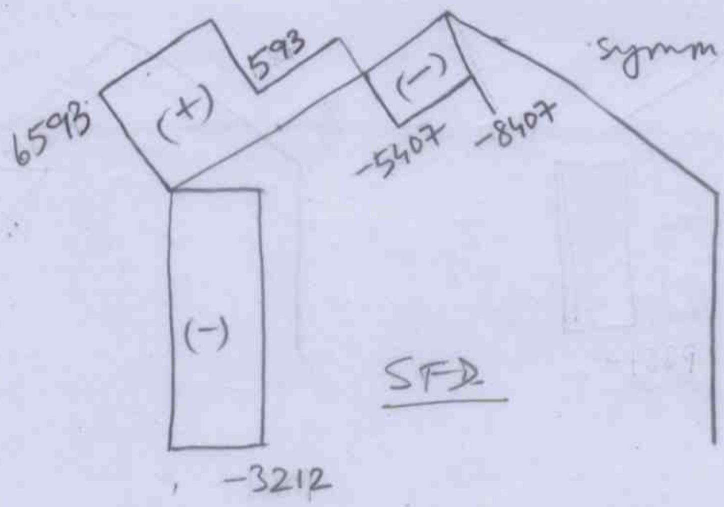
in S-D convention

$$M_{ab} = \frac{EI}{12}\left(2\theta_B - 6\left(-\frac{\Delta_b}{12}\right)\right) = 1.6394E4, \quad M_{ba} = M_{ab}+\frac{EI}{12}(2\theta_B) = 2.2146\,E4$$

$$M_{bc} = \frac{EI}{15}\left(4\theta_B - 6\left(\frac{\Delta_b}{9}\right)\right) - \frac{2\cdot1200\cdot5\cdot15}{9} = -2.2146\,E4$$ (check, ie same as M_{ba} for jt. equil).

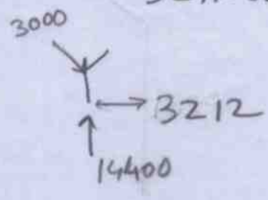
$$M_{cb} = \frac{EI}{15}\left(2\theta_B - 6\,\frac{\Delta_b}{9}\right) + \frac{2\cdot1200\cdot5\cdot15}{9} = 1.3252\,E4 \text{ kN.m}$$

⟨Results match with Mom^t distr method⟩

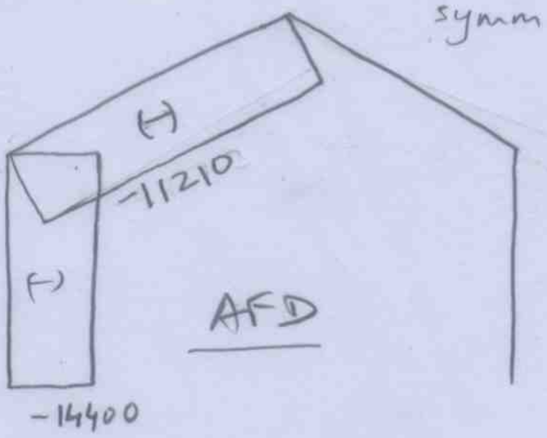


SFD

$$V_a = -\left(\frac{M_{ab} + M_{ba}}{12}\right) = -3211.68$$

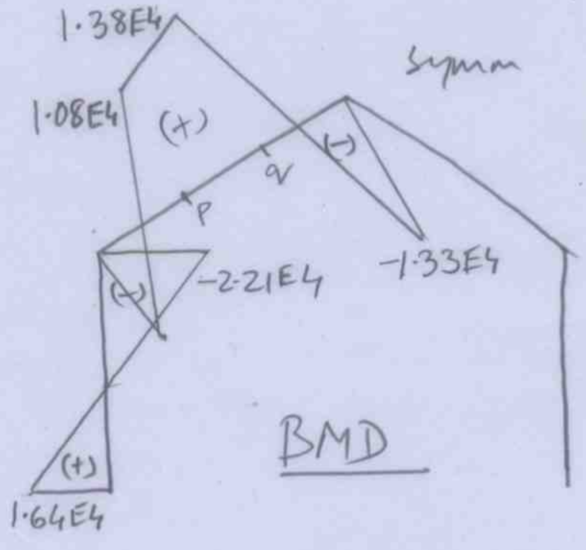


$$V_{b^+} = 14400 \cdot \frac{12}{15} - 3212 \cdot \frac{9}{15} - 3000 = 6592.8 \text{ kN}$$



AFD

$$N_{b^+} = 14400 \cdot \frac{9}{15} + 3212 \cdot \frac{12}{15} = 11209.6 \text{ kN}$$

