Department of Civil Engineering, IIT Bombay

CE317 **Structural Mechanics –II** **Tutorial 8**

1. Using stiffness method find joint deflections, member end forces, and reactions for the Grid shown in **Fig. 1**

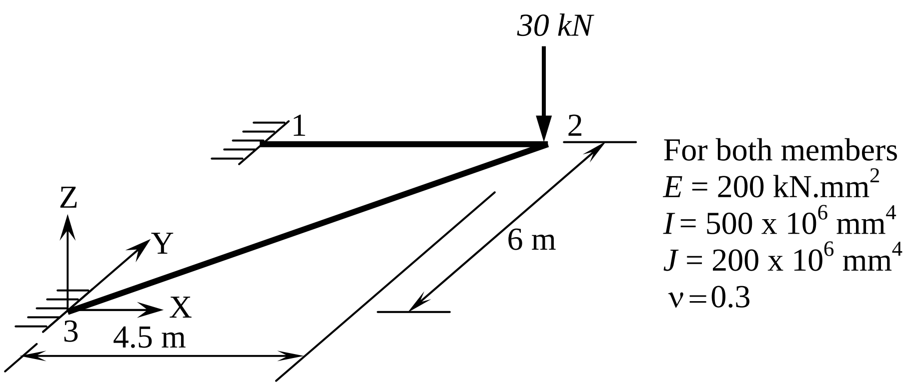

Fig.1

2. Using stiffness method find joint deflections, member end forces, and reactions for the Beam-Truss shown in **Fig. 2**. Use material and geometric properties from Fig.1 and I/A=3000mm^2

