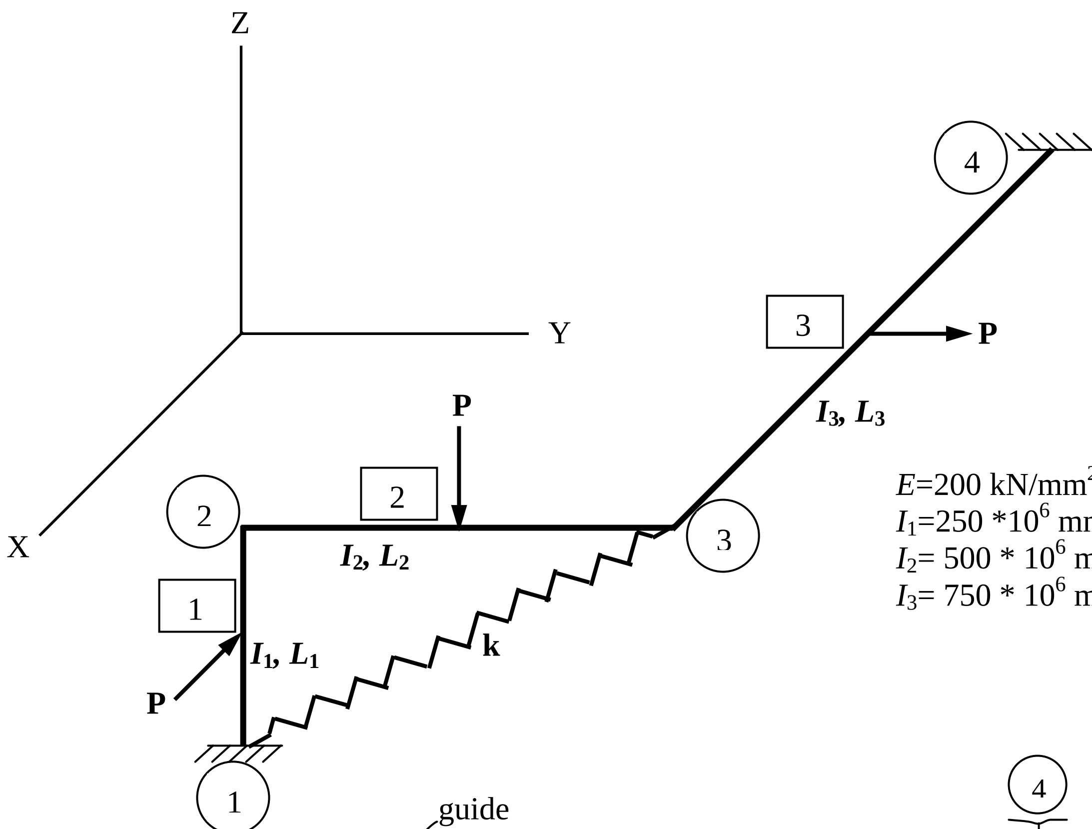
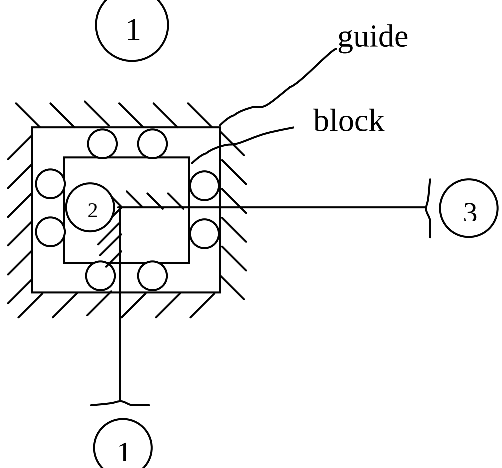
Department of Civil Engineering, IIT Bombay

