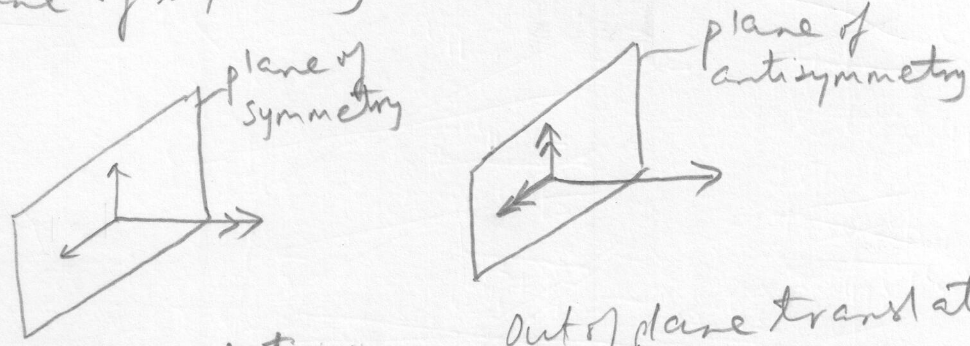
Symmetry and Antisymmetry:

Used to reduce size of problem & hence computation time.

Reflection symmetry : $\exists$ a plane(s) of symmetry of the structure & ~~supports~~. After reflection if loads dont need [need] to be reversed to bring problem into "self coincidence" then problem is symmetric [antisymmetric].

(1) To set up correct displacement BC's for nodes lying in the plane of symmetry, we have following rules:

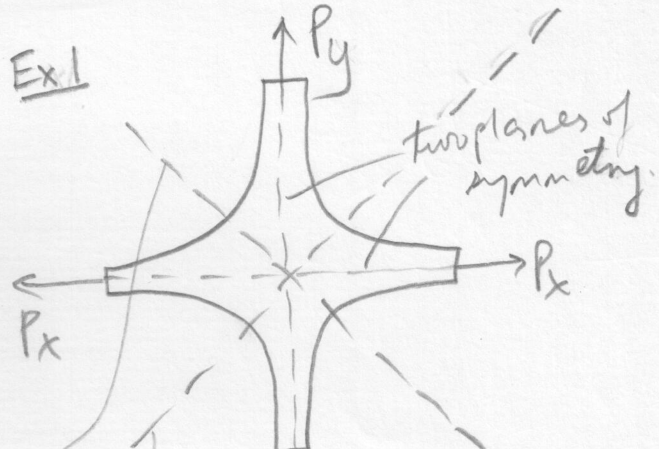


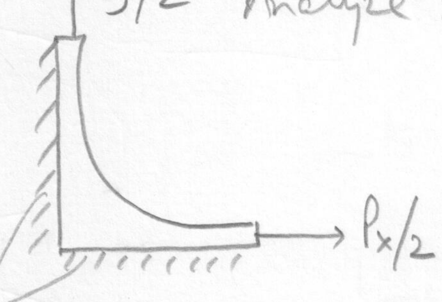
Inplane translations and rotation are nonzero

Out of plane translation and rotation are nonzero.

(2) For member lying in plane of symmetry, member properties (A, I, stiffness) and loads need to be halved. If member lies in 'n' planes of symmetry then props & loads need to be $\frac{1}{2n}$.