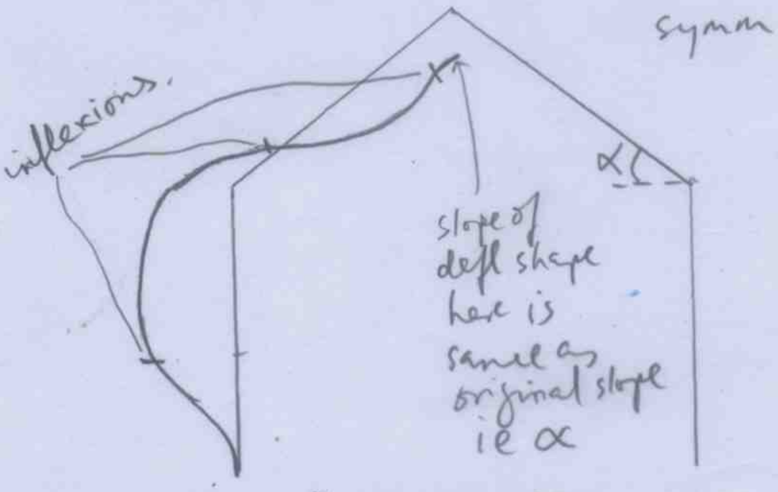


BMD

$$(BM)_p = -2.2146E4 + 6593 * 5 = 10819 \text{ kN.m}$$

$$(BM)_q = 10819 + 593 * 5 = 13784 \text{ kN.m}$$

$$(BM)_c = 13784 - 5407 * 5 = -13251 \text{ kN.m}$$



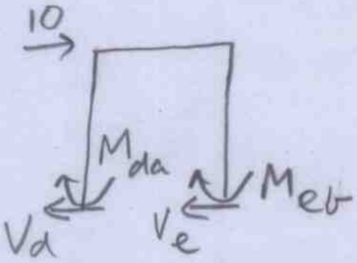
DEFL SHAPE

P2. Let $K = \frac{I}{L} = \frac{30}{5} = 6$. Use modified S-D eqns for far end SS.

$$EK\begin{bmatrix} 4\left(\frac{1}{2}+\frac{1}{2}\right) & 2\left(\frac{1}{2}\right) & 2\left(\frac{1}{2}\right) & 0 & -6\left(\frac{1}{2}\cdot\frac{1}{4}\right) \\ 2\left(\frac{1}{2}\right) & 4\left(\frac{1}{2}+\frac{1}{2}\right) & 0 & 2\left(\frac{1}{2}\right) & -6\left(\frac{1}{2}\cdot\frac{1}{4}\right) \\ 2\left(\frac{1}{2}\right) & 0 & 4\left(1+\frac{1}{2}+1\cdot\frac{3}{4}+1\right) & 2(1) & -6\left(\frac{1}{2}\cdot\frac{1}{4}\right) \\ 0 & 2\left(\frac{1}{2}\right) & 2(1) & 4\left(1+\frac{1}{2}+1\cdot\frac{3}{4}+1\right) & -6\left(\frac{1}{2}\cdot\frac{1}{4}\right) \\ (4+2)\left(\frac{1}{2}\right) & (4+2)\left(\frac{1}{2}\right) & (4+2)\left(\frac{1}{2}\right) & (4+2)\left(\frac{1}{2}\right) & -6\left(\frac{1}{2}\cdot\frac{1}{4}\right)\cdot 2\cdot 2 \end{bmatrix}\begin{Bmatrix}\theta_a \\ \theta_b \\ \theta_d \\ \theta_e \\ \Delta_a\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\\0\\-40\end{Bmatrix}$$

where we used $M_{ab} + M_{ad} = 0 \rightarrow (i)$; $M_{ba} + M_{be} = 0 \rightarrow (ii)$,
$M_{dc} + M_{da} + M_{de} + M_{dg} = 0 \rightarrow (iii)$; $M_{ed} + M_{ef} + M_{eb} + M_{eh} = 0 \rightarrow (iv)$

and sway eqn, $V_d + V_e = 10$



Soln is $EK\{\theta_a \ \theta_b \ \theta_d \ \theta_e \ \Delta_a\}^T = \{2.9787, 2.9787, 0.8511, 0.8511, 20.9929\}$

$; M_{gd} = M_{he} = 0$

$M_{ab} = EK(4(\frac{1}{2})\theta_a + 2(\frac{1}{2})\theta_b) = 8.9362$

$M_{ba} = EK(2(\frac{1}{2})\theta_a + 4(\frac{1}{2})\theta_b) = 8.9362$

$M_{ad} = EK(4(\frac{1}{2})\theta_a + 2(\frac{1}{2})\theta_d - 6(\frac{1}{2}\cdot\frac{1}{4})\Delta_a) = -8.9362$

$M_{da} = EK(2(\frac{1}{2})\theta_a + 4(\frac{1}{2})\theta_d - 6(\frac{1}{2}\cdot\frac{1}{4})\Delta_a) = -11.0638$

$M_{de} = EK(4(1)\theta_d + 2(1)\theta_e) = 5.1064$

$M_{ed} = EK(2(1)\theta_d + 4(1)\theta_e) = 5.1064$

$M_{be} = EK(4(\frac{1}{2})\theta_b + 2(\frac{1}{2})\theta_e - 6(\frac{1}{2}\cdot\frac{1}{4})\Delta_a) = -8.9362$

$M_{eb} = EK(2(\frac{1}{2})\theta_b + 4(\frac{1}{2})\theta_e - 6(\frac{1}{2}\cdot\frac{1}{4})\Delta_a) = -11.0638$

$M_{dc} = EK(4(1)\theta_a) = 3.4043$

$M_{cd} = EK(2(1)\theta_d) = 1.7021$

$M_{ef} = EK(4(1)\theta_e) = 3.4043$

$M_{fe} = EK(2(1)\theta_e) = 1.7021$

$M_{dg} = EK(3(1)\theta_d) = 2.5532$; $M_{eh} = EK(3(1)\theta_e) = 2.5532$

⟨Results match with Mom't distr — Tute 3, P.2⟩

$$V_{da} = -\frac{(-8.9362 - 11.0638)}{4} = 5 = V_{eb}$$

$$V_{ab} = -\frac{(8.9362 \times 2)}{5} = -3.5745$$

$$V_{cd} = -\frac{(3.4043 + 1.7021)}{5} = -1.0213$$

$$V_{gd} = -\frac{(2.5532)}{4} = -0.6383$$

$$V_{de} = -\frac{(5.1064 \times 2)}{5} = -2.0426$$