Department of Civil Engineering, IIT Bombay

CE317 Structural Mechanics –II Tutorial 7

P1. Use stiffness method to find joint deflections, member end forces, and reactions (Fig. 1). Use I/A=3000mm^2. Both members experience a constant temperature gradient through the thickness with $\Delta T = T_b - T_t = 30^o$ C.
Use $\alpha_t = 0.000012/^{o}$C, $h = 40$ mm, $EI = 500000$ kNm^2. Work the problem with and without mechanical loads.

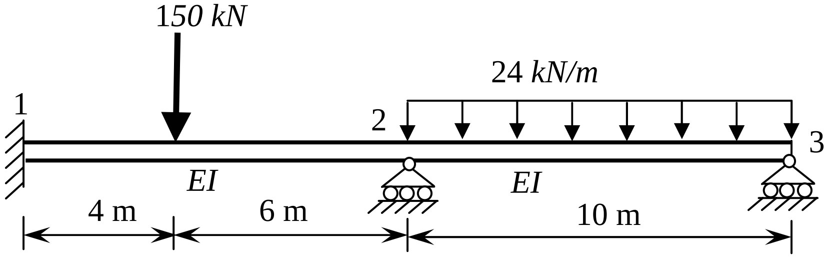

Fig. 1

P2. The structure shown in Fig. 2a is built with member 1-2 having initial crookedness as shown in Fig. 2b. Member 1-2 is forced to fit. Take $EI = 400000$ kNm2. Use stiffness method to find joint deflections, member end forces, and reactions.

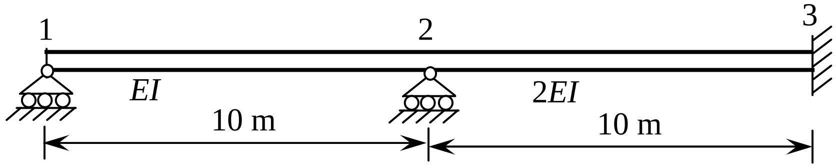

Fig2a

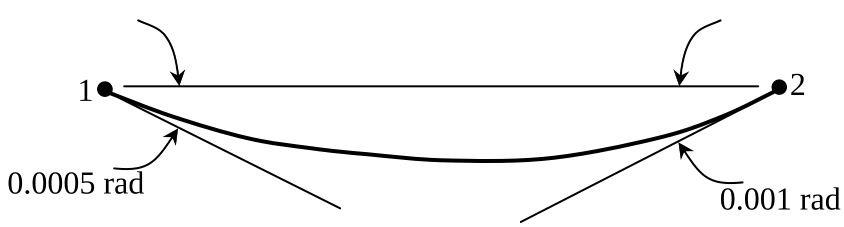

Fig. 2b

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%Tutorial 7 solution Problem 1

% no horizontal loading so can use beam formulation.
%g=A/I in m^2 for respective member, but not required

%solution in Matlab

a12=[-1 0; 0 1]; a21=[1 0; 0 1]; a23=a12; a32=a21;

L1=10; L2=10;
DelT12=30; DelT23=30; alpha=0.000012; h=40/1000; EI=5e5;

%Member stiffnesses…local
k112=EI/L1*[12/L1^2 -6/L1; -6/L1 4]; k221=k112;
k12=EI/L1*[12/L1^2 -6/L1; -6/L1 2]; k21=k12;
k223=EI/L2*[12/L2^2 -6/L2; -6/L2 4]; k332=k223;
k23=EI/L2*[12/L2^2 -6/L2; -6/L2 2]; k32=k23;

%Member stiffnesses…global
K11=a12'*k112*a12; K12=a12'*k12*a21; K21=K12';
K22=a21'*k221*a21 + a23'*k223*a23;
K33=a32'*k332*a32;
K23=a23'*k23*a32; K32=K23';
K13=zeros(2,2); K31= zeros(2,2);

%remove rows/cols 1,2,3,5 from K_Total
KII=[K22(2,2) K23(2,2); K32(2,2) K33(2,2)]

