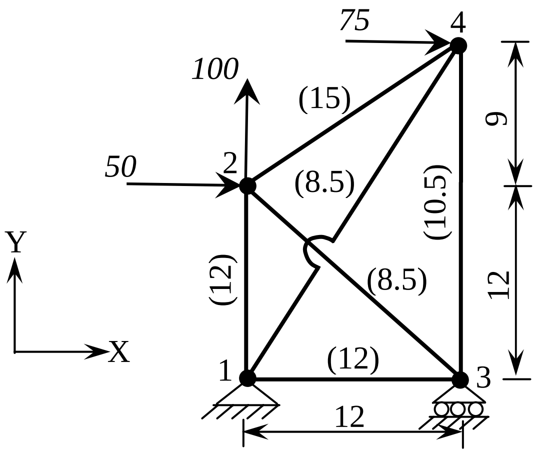
Department of Civil Engineering, IIT Bombay

CE317 **Structural Mechanics –II** **Tutorial 5**

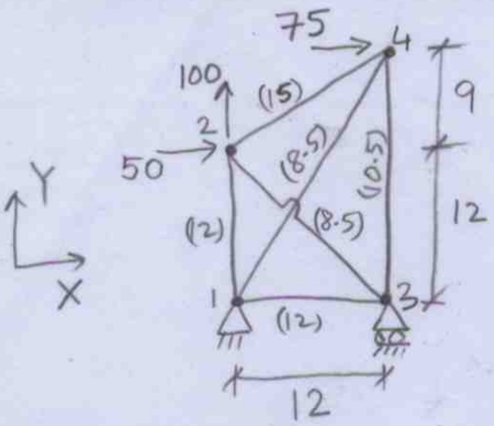
Use Stiffness matrix method. Use $E = 29E3$ kN/cm^2 and A as given in brackets for each member cm^2. All loads in *kN* and dimensions in *m*. Find deflections and member forces for following cases:

(i) Node 3 is roller, without settlement.
(ii) Node 3 is roller, with 0.1m downward settlement
(iii) Node 3 is hinge, without settlement.
(iv) Node 3 is hinge, with 0.1m downward settlement

P1



CE317 Tutorial 5



$E = 29E3$

Solve for two cases

Case(i) Node 3 is roller, no settlement

Case (ii) Node 3 is roller, 0.1m ↓ settlement.

Case(iii) Node 3 is hinge, no settlement

Case(iv) Node 3 is hinge, 0.1m ↓ settlement.

Transformation matrices: $\underline{\underline{a}}_{ij} = -\underline{\underline{a}}_{ji}$

$\underline{\underline{a}}_{12} = [0 \ -1]$, $\underline{\underline{a}}_{14} = \left[-\frac{12}{\sqrt{12^2+21^2}} \ \ -\frac{21}{\sqrt{12^2+21^2}}\right]$, $\underline{\underline{a}}_{13} = [-1 \ 0]$, $\underline{\underline{a}}_{32} = \left(\frac{1}{\sqrt{2}} \ -\frac{1}{\sqrt{2}}\right)$.

$\underline{\underline{a}}_{34} = [0 \ -1]$, $\underline{\underline{a}}_{24} = [-0.8 \ \ -0.6]$

Member stiffnesses: $\underline{\underline{k}}^{j}_{ii} = \underline{\underline{k}}^{i}_{jj}$, $\underline{\underline{k}}_{ij} = \underline{\underline{k}}_{ji}$

$\underline{\underline{k}}^{2}_{11} = \frac{E.12}{12} = \underline{\underline{k}}_{12} = \underline{\underline{k}}^{3}_{11} = \underline{\underline{k}}_{13}$, $\underline{\underline{k}}^{4}_{11} = \underline{\underline{k}}_{14} = \frac{E.(8.5)}{\sqrt{585}}$, $\underline{\underline{k}}^{2}_{33} = \underline{\underline{k}}_{32} = \frac{E.(8.5)}{12\sqrt{2}}$,

$\underline{\underline{k}}^{4}_{33} = \underline{\underline{k}}_{34} = E.\frac{(10.5)}{21}$, $\underline{\underline{k}}^{4}_{22} = \underline{\underline{k}}_{24} = E.\frac{15}{15}$.