CE317 Structural Mechanics –II Tutorial 9

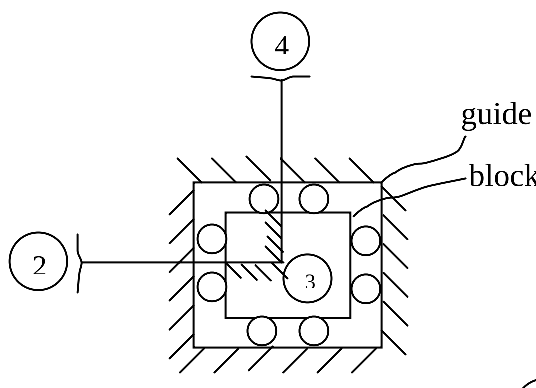
The figure shows a space frame. Nodes 2 and 3 are restrained by guides as shown below. Loads are applied at the midpoint of members in coordinate directions as shown. Use $P = 50$ kN. All members have circular cross-section. Member 2 has a linear temperature variation in the Z direction, with upper temperature 50 ^oC and lower temperature 25 ^oC. Use coefficient of thermal expansion as $\alpha_T = 12$ E-6 / ^oC. Member 1 is 100cm too long and its end at node 1 has a rotation misfit of -0.001 radians along Y axis. The support at node 4 settles 0.025m down (-Z). Use spring constant $k = 5$ E5 kN/m for the spring connecting nodes 1 and 3. Determine nodal displacements, reactions, member end forces and force in the spring, using the stiffness method.



E=200 kN/mm^2, $\nu = 0.3$
I_1=250 *10^6 mm^4, L_1= 10 m
I_2= 500 * 10^6 mm^4, L_2= 15 m
I_3= 750 * 10^6 mm^4, L_3= 20 m



Front view (along –X, in YZ plane at (2))



Top view (along –Z, in XY plane at (3))