%remove rows 4,6, and cols 1,2,3,5 from K_Total
KII_I=[K12(1,2) K13(1,2); K12(2,2) K13(2,2); K22(1,2)
K23(1,2); K32(1,2) K33(1,2)]
------------------------------------------------------------------------------
%members 1-2 and 2-3 cut on either side of node 2, i.e., stress free
thermal displacements applied at node 2 for members 1-2 and 2-3.
```

```
delta21s=-[-alpha*DelT12*L1^2/2/h -alpha*DelT12*L1/h]';
delta23s=-[alpha*DelT23*L1^2/2/h +alpha*DelT23*L1/h]';

%Fixed end forces due to thermal + mechanical loads
F12f=[150*6/10+150/L1^3*(6^2*4-6*4^2) ; -150*6^2*4/L1^2]
+k12* delta21s;
F21f=[-150*4/10+150/L1^3*(6^2*4-6*4^2) ; 150*4^2*6/L1^2]
+k221* delta21s;
F23f=[24*L2/2 ; -24*L2^2/12] +k223* delta23s;
F32f=[-24*L2/2 ; 24*L2^2/12] +k32* delta23s;

%Equivalent nodal loads due to thermal + mechanical loads
P1e= a12'*F12f; P2e= a21'*F21f + a23'*F23f; P3e=a32'*F32f;
Pa=[0 0]';

Pe=[P1e' P2e' P3e']'; Petilde=[Pe(4) Pe(6)]';
PI=Pa-Petilde; Pehat= [Pe(1) Pe(2) Pe(3) Pe(5)]';

%Nodal displacements
DeltaI=inv(KII)*(PI)

%Reactions
P_II=KII_I*DeltaI+Pehat

%Member end forces
DeltaII=[0 0 0 0]';
Delta1= [DeltaII(1); DeltaII(2)];
Delta2= [DeltaII(3); DeltaI(1)];
Delta3= [DeltaII(4); DeltaI(2)];
F12=k112*a12*Delta1 + k12*a21*Delta2+F12f
F21=k21*a12*Delta1 + k221*a21*Delta2+F21f
F23=k223*a23*Delta2 + k23*a32*Delta3+F23f
F32=k23*a23*Delta2 + k332*a32*Delta3+F32f

KII =
```

400000 100000
100000 200000

KII_I =

30000	0
100000	0
0	30000
-30000	-30000

DeltaI =
0.0069
-0.0269

P_II =
1.0e+003 *
0.1090
-4.0286
-0.9809
0.4819

F12 =
1.0e+003 *
-0.1090
-4.0286

F21 =
1.0e+003 *
-0.2590
6.0189

F23 =
1.0e+003 *
0.7219
-6.0189

F32 =

```
  481.8857
         0
```

```
%member 1-2 cut at node 2, member 2-3 cut at node 3, i.e., stress
free thermal displacements applied at node 2 for mem 1-2 and
node 3 for mem 2-3.
delta21s=-[-alpha*DelT12*L1^2/2/h -alpha*DelT12*L1/h]';
delta32s=delta21s;

%Fixed end forces due to thermal + mechanical loads
F12f=[150*6/10+150/L1^3*(6^2*4-6*4^2) ; -150*6^2*4/L1^2]
+k12* delta21s;
F21f=[-150*4/10+150/L1^3*(6^2*4-6*4^2) ; 150*4^2*6/L1^2]
+k221* delta21s;
F23f=[24*L2/2 ; -24*L2^2/12] +k23* delta32s;
F32f=[-24*L2/2 ; 24*L2^2/12] +k332* delta32s;

%Equivalent nodal loads due to thermal + mechanical loads
P1e= a12'*F12f; P2e= a21'*F21f + a23'*F23f; P3e=a32'*F32f;
Pa=[0 0]';

Pe=[P1e' P2e' P3e']'; Petilde=[Pe(4) Pe(6)]';
PI=Pa-Petilde; Pehat= [Pe(1) Pe(2) Pe(3) Pe(5)]';

%Nodal displacements
DeltaI=inv(KII)*(PI)

%Reactions
P_II=KII_I*DeltaI+Pehat

%Member end forces
DeltaII=[0 0 0 0]';
Delta1= [DeltaII(1); DeltaII(2)];
Delta2= [DeltaII(3); DeltaI(1)];
Delta3= [DeltaII(4); DeltaI(2)];
```