Structure stiffness & reduced stiffnesses

$$\underline{\underline{K}} = \begin{bmatrix} \underline{\underline{K}}_{11} & \underline{\underline{K}}_{12} & \underline{\underline{K}}_{13} & \underline{\underline{K}}_{14} \\ \hline \underline{\underline{K}}_{21} & \underline{\underline{K}}_{22} & \underline{\underline{K}}_{23} & \underline{\underline{K}}_{24} \\ \underline{\underline{K}}_{31} & \underline{\underline{K}}_{32} & \underline{\underline{K}}_{33} & \underline{\underline{K}}_{34} \\ \underline{\underline{K}}_{41} & \underline{\underline{K}}_{42} & \underline{\underline{K}}_{43} & \underline{\underline{K}}_{44} \end{bmatrix} \qquad \underline{\underline{K}}_{ij} = \underline{\underline{K}}_{ji}$$

Case(i),(ii)

$\underline{\underline{\Delta}}^{T}_{I} = \{ (\Delta_1)_2 \ (\Delta_2)_2 \ (\Delta_1)_3 \ (\Delta_1)_4 \ (\Delta_2)_4 \}^T$

For $\underline{\underline{K}}_{II}$ eliminate rows & cols 1, 2, 6 from $\underline{\underline{K}}$

$\underline{\underline{K}}_{I\,II}$ eliminate rows 1, 2, 6, cols 3, 4, 5, 7, 8.

So we need the block indicated

$$\underline{\underline{K}}_{22} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} E \, [0 \ 1] + \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} E \frac{8.5}{12\sqrt{2}} \left[-\frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}}\right] + \begin{bmatrix} -0.8 \\ -0.6 \end{bmatrix} E \, (-0.8 \ -0.6)$$

$$E \begin{bmatrix} \frac{8.5}{24\sqrt{2}} + 0.64 & \frac{-8.5}{24\sqrt{2}} + 0.48 \\ \text{Symm} & 1 + \frac{8.5}{24\sqrt{2}} + 0.36 \end{bmatrix}$$

②

$$\underline{\underline{K}}_{33} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} E \begin{pmatrix} 1 & 0 \end{pmatrix} + \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} \frac{E\,8\cdot5}{12\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \frac{E}{2} \begin{pmatrix} 0 & -1 \end{pmatrix} = E \begin{bmatrix} 1+\frac{8\cdot5}{24\sqrt{2}} & -\frac{8\cdot5}{24\sqrt{2}} \\ \text{symm} & 0.5+\frac{8\cdot5}{24\sqrt{2}} \end{bmatrix}$$

$$\underline{\underline{K}}_{44} = \begin{bmatrix} \frac{12}{\sqrt{585}} \\ \frac{21}{\sqrt{585}} \end{bmatrix} \frac{E\,8\cdot5}{\sqrt{585}} \begin{bmatrix} \frac{12}{\sqrt{585}} & \frac{21}{\sqrt{585}} \end{bmatrix} + \begin{bmatrix} 0\cdot8 \\ 0\cdot6 \end{bmatrix} E \begin{bmatrix} 0\cdot8 & 0\cdot6 \end{bmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{E}{2} \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1224}{585\sqrt{585}} + 0\cdot64 & \frac{2142}{585\sqrt{585}} + 0\cdot48 \\ \frac{2142}{585\sqrt{585}} + 0\cdot48 & \frac{3748\cdot5}{585\sqrt{585}} + 0\cdot36 + 0\cdot5 \end{bmatrix}$$

$$\underline{\underline{K}}_{23} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \frac{E\,8\cdot5}{12\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = E \begin{bmatrix} -\frac{8\cdot5}{24\sqrt{2}} & \frac{8\cdot5}{24\sqrt{2}} \\ \frac{8\cdot5}{24\sqrt{2}} & -\frac{8\cdot5}{24\sqrt{2}} \end{bmatrix}$$

$$\underline{\underline{K}}_{12} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} E \begin{bmatrix} 0 & 1 \end{bmatrix} = E \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\underline{\underline{K}}_{13} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} E \begin{bmatrix} 1 & 0 \end{bmatrix} = E \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{\underline{K}}_{24} = \begin{bmatrix} -0\cdot8 \\ -0\cdot6 \end{bmatrix} E \begin{pmatrix} 0\cdot8 & 0\cdot6 \end{pmatrix} = E \begin{bmatrix} -0\cdot64 & -0\cdot48 \\ -0\cdot48 & -0\cdot36 \end{bmatrix}$$

$$\underline{\underline{K}}_{14} = \begin{bmatrix} \frac{-12}{\sqrt{585}} \\ \frac{-21}{\sqrt{585}} \end{bmatrix} \frac{E\,8\cdot5}{\sqrt{585}} \begin{bmatrix} \frac{12}{\sqrt{585}} & \frac{21}{\sqrt{585}} \end{bmatrix} = E \begin{bmatrix} \frac{-1224}{585^{3/2}} & \frac{-2142}{585^{3/2}} \\ \frac{-2142}{585^{3/2}} & \frac{-3748\cdot5}{585^{3/2}} \end{bmatrix}$$