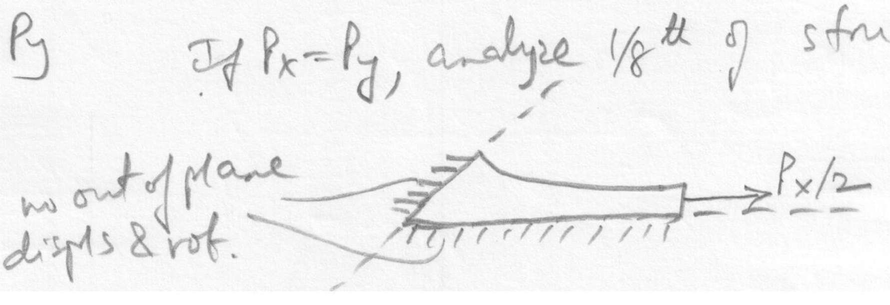
Ex 1



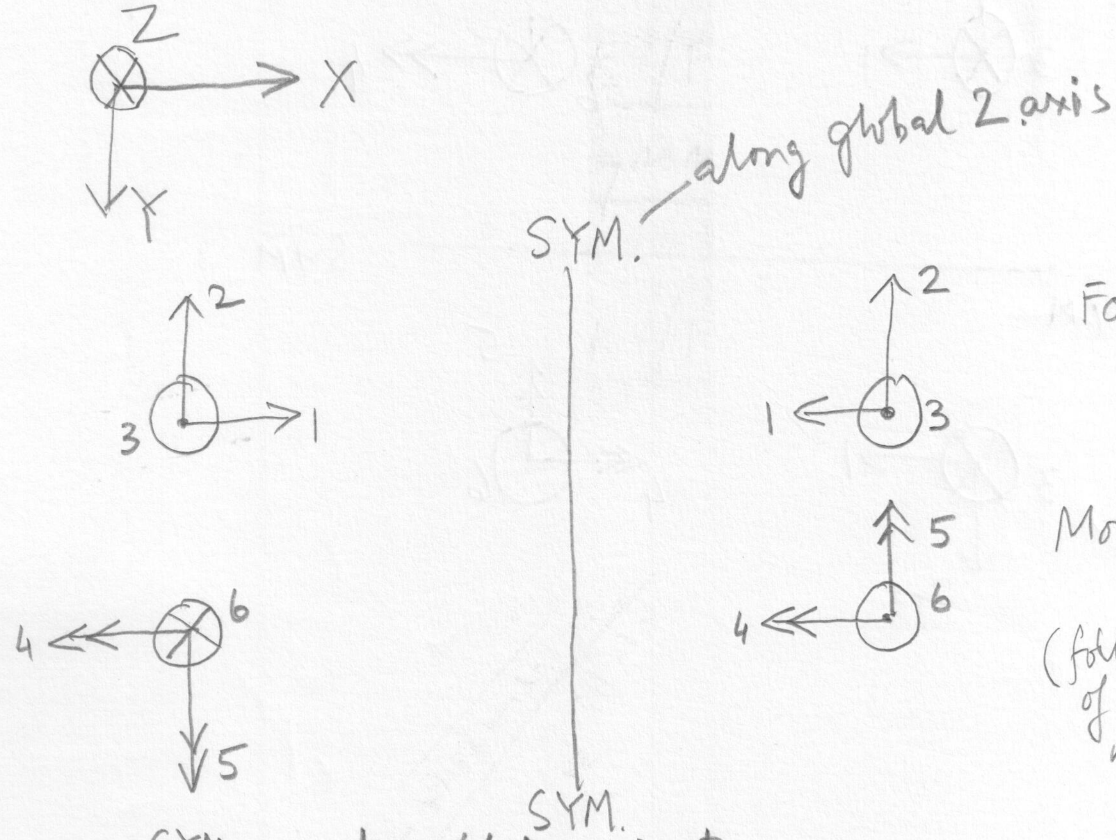


Analyze 1/4 structure

If $P_x = P_y$, analyze 1/8th of structure.



Global DOF's (Displacements / Forces / Reactions) for Symmetry and Antisymmetry.



Forces/linear displs

Moments/Rotations ie, angular

(follow directions of curved arrows not double arrows)

Thus when SYM axis along global 2 direction:

⇒ Dof 1, 5, 6 (ie, Δ_X, θ_Y, θ_Z, P_X, M_Y, M_Z) have sign reversals

Dof 2, 3, 4 (ie Δ_Y, Δ_Z, θ_X, P_Y, P_Z, M_X) DO NOT have sign reversals

when structure & load is SYM

⇒ vice-versa when str + load is ASYM.

Thus when SYM axis along global 1 direction:

⇒ 2, 4, 6 have sign reversal

1, 3, 5 DO NOT have sign reversal.

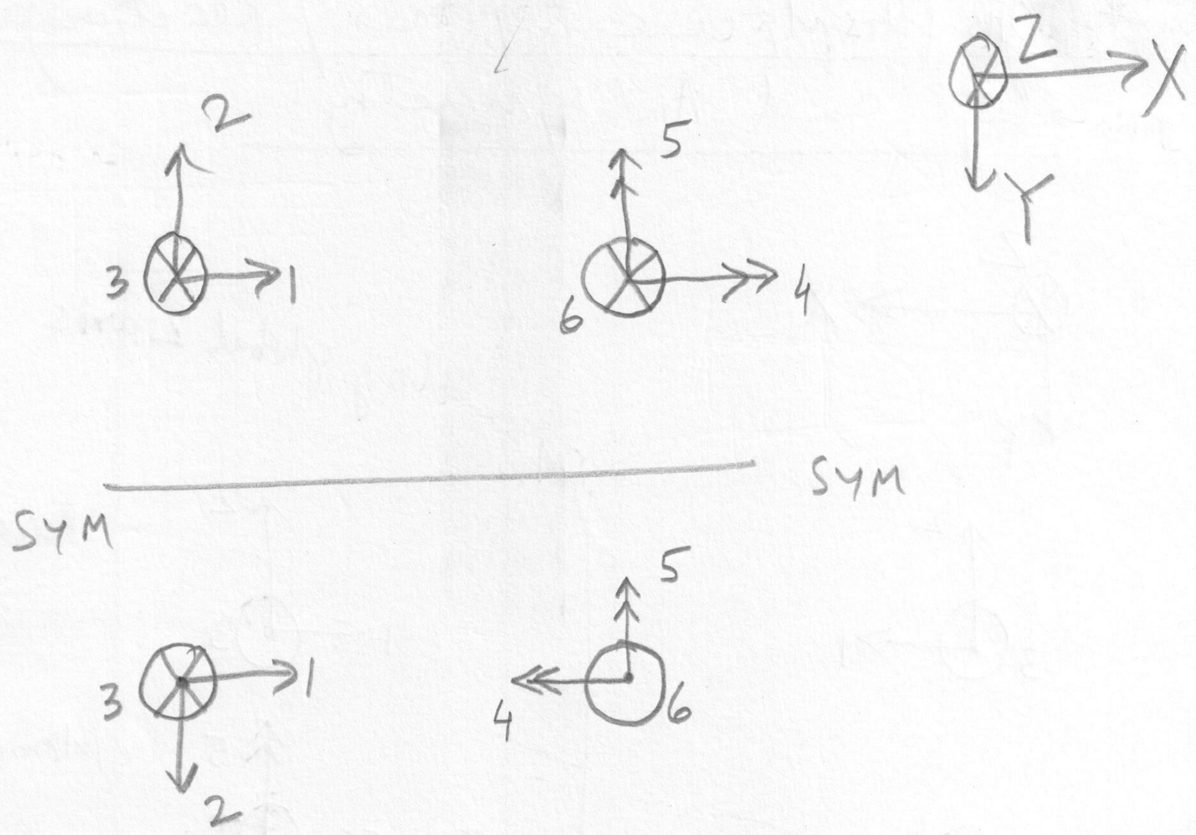
when str & loads are SYM.