```
F12=k112*a12*Delta1 + k12*a21*Delta2+F12f
F21=k21*a12*Delta1 + k221*a21*Delta2+F21f
F23=k223*a23*Delta2 + k23*a32*Delta3+F23f
F32=k23*a23*Delta2 + k332*a32*Delta3+F32f

DeltaI =
    0.0069
   -0.0269

P_II =
  1.0e+003 *
    0.1090
   -4.0286
   -0.9809
    0.4819

F12 =
  1.0e+003 *
   -0.1090
   -4.0286

F21 =
  1.0e+003 *
   -0.2590
    6.0189

F23 =
  1.0e+003 *
    0.7219
   -6.0189

F32 =
  481.8857
    0.0000
--------------------------------------------------------------------------------
```

%members 1-2 and 2-3 cut at node 1 and 3, respectively, i.e., stress free thermal displacements applied at node 1 for members 1-2 and 2-3.

```
delta12s=-[alpha*DelT12*L1^2/2/h alpha*DelT12*L1/h]';
delta32s=-[-alpha*DelT23*L1^2/2/h -alpha*DelT23*L1/h]';

%Fixed end forces due to thermal + mechanical loads
F12f=[150*6/10+150/L1^3*(6^2*4-6*4^2) ; -150*6^2*4/L1^2]
+k112* delta12s;
F21f=[-150*4/10+150/L1^3*(6^2*4-6*4^2) ; 150*4^2*6/L1^2]
+k21* delta12s;
F23f=[24*L2/2 ; -24*L2^2/12] +k23* delta32s;
F32f=[-24*L2/2 ; 24*L2^2/12] +k332* delta32s;

%Equivalent nodal loads due to thermal + mechanical loads
P1e= a12'*F12f; P2e= a21'*F21f + a23'*F23f; P3e=a32'*F32f;
Pa=[0 0]';

Pe=[P1e' P2e' P3e']'; Petilde=[Pe(4) Pe(6)]';
PI=Pa-Petilde; Pehat= [Pe(1) Pe(2) Pe(3) Pe(5)]';

%Nodal displacements
DeltaI=inv(KII)*(PI)

%Reactions
P_II=KII_I*DeltaI+Pehat

%Member end forces
DeltaII=[0 0 0 0]';
Delta1= [DeltaII(1); DeltaII(2)];
Delta2= [DeltaII(3); DeltaI(1)];
Delta3= [DeltaII(4); DeltaI(2)];
F12=k112*a12*Delta1 + k12*a21*Delta2+F12f
F21=k21*a12*Delta1 + k221*a21*Delta2+F21f
F23=k223*a23*Delta2 + k23*a32*Delta3+F23f
```

```
F32=k23*a23*Delta2 + k332*a32*Delta3+F32f

DeltaI =
    0.0069
   -0.0269

P_II =
  1.0e+003 *
    0.1090
   -4.0286
   -0.9809
    0.4819

F12 =
  1.0e+003 *
   -0.1090
   -4.0286

F21 =
  1.0e+003 *
   -0.2590
    6.0189

F23 =
  1.0e+003 *
    0.7219
   -6.0189

F32 =
  481.8857
         0
-------------------------------------------------------------------------------
%Fixed end forces due to thermal loads only, no mechanical loads
F12f= k12* delta21s;
F21f= k221* delta21s;
```

```
F23f= k223* delta23s;
F32f= k32* delta23s;

%Equivalent nodal loads due to thermal + mechanical loads
P1e= a12'*F12f; P2e= a21'*F21f + a23'*F23f; P3e=a32'*F32f;
Pa=[0 0]';

Pe=[P1e' P2e' P3e']'; Petilde=[Pe(4) Pe(6)]';
PI=Pa-Petilde; Pehat= [Pe(1) Pe(2) Pe(3) Pe(5)]';

%Nodal displacements
DeltaI=inv(KII)*(PI)

%Reactions
P_II=KII_I*DeltaI+Pehat

%Member end forces
DeltaII=[0 0 0 0]';
Delta1= [DeltaII(1); DeltaII(2)];
Delta2= [DeltaII(3); DeltaI(1)];
Delta3= [DeltaII(4); DeltaI(2)];
F12=k112*a12*Delta1 + k12*a21*Delta2+F12f
F21=k21*a12*Delta1 + k221*a21*Delta2+F21f
F23=k223*a23*Delta2 + k23*a32*Delta3+F23f
F32=k23*a23*Delta2 + k332*a32*Delta3+F32f