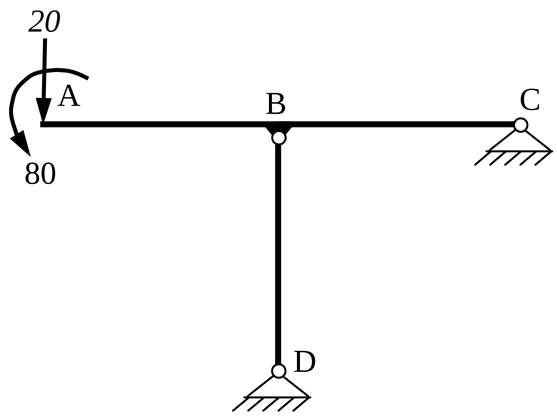

Fig. 2

```
%Tutorial 8 solution Problem 1
% Grid problem
%solution in Matlab

format long

a12=[1 0 0; 0 -1 0; 0 0 -1]; a21=eye(3,3);
a23=[1 0 0; 0 3/5 4/5; 0 -4/5 3/5];
a32=[1 0 0; 0 -3/5 -4/5; 0 4/5 -3/5];

%units Kn, mm

L1=4500; L2=7500; E1=200; E2=200; I1=5e8; I2=5e8; J1=2e8;
J2=2e8; nu1=0.3; nu2=0.3; G1=E1/2/(1+nu1); G2=E2/2/(1+nu2);
alf1=G1*J1/E1/I1; alf2=G2*J2/E2/I2;

%Member stiffnesses…local
k112= E1*I1/L1*[12/L1^2  0 6/L1; 0 alf1 0; 6/L1 0 4];
k221=k112;
k12= E1*I1/L1*[-12/L1^2  0 -6/L1; 0 alf1 0; -6/L1 0 -2];
k21=k12;
k223= E2*I2/L2*[12/L2^2  0 6/L2; 0 alf2 0; 6/L2 0 4];
k332=k223;
k23= E2*I2/L2*[-12/L2^2  0 -6/L2; 0 alf2 0; -6/L2 0 -2];
k32=k23;

%Member stiffnesses…global
K11=a12'*k112*a12; K12=a12'*k12*a21; K21=K12';
K22=a21'*k221*a21 + a23'*k223*a23;
K33=a32'*k332*a32;
K23=a23'*k23*a32; K32=K23';
K13=zeros(3,3); K31= K13';

%remove rows/cols 1,2,3,7,8,9 from K_Total
KII=K22
```

```
%remove rows 4,5,6, and cols 1,2,3,7,8,9 from K_Total
KII_I=[K12; K32]

PI=[-30 0 0]';

%Nodal displacements
DeltaI=inv(KII)*(PI)

%Reactions
P_II=KII_I*DeltaI

%Member end forces
DeltaII=[0 0 0 0 0 0]';
Delta1= [DeltaII(1); DeltaII(2); DeltaII(3)];
Delta2= DeltaI;
Delta3= [DeltaII(4); DeltaII(5); DeltaII(6)];
F12=k112*a12*Delta1 + k12*a21*Delta2
F21=k21*a12*Delta1 + k221*a21*Delta2
F23=k223*a23*Delta2 + k23*a32*Delta3
F32=k23*a23*Delta2 + k332*a32*Delta3

KII =
  1.0e+008 *
   0.00000016013169  -0.00008533333333   0.00036029629630
  -0.00008533333333   0.38290598290598  -0.24615384615385
   0.00036029629630  -0.24615384615385   1.09401709401709

KII_I =
  1.0e+007 *
  -0.00000131687243                  0  -0.00296296296296
                  0  -0.34188034188034                  0
   0.00296296296296                  0   4.44444444444444
  -0.00000028444444   0.00085333333333  -0.00064000000000
  -0.00085333333333   1.63282051282051  -1.37846153846154
   0.00064000000000  -1.37846153846154   0.82871794871795
```

```
DeltaI =
  -7.24313585253824
  -0.00009435734388
   0.00236417567173

P_II =
  1.0e+005 *
   0.00025333209431
   0.00322589209856
  -1.09536958368734
   0.00004666790569
   0.27678154205608
  -0.25463041631266

F12 =
  1.0e+005 *
   0.00025333209431
  -0.00322589209856
   1.09536958368734

F21 =
  1.0e+003 *
  -0.02533320943076
  -0.32258920985556
  -4.46248406966866

F23 =
  1.0e+003 *
  -0.00466679056924
   3.76354078164826
   2.41941907391671

F32 =
  1.0e+004 *
   0.00046667905692
   0.37635407816483
   3.74203483432456
```

```
%Tutorial 8 solution Problem 2
%beam-truss. Done as beam-truss, so K (7x7)
% no horizontal loading so can use beam formulation.
%g=A/I in m^2 for respective member, but not required
%Solution in matlab

a12=[-1 0; 0 1]; a21=[1 0; 0 1]; a23=a12; a32=a21; a24=[0 -1];
a42=-a24;

%EI/EA=1/g

L1=10; L2=10; L3=10; EI1=1e5; EI2=1e5; g=1/30; EA3=1e5*g;

%Member stiffnesses…local
k112=EI1/L1*[12/L1^2 -6/L1; -6/L1 4]; k221=k112;
k12=EI1/L1*[12/L1^2 -6/L1; -6/L1 2]; k21=k12;
k223=EI2/L2*[12/L2^2 -6/L2; -6/L2 4]; k332=k223;
k23=EI2/L2*[12/L2^2 -6/L2; -6/L2 2]; k32=k23;
k224=EA3/L3*[1]; k442=k224; k24=k224; k42=k224;


%Member stiffnesses…global
K11=a12'*k112*a12; K12=a12'*k12*a21; K21=K12';
K221=a21'*k221*a21; K223=a23'*k223*a23;
K224= a24'*k224*a24; K442=K224;
K22= K221+K223+[K224(2,2) 0; 0 0]
K23=a23'*k23*a32;
K24truss= a24'*k24*a42; K24=[K24truss(2,2); 0]
K33=a32'*k332*a32; K32=K23';
K13=zeros(2,2); K31= zeros(2,2);
K442=K224; K44=[K442(2,2)]; K42= K24';
K14=zeros(2,1); K41= K14';  K34=zeros(2,1); K43= K34';
```

```
%remove rows/cols 5, 7 from K_Total
KII=[K11 K12 K13(:,2); K21 K22 K23(:,2); K31(2,:)  K32(2,:)
K33(2,2)]

%remove rows 1,2,3,4,6 and cols 5, 7 from K_Total
KII_I=[K31(1,:) K32(1,:) K33(1,2); K41(1,:) K42(1,:) K43(1,2);]

PI=[20 -80 0 0 0]';

%Nodal displacements
DeltaI=inv(KII)*(PI)

%Reactions
P_II=KII_I*DeltaI

%Member end forces
DeltaII=[0 0]';
Delta1= [DeltaI(1); DeltaI(2)];
Delta2= [DeltaI(3); DeltaI(4)];
Delta3= [DeltaII(1); DeltaI(5)];
Delta4= [DeltaII(2)];
F12=k112*a12*Delta1 + k12*a21*Delta2
F21=k21*a12*Delta1 + k221*a21*Delta2
F23=k223*a23*Delta2 + k23*a32*Delta3
F32=k23*a23*Delta2 + k332*a32*Delta3
F24=k224*a24*Delta2 + k24*a42*Delta4
F42=k42*a24*Delta2 + k442*a42*Delta4
```

KII =
  1.0e+004 *
   0.12000000000000   0.60000000000000 -0.12000000000000
0.60000000000000   0
   0.60000000000000   4.00000000000000 -0.60000000000000
2.00000000000000   0
  -0.12000000000000 -0.60000000000000   0.27333333333333
0   0.60000000000000
   0.60000000000000   2.00000000000000   0
8.00000000000000   2.00000000000000
                  0   0   0.60000000000000   2.00000000000000
4.00000000000000

KII_I =
  1.0e+003 *
                  0   0 -1.20000000000000 -
6.00000000000000 -6.00000000000000
                  0   0 -0.33333333333333   0
0

DeltaI =
   0.48800000000000
  -0.04173333333333
   0.14400000000000
  -0.02373333333333
  -0.00973333333333

P_II =
  28.00000000000007
 -48.00000000000003

F12 =
 -20.00000000000000
 -79.99999999999977

```
F21 =
  1.0e+002 *
  -0.20000000000000
   2.80000000000000

F23 =
  1.0e+002 *
   0.28000000000000
  -2.80000000000001

F32 =
  28.00000000000007
  -0.00000000000017

F24 =
   7.91111111111112   7.91111111111112

F42 =
   7.91111111111112   7.91111111111112
```