$$\underline{\underline{K}}_{34} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \frac{E}{2} \begin{bmatrix} 0 & 1 \end{bmatrix} = E \begin{bmatrix} 0 & 0 \\ 0 & -0\cdot5 \end{bmatrix}$$

$$\underline{\underline{K}}_{II} = E \begin{bmatrix} \frac{8\cdot5}{24\sqrt{2}} + 0\cdot64 & -\frac{8\cdot5}{24\sqrt{2}} + 0\cdot48 & -\frac{8\cdot5}{24\sqrt{2}} & -0\cdot64 & -0\cdot48 \\ -\frac{8\cdot5}{24\sqrt{2}} + 0\cdot48 & 1+\frac{8\cdot5}{24\sqrt{2}} + 0\cdot36 & \frac{8\cdot5}{24\sqrt{2}} & -0\cdot48 & -0\cdot36 \\ -\frac{8\cdot5}{24\sqrt{2}} & \frac{8\cdot5}{24\sqrt{2}} & 1+\frac{8\cdot5}{24\sqrt{2}} & 0 & 0 \\ -0\cdot64 & -0\cdot48 & 0 & \frac{1224}{585^{3/2}} + 0\cdot64 & \frac{2142}{585^{3/2}} + 0\cdot48 \\ -0\cdot48 & -0\cdot36 & 0 & \frac{2142}{585^{-3/2}} + 0\cdot48 & \frac{3748\cdot5}{585^{-3/2}} + 0\cdot36 + 0\cdot5 \end{bmatrix}$$

$$\underline{\underline{K}}_{I\,II} = E \begin{bmatrix} 0 & 0 & \frac{8\cdot5}{24\sqrt{2}} \\ 0 & -1 & -\frac{8\cdot5}{24\sqrt{2}} \\ -1 & 0 & -\frac{8\cdot5}{24\sqrt{2}} \\ \frac{-1224}{585^{-3/2}} & \frac{-2142}{585^{-3/2}} & 0 \\ \frac{-2142}{585^{-3/2}} & \frac{-3748\cdot5}{585^{3/2}} & -0\cdot5 \end{bmatrix}$$

$$\underline{\Delta}_{I} = \underline{\underline{K}}_{II}^{-1}\left(\underline{P}_{I} - \underline{\underline{K}}_{I II}\,\underline{\Delta}_{II}\right) \longrightarrow (1)$$

Case(i) $\underline{P}_I^T = \{50 \;\; 100 \;\; 0 \;\; 75 \;\; 0\}^T$, $\underline{\Delta}_{II}^T = \{0 \;\; 0 \;\; 0\}^T$, use (1), get

$$\Delta_I^T = \frac{1}{E}\{\underset{(\Delta_1)_2}{579.2529} \quad \underset{(\Delta_2)_2}{196.6106} \quad \underset{(\Delta_1)_3}{76.6346} \quad \underset{(\Delta_1)_4}{925.2505} \quad \underset{(\Delta_2)_4}{-209.2307}\}^T$$

$$= \{0.02 \quad 0.0068 \quad 0.0026 \quad 0.0319 \quad -0.0072\}^T$$

$$\underline{F}_{12} = E\,[0 \;\; 1]\frac{1}{E}\begin{bmatrix}579.2529\\196.6106\end{bmatrix} = 196.61$$

$$\underline{F}_{13} = E\,[-1 \;\; 0]\frac{1}{E}\begin{bmatrix}76.6346\\0\end{bmatrix} = 76.63$$

$$\underline{F}_{14} = \frac{E(8.5)}{\sqrt{585}}\begin{bmatrix}\frac{12}{\sqrt{585}} & \frac{21}{\sqrt{585}}\end{bmatrix}\frac{1}{E}\begin{bmatrix}925.2505\\-209.2307\end{bmatrix} = 97.48$$

$$\underline{F}_{32} = \frac{E(8.5)}{12\sqrt{2}}\left[\begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\frac{1}{E}\begin{bmatrix}76.6346\\0\end{bmatrix} + \begin{bmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\frac{1}{E}\begin{bmatrix}579.2529\\196.6106\end{bmatrix}\right] = -108.38$$

$$\underline{F}_{34} = \frac{E}{2}\left[[0 \;\; -1]\frac{1}{E}\begin{bmatrix}76.6346\\0\end{bmatrix} + [0 \;\; 1]\frac{1}{E}\begin{bmatrix}925.2505\\-209.2307\end{bmatrix}\right] = -104.61$$

$$\underline{F}_{24} = E\left[[-0.8 \;\; -0.6]\frac{1}{E}\begin{bmatrix}579.2529\\196.6106\end{bmatrix} + [0.8 \;\; 0.6]\frac{1}{E}\begin{bmatrix}925.2505\\-209.2307\end{bmatrix}\right] = 33.29$$