DeltaI =
    0.0064
   -0.0257

P_II =
  1.0e+003 *
    0.1929
   -3.8571
   -0.7714
```

```
    0.5786

F12 =
  1.0e+003 *
   -0.1929
   -3.8571

F21 =
  1.0e+003 *
   -0.1929
    5.7857

F23 =
  1.0e+003 *
    0.5786
   -5.7857

F32 =
  578.5714
         0
```

```
%Tutorial 7 solution Problem 2

% no horizontal loading so can use beam formulation.
%g=A/I in m^2 for respective member, but not required

%Solution in matlab

a12=[-1 0; 0 1]; a21=[1 0; 0 1]; a23=a12; a32=a21;

L1=10; L2=10; EI=4e5;

%Member stiffnesses…local
k112=EI/L1*[12/L1^2 -6/L1; -6/L1 4]; k221=k112;
k12=EI/L1*[12/L1^2 -6/L1; -6/L1 2]; k21=k12;
k223=2*EI/L2*[12/L2^2 -6/L2; -6/L2 4]; k332=k223;
k23=2*EI/L2*[12/L2^2 -6/L2; -6/L2 2]; k32=k23;

%Member stiffnesses…global
K11=a12'*k112*a12; K12=a12'*k12*a21; K21=K12';
K22=a21'*k221*a21 + a23'*k223*a23;
K33=a32'*k332*a32;
K23=a23'*k23*a32; K32=K23';
K13=zeros(2,2); K31= zeros(2,2);

%remove rows/cols 1,3,5,6 from K_Total
KII=[K11(2,2) K12(2,2); K21(2,2) K22(2,2)]

%remove rows 2,4 and cols 1,3,5,6 from K_Total
KII_I=[K11(1,2) K12(1,2); K21(1,2) K22(1,2); K31(1,2)
K32(1,2); K31(2,2) K32(2,2)]

%self straining displacements (local) due to misfit.
delta21s=-[0 -0.001]'; delta12s=-[0 0.0005]';
delta23s=[0 0]'; delta32s=[0 0]';
```

```
%Fixed end forces due to misfit
F12f= k112*delta12s+k12* delta21s;
F21f= k21*delta12s+k221* delta21s;
F23f= k223* delta23s+ k23* delta32s
F32f= k32* delta23s+ k332* delta32s

%Equivalent nodal loads due to misfit
P1e= a12'*F12f; P2e= a21'*F21f + a23'*F23f; P3e=a32'*F32f;
Pa=[0 0]';

Pe=[P1e' P2e' P3e']'; Petilde=[Pe(2) Pe(4)]';
PI=Pa-Petilde; Pehat= [Pe(1) Pe(3) Pe(5) Pe(6)]';

%Nodla displacements
DeltaI=inv(KII)*(PI)

%Reactions
P_II=KII_I*DeltaI+Pehat

%Member end forces
DeltaII=[0 0 0 0]';
Delta1= [DeltaII(1); DeltaI(1)];
Delta2= [DeltaII(2); DeltaI(2)];
Delta3= [DeltaII(3); DeltaII(4)];
F12=k112*a12*Delta1 + k12*a21*Delta2+F12f
F21=k21*a12*Delta1 + k221*a21*Delta2+F21f
F23=k223*a23*Delta2 + k23*a32*Delta3+F23f
F32=k23*a23*Delta2 + k332*a32*Delta3+F32f

KII =
      160000       80000
       80000      480000

KII_I =
       24000       24000
```

```
   -24000      24000
        0     -48000
        0     160000

F23f =
     0
     0

F32f =
     0
     0

DeltaI =
  1.0e-003 *
    0.1364
   -0.2727

P_II =
    8.7273
  -21.8182
   13.0909
  -43.6364

F12 =
   -8.7273
         0

F21 =
   -8.7273
   87.2727

F23 =
   13.0909
  -87.2727
```

F32 =

13.0909

-43.6364