%Tutorial 9 solution Problem 1
% Space frame problem
%solution in Matlab
%ambient temp of 25^{0}C assumed.
%height of members for temperature straining calculations assumed same as diameter of the circular member.

```
format long

%transformation matrices
a12=[0 0 -1; 0 -1 0; -1 0 0]; a21=[0 0 1; 0 1 0; -1 0 0];
a23=[0 -1 0; 0 0 1; -1 0 0]; a32=[0 1 0; 0 0 -1; -1 0 0];
a34=[1 0 0; 0 -1 0; 0 0 -1]; a43=[-1 0 0; 0 1 0; 0 0 -1];
a13=[0 -15/325^.5 -10/325^.5]; a31=-a13;

trans12=[a12 zeros(3,3); zeros(3,3) a12];
trans21=[a21 zeros(3,3); zeros(3,3) a21];
trans23=[a23 zeros(3,3); zeros(3,3) a23];
trans32=[a32 zeros(3,3); zeros(3,3) a32];
trans34=[a34 zeros(3,3); zeros(3,3) a34];
trans43=[a43 zeros(3,3); zeros(3,3) a43];

%units Kn, mm

%circular section members
L1=10000; L2=15000; L3=20000;
E1=200; E2=200; E3=200;
I1=2.5e8; I2=5e8; I3=7.5e8;
J1=2*I1; J2=2*I2; J3=2*I3;
r1=(4*I1/pi)^0.25; r2=(4*I2/pi)^0.25; r3=(4*I3/pi)^0.25;
h1=r1; h2=r2; h3=r3;
A1=r1^2*pi; A2=r2^2*pi; A3=r3^2*pi;
nu1=0.3; nu2=0.3; nu3=0.3;
G1=E1/2/(1+nu1); G2=E2/2/(1+nu2); G3=E3/2/(1+nu3);
alfai1=A1/I1; alfai2=A2/I2; alfai3=A3/I3;
```

```
%alfai1=0; alfai2=0; alfai3=0;
alfgjei1=G1*J1/(E1*I1); alfgjei2=G2*J2/(E2*I2);
alfgjei3=G3*J3/(E3*I3);
k=5e5/1e3;

%coefficient of thermal expansion
alpha2=12e-6;

%applied mechanical loads at nodes.
%None

%applied mechanical load on members
pam_mem1= 50; pam_mem2= 50; pam_mem3= 50;

%settlement nodal displacements in local coordinates, node_coord
deltast4_3=0.025*1000;
%deltast4_3=0;

%applied thermal load for members
DelT2=-25; Tav2=25/2;
%DelT2=0; Tav2=0;

%misfit nodal displacements in local coordinates, node_coord
deltam1_1=100*10; deltam1_5=0.001;
%deltam1_1=0; deltam1_5=0

%Member stiffnesses…local
%frame elements
k112= E1*I1/L1*[alfai1 0 0 0 0 0; 0 12/L1^2 0 0 0 -6/L1; 0 0
12/L1^2 0 6/L1 0; 0 0 0 alfgjei1 0 0; 0 0 6/L1 0 4 0; 0 -6/L1 0 0 0
4];
k221=k112;
k12= E1*I1/L1*[alfai1 0 0 0 0 0; 0 12/L1^2 0 0 0 -6/L1; 0 0 -
12/L1^2 0 -6/L1 0; 0 0 0 alfgjei1 0 0; 0 0 -6/L1 0 -2 0; 0 -6/L1 0 0
0 2];
```

```
k21=k12;
k223= E2*I2/L2*[alfai2 0 0 0 0 0; 0 12/L2^2 0 0 0 -6/L2; 0 0
12/L2^2 0 6/L2 0; 0 0 0 alfgjei2 0 0; 0 0 6/L2 0 4 0; 0 -6/L2 0 0 0
4];
k332=k223;
k23= E2*I2/L2*[alfai2 0 0 0 0 0; 0 12/L2^2 0 0 0 -6/L2; 0 0 -
12/L2^2 0 -6/L2 0; 0 0 0 alfgjei2 0 0; 0 0 -6/L2 0 -2 0; 0 -6/L2 0 0
0 2];
k32=k23;
k334= E3*I3/L3*[alfai3 0 0 0 0 0; 0 12/L3^2 0 0 0 -6/L3; 0 0
12/L3^2 0 6/L3 0; 0 0 0 alfgjei3 0 0; 0 0 6/L3 0 4 0; 0 -6/L3 0 0 0
4];
k443=k334;
k34= E3*I3/L3*[alfai3 0 0 0 0 0; 0 12/L3^2 0 0 0 -6/L3; 0 0 -
12/L3^2 0 -6/L3 0; 0 0 0 alfgjei3 0 0; 0 0 -6/L3 0 -2 0; 0 -6/L3 0 0
0 2];
k43=k34;
%spring elements
k113=[k]; k331=[k]; k13=[k]; k31=[k];

%Member stiffnesses…global
K113spring= a13'*k113*a13; K331spring= a31'*k331*a31;
K13spring= a13'*k13*a31; K31spring= a31'*k31*a13;
K11=trans12'*k112*trans12 + [K113spring zeros(3,3); zeros(3,6)];
K12=trans12'*k12*trans21; K21=K12';
K13=[K13spring zeros(3,3); zeros(3,6)]; K31= K13';
K14=zeros(6,6); K41= K14';
K22= trans21'*k221*trans21 + trans23'*k223*trans23;
K23= trans23'*k23*trans32; K32=K23';
K24=zeros(6,6); K42=K24';
K33= trans32'*k332*trans32 + trans34'*k334*trans34 +
[K331spring zeros(3,3); zeros(3,6)];
K34=trans34'*k34*trans43; K43=K34';
K44=trans43'*k443*trans43;
```

```
%retain only rows/cols 7, 15 from K_Total
KII=[K22(1,1) K23(1,3); K32(3,1) K33(3,3)]

%remove rows 7,15 and retain cols 7, 15 from K_Total
KII_I=[K12(:,1) K13(:,3); K22(2:6,1) K23(2:6,3); K32(1:2,1)
K33(1:2,3); K32(4:6,1) K33(4:6,3); K42(:,1) K43(:,3)]

%straightforward way of getting KII, KII_I, KI_II, KII_II, from
K_Total, not recommended for hand calculations. This is done to
show that the axial stiffness terms affect on KII_II. In this regard