Case(ii) P_I^T = same as before, $\Delta_{II}^T = \{0 \;\; 0 \;\; -0.1\}^T$, use (1), get

We get,

$$\Delta_I^T = \frac{1}{E}\{3.4793, \quad 0.1966 \quad 0.0766 \quad 6.0003 \quad -3.1092\}^T * 10^3$$

From above it is evident that $\underline{F}_{12}$ $\underline{F}_{13}$ remain same. Further,

$$\underline{F}_{32} = \frac{E(8.5)}{12\sqrt{2}}\left[\begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\frac{1}{E}\begin{bmatrix}76.6\\-E(0.1)\end{bmatrix} + \begin{bmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\frac{1}{E}\begin{bmatrix}3479.3\\196.6\end{bmatrix}\right] = -108.41$$

Note that $(\Delta_2)_3 = -0.1$ used.

$$\underline{F}_{14} = \frac{E(8.5)}{\sqrt{585}}\begin{bmatrix}\frac{12}{\sqrt{585}} & \frac{21}{\sqrt{585}}\end{bmatrix}\frac{1}{E}\begin{bmatrix}6000.3\\-3109.2\end{bmatrix} = 97.50$$

$$\underline{F}_{34} = \frac{E}{2}\left[[0 \;\; -1]\frac{1}{E}\begin{bmatrix}76.6\\-E(0.1)\end{bmatrix} + [0 \;\; 1]\frac{1}{E}\begin{bmatrix}6000.3\\-3109.2\end{bmatrix}\right] = 104.6.$$

$$\underline{F}_{24} = E\left[[-0.8 \;\; -0.6]\frac{1}{E}\begin{bmatrix}3479.3\\196.6\end{bmatrix} + [0.8 \;\; 0.6]\frac{1}{E}\begin{bmatrix}6000.3\\-3109.2\end{bmatrix}\right] = 33.32$$

So, as expected, displ's in case (ii) different from those in case (i) but forces remain same, since if jt. 3 is roller then its settlement contributes to rbm only & not to straining, so mem forces unaltered

Case (iii), (iv)

$$\underline{\Delta}_I^T = \{(\Delta_1)_2 \ (\Delta_2)_2 \ (\Delta_1)_4 \ (\Delta_2)_4\}^T$$

For $\underline{\underline{K}}_{II}$ eliminate rows & cols 1, 2, 5, 6 from $\underline{\underline{K}}$

$K_{I II}$ eliminate rows 1, 2, 5, 6, & cols 3, 4, 7, 8 from $\underline{\underline{K}}$

$\underline{\underline{K}}_{II}$ = use $\underline{\underline{K}}_{II}$ of cases (i), (ii) with 3rd row & col deleted, ie, $\underline{\underline{K}}_{II}$ is 4×4.

$$\underline{\underline{K}}_{I II} = \begin{bmatrix} \underline{\underline{K}}_{21} & \underline{\underline{K}}_{23} \\ \underline{\underline{K}}_{41} & \underline{\underline{K}}_{43} \end{bmatrix} = E \begin{bmatrix} 0 & 0 & -\frac{8.5}{24\sqrt{2}} & \frac{8.5}{24\sqrt{2}} \\ 0 & -1 & \frac{8.5}{24\sqrt{2}} & -\frac{8.5}{24\sqrt{2}} \\ \frac{-1224}{585^{3/2}} & \frac{-2142}{585^{3/2}} & 0 & 0 \\ \frac{-2142}{585^{3/2}} & \frac{-3748.5}{585^{3/2}} & 0 & -0.5 \end{bmatrix}$$