⇒ vice-versa when str + load is ASYM.

easily visualized by rotating above fig by 90° and replacing $1 \rightleftharpoons 2$, $4 \rightleftharpoons 5$

ie $X \rightleftharpoons Y$.

(or see reverse page)

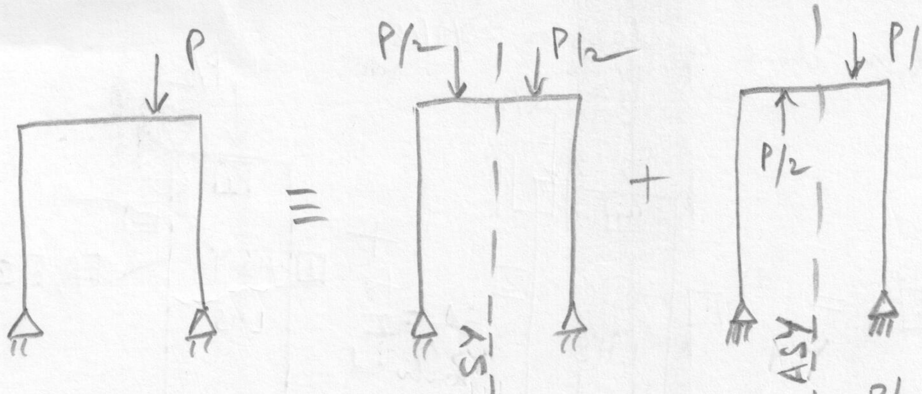


<u>Details of Ex3 superposition table for region R4:</u>

$\underline{III} \to \underline{IV}$, vertical plane of SY, ASY SY, ASY applied to solns of Prob1, Prob2, Prob3, Prob4, as we go from $\underline{III} \to \underline{IV}$. Hence wrt signs of R3, for dof (1,6) reverse signs when SY and dont reverse when ASY, and for dof 2 reverse signs when ASY and dont reverse when SY.

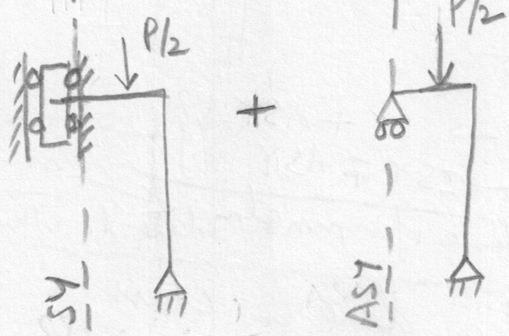
Alternatively do $\underline{II} \to \underline{IV}$, horizontal plane of SY, SY, ASY, ASY applied to solns of Prob1, Prob2, Prob3, Prob4 as we go from $\underline{II} \to \underline{IV}$. Hence wrt signs of R2, for dof (2,6) reverse signs when SY and dont reverse when ASY, and for dof 1 reverse signs when ASY and dont reverse when SY. Both $\underline{III} \to \underline{IV}$ & $\underline{II} \to \underline{IV}$ yield same result

Ex2



See modification on reverse.

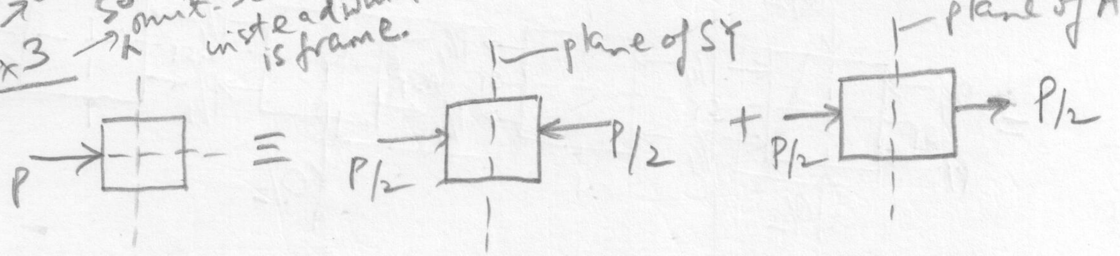
so analyze 1/2*2



Superposition
Right half: SY + ASY
Left half: ±SY∓ASY for dof $\frac{2}{(1,6)}$

Ex3

Generic structure so omit. Do (3a) instead which is frame.