see above for ratios alfai1, alfai2, alfai3, taken as zero or given
values. Here K=K_Total
%K=[K11 K12 K13 K14; K21 K22 K23 K24; K31 K32 K33 K34;
%K41 K42 K43 K44];
%KII=[K(7,7) K(7,15); K(15,7) K(15,15)];
%KII_I=[K(1:6,7) K(1:6,15); K(8:14,7) K(8:14,15); K(16:24,7)
%K(16:24,15)];
%KI_II=[K(7,1:6) K(7,8:14) K(7,16:24); K(15,1:6) K(15,8:14)
%K(15,16:24)];
%KII_II=[K(1:6,1:6) K(1:6,8:14) K(1:6,16:24); K(8:14,1:6)
%K(8:14,8:14) K(8:14,16:24); K(16:24,1:6) K(16:24,8:14)
%K(16:24,16:24)];

%applied nodal loads in global coordinates
PIa=[0, 0]';
PIIa=zeros(22,1);

%fixed end forces due to mech loads, local coords
F12fmech=[0, 0, -pam_mem1/2, 0, -pam_mem1*L1/8, 0]';
F21fmech=[0, 0, -pam_mem1/2, 0, -pam_mem1*L1/8, 0]';
F23fmech=[0, pam_mem2/2, 0, 0, 0, -pam_mem2*L2/8]';
F32fmech=[0, -pam_mem2/2, 0, 0, 0, pam_mem2*L2/8]';
F34fmech=[0, pam_mem3/2, 0, 0, 0, -pam_mem3*L3/8]';
F43fmech=[0, -pam_mem3/2, 0, 0, 0, pam_mem3*L3/8]';
```

```
%selfstraining displs, local coords, st-settlement, t-temp, m-misfit
delta12st=zeros(6,1); delta12t=zeros(6,1);
delta12m=-[deltam1_1, 0, 0, 0, deltam1_5, 0]';
delta21st=zeros(6,1); delta21t=zeros(6,1); delta21m=zeros(6,1);
delta23st=zeros(6,1); delta23m=zeros(6,1);
delta23t=-[alpha2*Tav2*L2, alpha2*DelT2*L2^2/2/h2, 0, 0, 0,
alpha2*DelT2*L2/h2]';
delta32st=zeros(6,1); delta32t=zeros(6,1); delta32m=zeros(6,1);
delta34st=zeros(6,1); delta34t=zeros(6,1); delta34m=zeros(6,1);
delta43st=[0, 0, deltast4_3, 0, 0, 0]';
delta43t=zeros(6,1); delta43m=zeros(6,1);

%total selfstraining displs, local coords
delta12s=delta12st+delta12t+delta12m;
delta21s=delta21st+delta21t+delta21m;
delta23s=delta23st+delta23t+delta23m;
delta32s=delta32st+delta32t+delta32m;
delta34s=delta34st+delta34t+delta34m;
delta43s=delta43st+delta43t+delta43m;

%fixed end forces due to selfstraining loads, local coords
F12fs=k112*delta12s+k12*delta21s;
F21fs=k21*delta12s+k221*delta21s;
F23fs=k223*delta23s+k23*delta32s;
F32fs=k32*delta23s+k332*delta32s;
F34fs=k334*delta34s+k34*delta43s;
F43fs=k43*delta34s+k443*delta43s;

%total fixed end forces due to selfstraining+mech loads, local
coords
F12f=F12fmech+F12fs; F21f=F21fmech+F21fs;
F23f=F23fmech+F23fs; F32f=F32fmech+F32fs;
F34f=F34fmech+F34fs; F43f=F43fmech+F43fs;
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```
%nodal loads due to selfstraining+mech loads, global coords
Pnode1f=trans12'*F12f;
Pnode2f=trans21'*F21f+trans23'*F23f;
Pnode3f=trans32'*F32f+trans34'*F34f;
Pnode4f=trans43'*F43f;

%equivalent nodal load vector due to selfstraining+mech loads,
global coords
Pef=[Pnode1f; Pnode2f; Pnode3f; Pnode4f];

%splitting into free and reactive equivalent loads due to
selfstraining+mech loads, global coords
PIe=[Pef(7,1); Pef(15,1)];
PIIe=[Pef(1:6,:); Pef(8:14,:); Pef(16:24,:)];

%total applied free joint loads
PI=PIa-PIe;

%Nodal displacements
DeltaI=inv(KII)*(PI)

%Reactions
P_II=KII_I*DeltaI+PIIe

%member end forces and spring force
DeltaII=zeros(22,1);
Delta1= [DeltaII(1:6)];
Delta2= [DeltaI(1); DeltaII(7:11)]
Delta3= [DeltaII(12:13); DeltaI(2); DeltaII(14:16)];
Delta4= [DeltaII(17:22)];
F12=k112*trans12*Delta1 + k12*trans21*Delta2 + F12f
F21=k21*trans12*Delta1 + k221*trans21*Delta2 + F21f
F23=k223*trans23*Delta2 + k23*trans32*Delta3 + F23f
F32=k32*trans23*Delta2 + k332*trans32*Delta3 + F32f
F34=k334*trans34*Delta3 + k34*trans43*Delta4 + F34f
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```
F43=k43*trans34*Delta3 + k443*trans43*Delta4 + F43f
F13=k113*a13*[Delta1(1:3)] + k13*a31*[Delta3(1:3)]
F31=k31*a13*[Delta1(1:3)] + k331*a31*[Delta3(1:3)]

%total reaction check
R_X=P_II(1)+P_II(1+6+6-1)+P_II(1+6+6-1+6-1)
R_Y=P_II(2)+P_II(2+6-1)+P_II(2+6-1+6)+ P_II(2+6-1+6+6-1)
R_Z=P_II(3)+P_II(3+6-1)+P_II(3+6-1+6+6-1)
```