Case (iii) $\underline{P}_I^T = \{50 \ 100 \ 75 \ 0\}^T$, $\underline{\Delta}_{II}^T = \{0 \ 0 \ 0 \ 0\}^T$, use (1), get

$$\underline{\Delta}_I^T = \frac{1}{E}\{522.4632 \ \ 202.6576 \ \ 873.2120 \ \ -202.3199\}^T$$

$$\underline{F}_{12} = E[0 \ 1]\frac{1}{E}\begin{bmatrix}522.4632 \\ 202.6576\end{bmatrix} = 202.6576$$

$$\underline{F}_{13} = E[1 \ 0]\frac{1}{E}\begin{bmatrix}0 \\ 0\end{bmatrix} = 0$$

$$\underline{F}_{14} = E\frac{8.5}{\sqrt{585}}\begin{bmatrix}\frac{12}{\sqrt{585}} & \frac{21}{\sqrt{585}}\end{bmatrix}\frac{1}{E}\begin{bmatrix}873.2120 \\ -202.3199\end{bmatrix} = 90.52$$

$$\underline{F}_{32} = E\frac{8.5}{12\sqrt{2}}\begin{bmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\frac{1}{E}\begin{bmatrix}522.4632 \\ 202.6576\end{bmatrix} = -113.26$$

$$\underline{F}_{34} = \frac{E}{2}[0 \ 1]\frac{1}{E}\begin{bmatrix}873.2120 \\ -202.3199\end{bmatrix} = -101.16$$

$$\underline{F}_{24} = E\left[[-0.8 \ -0.6]\frac{1}{E}\begin{bmatrix}522.4632 \\ 202.6576\end{bmatrix} + [0.8 \ 0.6]\begin{bmatrix}873.2120 \\ -202.3199\end{bmatrix}\right] = 37.61$$

Case (iv) $\underline{P}_I^T = \{50 \quad 100 \quad 75 \quad 0\}^T$, $\underline{\Delta}_{II}^T = \{0 \quad 0 \quad 0 \quad -0.1\}^T$, use (1), get,

$$\underline{\Delta}_I^T = \frac{1}{E}\{3422.5 \quad 202.7 \quad 5948.2 \quad -3102.3\}$$

$$\underline{F}_{12} = E[0 \quad 1]\frac{1}{E}\begin{bmatrix} 3422.5 \\ 202.7 \end{bmatrix} = 202.7$$

$$\underline{F}_{13} = E[1 \quad 0]\frac{1}{E}\begin{Bmatrix} 0 \\ -0.1E \end{Bmatrix} = 0$$

$$\underline{F}_{14} = E\frac{(8.5)}{\sqrt{585}}\left[\frac{12}{\sqrt{585}} \quad \frac{21}{\sqrt{585}}\right]\frac{1}{E}\begin{bmatrix} 5948.2 \\ -3102.3 \end{bmatrix} = 90.52$$

$$\underline{F}_{32} = \frac{E(8.5)}{12\sqrt{2}}\left[\left[\frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}}\right]\frac{1}{E}\begin{bmatrix} 0 \\ -0.1E \end{bmatrix} + \left[-\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right]\frac{1}{E}\begin{bmatrix} 3422.5 \\ 202.7 \end{bmatrix}\right] = -1140.31$$

$$\underline{F}_{34} = \frac{E}{2}\left[[0 \quad -1]\frac{1}{E}\begin{bmatrix} 0 \\ -0.1E \end{bmatrix} + [0 \quad 1]\frac{1}{E}\begin{bmatrix} 5948.2 \\ -3102.3 \end{bmatrix}\right] = -1551.1$$

$$\underline{F}_{24} = E\left[[-0.8 \quad -0.6]\frac{1}{E}\begin{bmatrix} 3422.5 \\ 202.7 \end{bmatrix} + [0.8 \quad 0.6]\frac{1}{E}\begin{bmatrix} 5948.2 \\ -3102.3 \end{bmatrix}\right] = 37.56$$

Cases (i), (ii)

$[K]_{I\,I}$ =

0.8904	0.2296	-0.2504	-0.6400	-0.4800
0.2296	1.6104	0.2504	-0.4800	-0.3600
-0.2504	0.2504	1.2504	0	0
-0.6400	-0.4800	0	0.7265	0.6314
-0.4800	-0.3600	0	0.6314	1.1249

$[K]_{I\,II}$ =

0	0	0.2504
0	-1.0000	-0.2504
-1.0000	0	-0.2504
-0.0865	-0.1514	0
-0.1514	-0.2649	-0.5000

Cases (i), (ii)

$[K]_{I\,I}$ =

0.8904	0.2296	-0.6400	-0.4800
0.2296	1.6104	-0.4800	-0.3600
-0.6400	-0.4800	0.7265	0.6314
-0.4800	-0.3600	0.6314	1.1249

$[K]_{I\,II}$ =

0	0	-0.2504	0.2504
0	-1.0000	0.2504	-0.2504
-0.0865	-0.1514	0	0
-0.1514	-0.2649	0	-0.5000