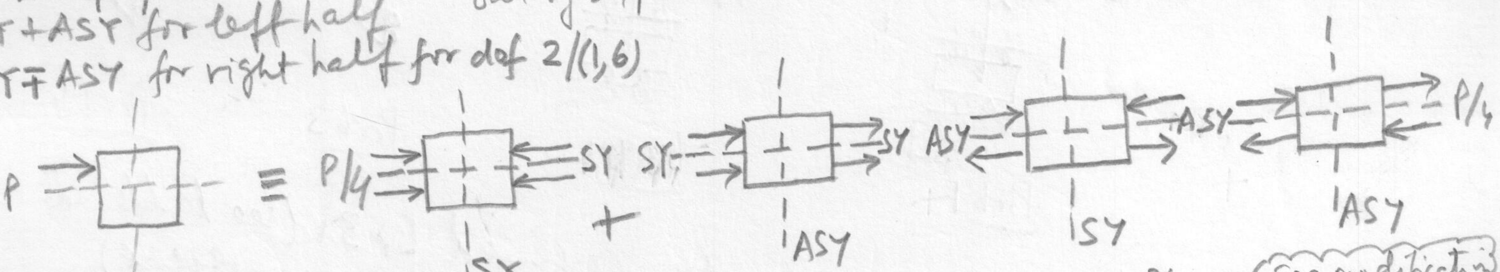


so analyze 1/2*2

no displ/rot out of SY plane

no displ/rot in ASY plane

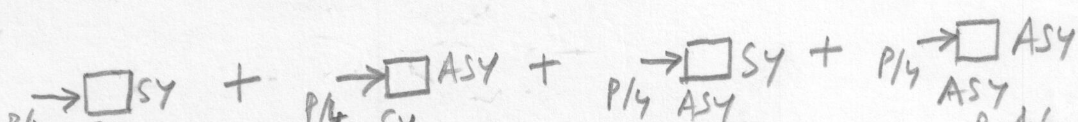
Then superpose
SY+ASY for left half
±SY∓ASY for right half for dof 2/(1,6)



All loads shown are P/4

See modification on reverse

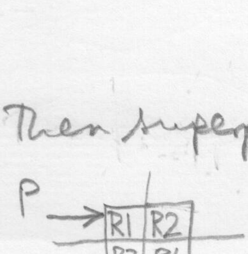
so analyze 1/4*4



Prob1 Prob2 Prob3 Prob4

So we analyze $1/4^{th}$ structure for four different sets of BC's ie SY-SY, SY-ASY, ASY-SY, ASY-ASY.

Then superpose as

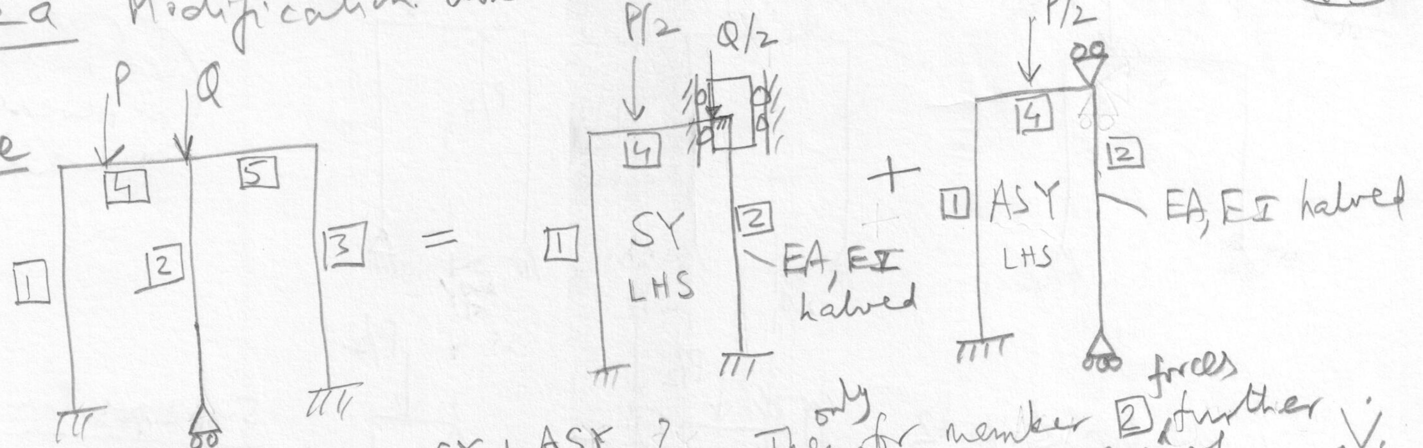


Prob1	Prob2	Prob3	Prob4	
+	+	+	+	→ R1
+/−	−/+	+/−	−/+	→ R2 for dof 2/(1,6)
+/−	+/−	−/+	−/+	→ R3 for dof 1/(2,6)
−/+	+/−	+/−	−/+	→ R4 for dof (1,2)/6