KII =
1.0e+002 *

0.00955555555556	0
0	1.54426709401709

KII_I =
1.0e+003 *

-0.00060000000000	0
0	-0.23076923076923
0	-0.15384615384615
0	0
-3.00000000000000	0
0	0
0	0
0	-0.00035555555556
0	-2.66666666666667
-3.00000000000000	0
-2.66666666666667	0
-0.00035555555556	0
0	0.23076923076923
0	-2.66666666666667
0	2.25000000000000
-2.66666666666667	0
0	0
0	0
0	-0.00022500000000
0	0
0	2.25000000000000
0	0

%Results with self straining

DeltaI =
 -23.02325581395349
 -0.19831413955947

P_II =
 1.0e+006 *
 0.00004181395349
 0.00004576480144
 1.12102875314721
 0
 0.15156976744186
 0
 0.00237799637856
 -1.12097317276789
 -0.09458604965702
 0.01656976744186
 0.06139534883721
 0.00000818604651
 -0.00244876118000
 0.09564372506800
 0.05580379318599
 0.18639534883721
 0
 -0.00002500000000
 -0.00000558037932
 0
 0.05580379318599
 -0.12500000000000

Delta2 =
 -23.02325581395349
 0
 0

0
0
0

F12 =
1.0e+006 *
-1.12099824327959
0
-0.00004181395349
0
-0.15156976744186
0

F21 =
1.0e+006 *
-1.12099824327959
0
-0.00000818604651
0
0.01656976744186
0

F23 =
1.0e+004 *
-0.23779963785636
0.00250705116941
0.00081860465116
0
6.13953488372093
9.45860496570194

F32 =
1.0e+004 *
-0.23779963785636
-0.00249294883059

-0.00081860465116
0
-6.13953488372093
-9.56437250680033

F34 =
1.0e+005 *
0
0.00025000000000
-0.00005580379319
0
-0.55803793185991
-1.25000000000000

F43 =
1.0e+005 *
0
-0.00025000000000
0.00005580379319
0
0.55803793185991
1.25000000000000

F13 =
-55.00244606393718

F31 =
-55.00244606393718

R_X =
50
R_Y =
-50
R_Z =
50.00000000007963

%Results without selfstraining
DeltaI =
-26.16279069767442
-0.16188909351793

P_II =
1.0e+005 *
0.00040697674419
0.00037359021581
0.00024906014387
0
1.40988372093023
0
0
0.00025057560567
0.94181704249381
0.15988372093023
0.69767441860465
0.00009302325581
-0.00062359021581
-0.93318295750619
-0.00364250460415
1.94767441860465
0
-0.00025000000000
0.00000036425046
0
-0.00364250460415
-1.25000000000000

Delta2 =
-26.16279069767442
0
0
0

0
0

F12 =
1.0e+005 *
0
0
-0.00040697674419
0
-1.40988372093023
0

F21 =
1.0e+004 *
0
0
-0.00093023255814
0
1.59883720930233
0

F23 =
1.0e+004 *
0
0.00250575605666
0.00093023255814
0
6.97674418604651
-9.41817042493812

F32 =
1.0e+004 *
0
-0.00249424394334
-0.00093023255814

```
              0
 -6.97674418604651
  9.33182957506188

F34 =
  1.0e+005 *
              0
  0.00025000000000
  0.00000036425046
              0
  0.00364250460415
 -1.25000000000000

F43 =
  1.0e+005 *
              0
 -0.00025000000000
 -0.00000036425046
              0
 -0.00364250460415
  1.25000000000000

F13 =
 -44.89995597056096

F31 =
 -44.89995597056096

R_X =
    50
R_Y =
   -50
R_Z =
  50.00000000000001
```