details on reverse of p.2 ←

Ex 2a Modification done in class on 28/9/18

Frame

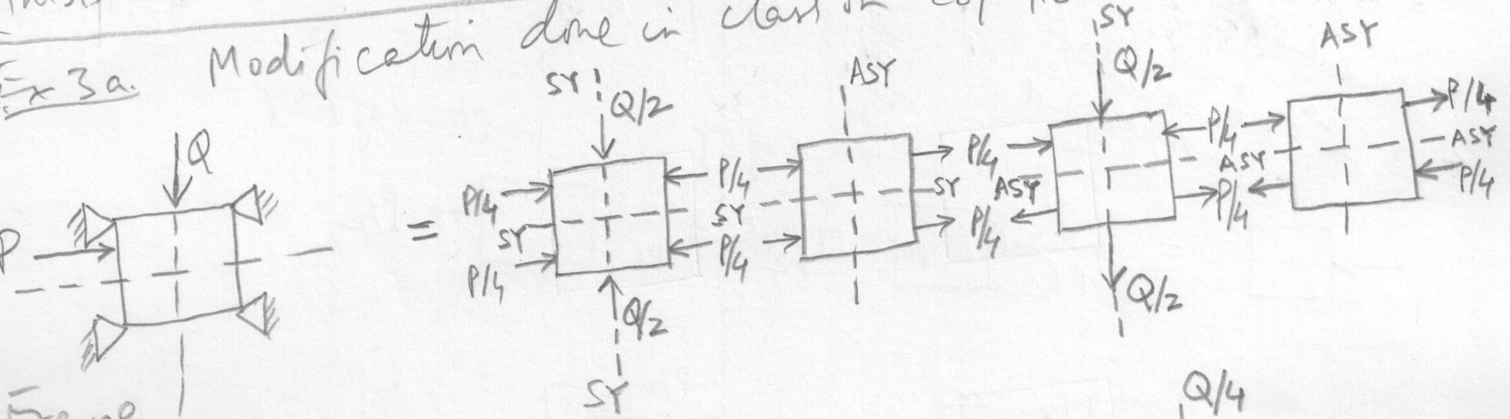


Superposition: { LHS → SY + ASY
RHS → ±SY ∓ ASY } → Then for member [2] only forces, further superpose LHS+RHS soln.

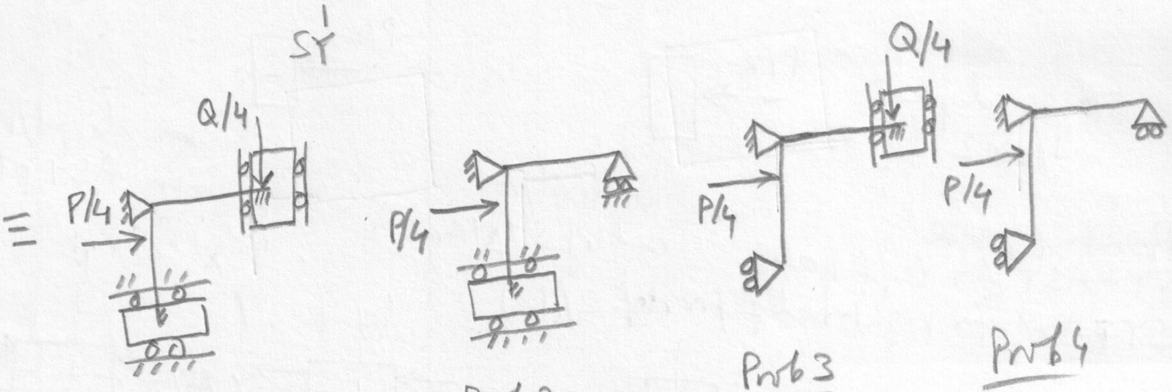
sufficient for displs ←

{This is done ∵ [2] lies on plane of symmetry due to which we halved its EA, EI and Q}

Ex 3a. Modification done in class on 28/9/18



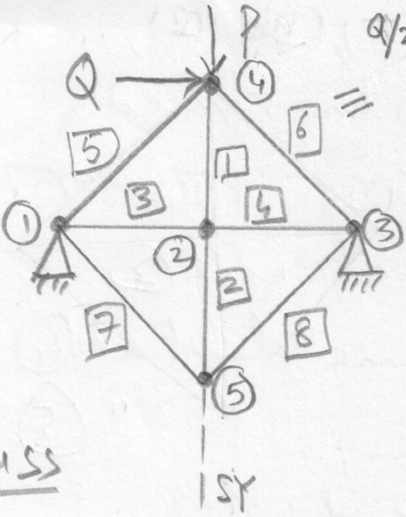
Frame



Prob 1 Prob 2 Prob 3 Prob 4

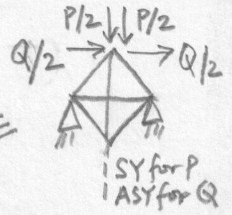
Superposition is same as for original Ex3 (see table on reverse).

Ex 4

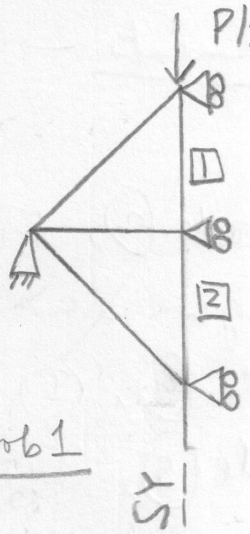


Truss

$\equiv$

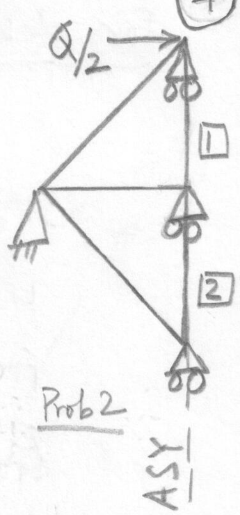


$\Rightarrow$ So analyze as



Prob 1

+



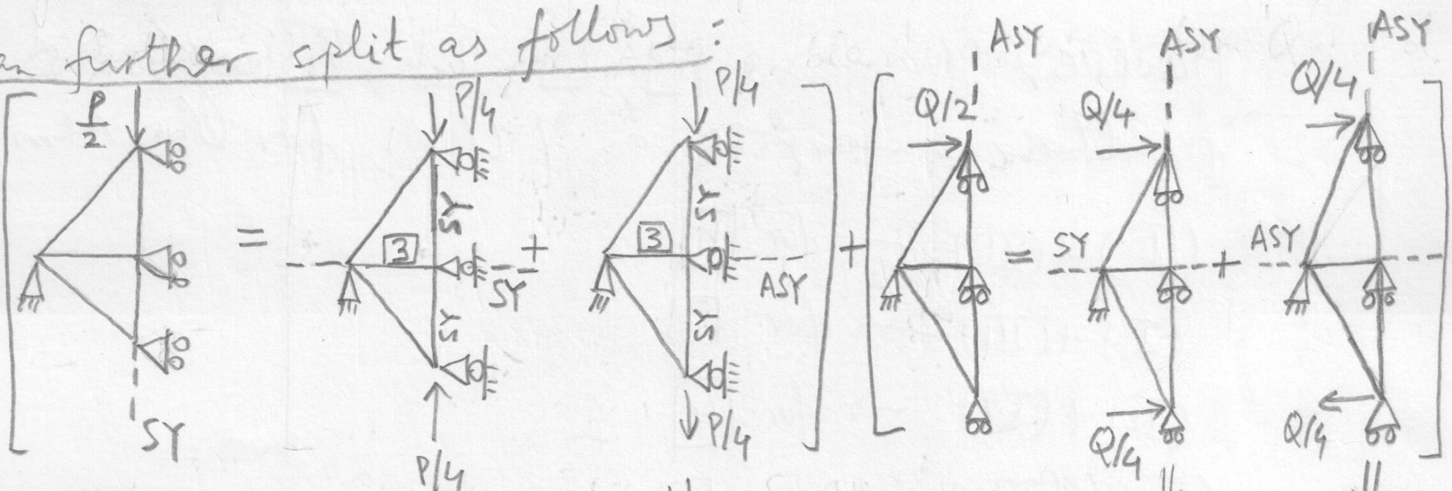
Prob 2

Properties of $\boxed{1}$, $\boxed{2}$, i.e. members lying in plane of SYM/ASYM should be halved. Same true for loads P, Q.

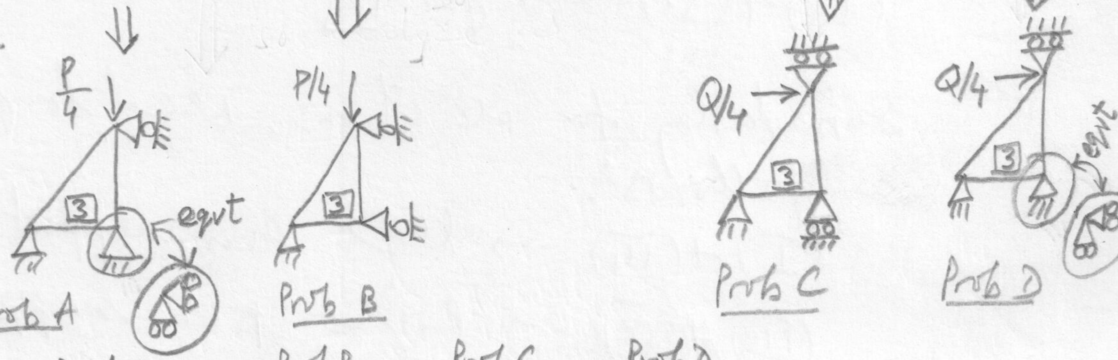
Superpose as

	Prob 1	Prob 2	
	+	+	$\rightarrow$ for LHS half
	+/−	−/+	$\rightarrow$ for RHS half for dof 2/1

Can further split as follows:



So analyze as



Prob A Prob B Prob C Prob D

A, I of mem $\boxed{3}$ to be halved in Probs A–D

Then superpose as

	Prob A	Prob B	Prob C	Prob D	
(I)	+	+	+	+	upper left 1/4th
(II)	+/−	+/−	−/+	−/+	upper right 1/4th dof 2/1
(III)	+/−	−/+	+/−	−/+	lower left 1/4th dof 1/2
(IV)	−	+	+	−	lower right 1/4th both dof (1,2)

<u>For displacements</u> → Superposition (I), (II), (III), (IV), i.e. no further superposition required.

So for node (2), superposition (I) or (II) or (III) or (IV) would give same displacements at node (2).

For node (4), (I) or (II) give same displ at (4)

For node (5), (III) or (IV) " " " " (5)

We dont further superpose for displs.

<u>For Member forces and reactions</u>

If member or support doesent lie in plane of symmetry, no further superposition required.

So for mem's $\boxed{5}$, $\boxed{6}$, $\boxed{7}$, $\boxed{8}$, superpositions (I), (II), (III), (IV), respectively, give member forces.

However for mem's $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$ we need to further superpose as follows for the mem force.

(I) + (II) → for $\boxed{1}$

(I) + (III) → for $\boxed{3}$

(II) + (IV) → for $\boxed{4}$

(III) + (IV) → for $\boxed{2}$

Similarly for reactions, need to further superpose as follows:

(I) + (III) → left pin support

(II) + (IV) → Right pin support.

This is because we halved EA, EI for $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, while consistently halving Q for $\boxed{1}$, $\boxed{2}$

Ex5

All 16 mem's identical.

4 ASY Planes as shown.

So need to analyze only

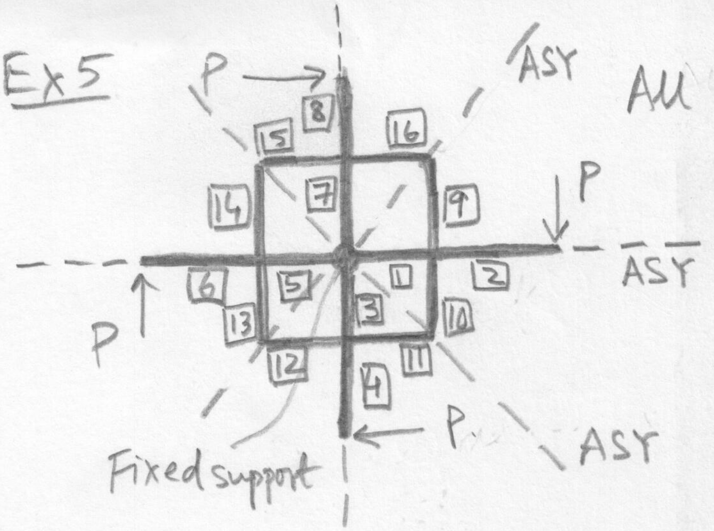


Fig 1

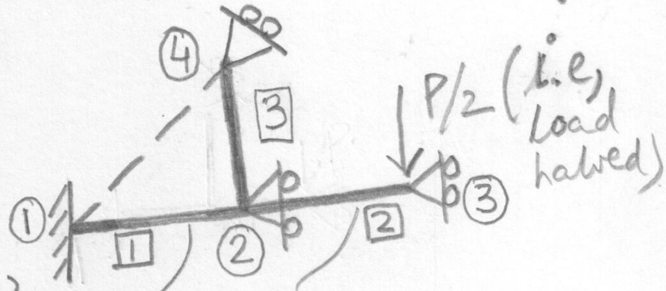


Fig 2

EI, EA halved

This is physical reasoning for halving loads and stiffnesses. ⟹

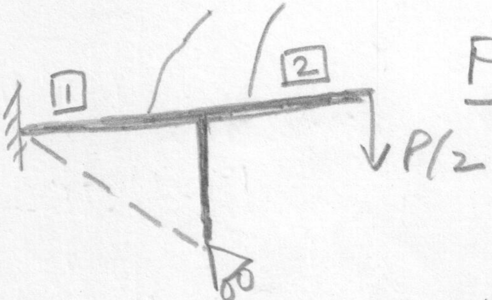
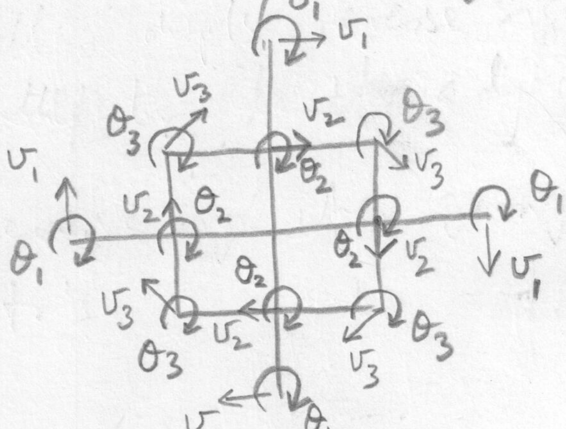


Fig 3

No superpositions required ∵ we solve only a single 1/8th problem.

Displacements :

Similarly for mem forces.



For 1, 2 ... 8 you need to further superpose mem forces from Fig 2 + Fig 3, since we halved load and EA, EI. Due to ASY, this amounts to doubling of shear & BM obtained from Fig 2 or Fig 3, and axial force becomes zero.

```
%Symmetry Antisymmetry –                    – done as 1/8th structure

a24=[0 1 0; -1 0 0; 0 0 1]; a42=[1/sqrt(2) -1/sqrt(2)  0; 1/sqrt(2) 1/sqrt(2) 0; 0 0 1]; a12=[-1 0 0; 0 -1 0; 0 0 1]; a23=a12; a21=[1 0 0; 0 1 0; 0 0 1]; a32=a21;

ei=2e14; ea=2.19122e10; g=ea/ei; L=3000; P=100000; f=P/2;

%Load on node in plane of SY/ASY is halved in the above

kiij=ei*[g/L 0 0 ; 0 12/L^3 -6/L^2 ; 0 -6/L^2 4/L]; kij=ei*[g/L 0 0; 0 12/L^3 -6/L^2; 0 -6/L^2 2/L];

K1=0.5*[a12'*kiij*a12 a12'*kij*a21; a21'*kij*a12 a21'*kiij*a21]; K2=K1; K3=[a24'*kiij*a24 a24'*kij*a42; a42'*kij*a24 a42'*kiij*a42];

%EI EA on node in plane of SY/ASY is halved in the above

Kone=[K1 zeros(6,6); zeros(6,12)];

Ktwo=[zeros(3,12); zeros(6,3) K2 zeros(6,3); zeros(3,12)];

 Kthree=[zeros(3,12); zeros(3,3) a24'*kiij*a24 zeros(3,3) a24'*kij*a42; zeros(3,12); zeros(3,3) a42'*kij*a24 zeros(3,3) a42'*kiij*a42];

K=Kone+Ktwo+Kthree;

KII=K([5,6,8,9,11,12], [5,6,8,9,11,12]);

PI=[0;0;f;0;0;0];

 inv(KII)*PI

8.2530
0.0038
24.0122
0.0060
11.6845
0.0023
```

```
%Symmetry Antisymmetry –  – done as full structure without using symm/asymm

a12=[-1 0 0; 0 -1 0; 0 0 1];a23=a12; a910=a12; a101=a12; a67=a12; a78=a12; a1112=a12; a124=a12;
a21=eye(3,3); a32=a21; a109=a21; a110=a21; a76=a21; a87=a21; a1211=a21; a412=a21;

a24=[0 1 0; -1 0 0; 0 0 1]; a82=a24; a57=a24; a71=a24; a112=a24; a1213=a24; a610=a24; a1011=a24;
a42=[0 -1 0; 1 0 0; 0 0 1]; a28=a42; a75=a42; a17=a42; a121=a42; a1312=a42; a106=a42; a1110=a42;

ei=2e14; ea=2.19122e10; g=ea/ei; L=3000; P=100000; f=P;

kiij=ei*[g/L 0 0 ; 0 12/L^3 -6/L^2 ; 0 -6/L^2 4/L]; kij=ei*[g/L 0 0; 0 12/L^3 -6/L^2; 0 -6/L^2 2/L];

K11=a12'*kiij*a12+a112'*kiij*a112+a110'*kiij*a110+a17'*kiij*a17; K12=a12'*kij*a21;
K112=a112'*kij*a121; K110=a110'*kij*a101; K17=a17'*kij*a71;

K22=a21'*kiij*a21+a28'*kiij*a28+a23'*kiij*a23+a24'*kiij*a24; K28=a28'*kij*a82; K23=a23'*kij*a32;
K24=a24'*kij*a42;

K33=a32'*kiij*a32;

K44=a42'*kiij*a42+a412'*kiij*a412; K412=a412'*kij*a124;

K55=a57'*kiij*a57; K57=a57'*kij*a75;

K66=a67'*kiij*a67+a610'*kiij*a610; K67=a67'*kij*a76; K610=a610'*kij*a106;

K77=a71'*kiij*a71+a76'*kiij*a76+a75'*kiij*a75+a78'*kiij*a78; K78=a78'*kij*a87;

K88=a87'*kiij*a87+a82'*kiij*a82;

K99=a910'*kiij*a910; K910=a910'*kij*a109;

K1010=a101'*kiij*a101+a1011'*kiij*a1011+a109'*kiij*a109+a106'*kiij*a106;
K1011=a1011'*kij*a1110;

K1111=a1110'*kiij*a1110+a1112'*kiij*a1112; K1112=a1112'*kij*a1211;

K1212=a121'*kiij*a121+a124'*kiij*a124+a1213'*kiij*a1213+a1211'*kiij*a1211;
K1213=a1213'*kij*a1312;

K1313=a1312'*kiij*a1312;

Ktophalf=[K11 K12 zeros(3,12) K17 zeros(3,6) K110 zeros(3,3) K112 zeros(3,3); zeros(3,3) K22 K23
K24 zeros(3,9) K28 zeros(3,15) ; zeros(3,6) K33 zeros(3,30) ; zeros(3,9) K44 zeros(3,21) K412
zeros(3,3) ; zeros(3,12) K55 zeros(3,3) K57 zeros(3,18) ; zeros(3,15) K66 K67 zeros(3,6) K610
zeros(3,9); zeros(3,18) K77 K78 zeros(3,15) ;zeros(3,21) K88 zeros(3,15) ;zeros(3,24) K99 K910
zeros(3,9) ;zeros(3,27) K1010 K1011 zeros(3,6) ; zeros(3,30) K1111 K1112 zeros(3,3) ; zeros(3,33)
K1212 K1213; zeros(3,36) K1313];

Kuppertriangular=triu(Ktophalf);

D=diag((diag(Kuppertriangular))');
```

```
K=Kuppertriangular +Kuppertriangular' -D;

KII=K([4:39],[4:39]);

PI=[zeros(4,1); f; zeros(4,1); -f; zeros(12,1); -f; zeros(10,1); f; zeros(2,1)];

inv(KII)*PI

ans =

    0.0000
    8.2530
    0.0038
    0.0000
   24.0122
    0.0060
    8.2622
    8.2622
    0.0023
  -24.0122
   -0.0000
    0.0060
   -8.2622
   -8.2622
    0.0023
   -8.2530
   -0.0000
    0.0038
   -8.2622
    8.2622
    0.0023
   -0.0000
  -24.0122
    0.0060
   -0.0000
   -8.2530
    0.0038
    8.2622
   -8.2622
    0.0023
    8.2530
   -0.0000
    0.0038
   24.0122
   -0.0000
    0.0060
```