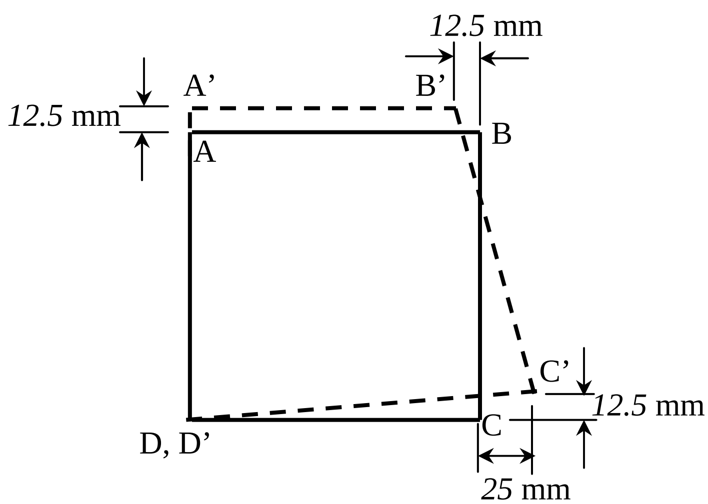
CE469 Advanced Mechanics of Solids Quiz-1 7/2/12

1. The square plate of side 1m, shown in **Fig. 1**, is loaded so that it is in a state of plane strain (i.e., $\varepsilon_{13} = \varepsilon_{23} = \varepsilon_{33} = 0$). The deformed plate is shown by dotted straight lines. Assume small displacement gradients. **At corner *B* determine the following**:

a) Maximum and minimum engineering extensional strains and the direction of the corresponding line elements on which they act.
b) Maximum change in angle between two elements originally at right angles. Also sketch the direction of these elements.

Fig. 1



2. The state of stress at a point, expressed in two Cartesian systems is,

$$\sigma_{ij} = \begin{pmatrix} 5 & a & -a \\ a & 0 & b \\ -a & b & 0 \end{pmatrix}, \qquad \sigma'_{ij} = \begin{pmatrix} c & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & d \end{pmatrix}$$

where $a,\ b,\ c,\ d$ are constants. If the magnitude of the maximum shearing stress at this point is 5.5 MPa,

a) Determine the principal stresses at this point
b) Also find a and b (you need not solve for them, just give the equation(s) that yields their solution)

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1) $u_1 = ax_1 + bx_2 + cx_1x_2 + d$

$u_2 = fx_1 + gx_2 + hx_1x_2 + i$

$u_1(0,0) = 0, \quad u_2(0,0) = 0 \Rightarrow d = i = 0.$

$u_1(1000,0) = 25 = 1000a \Rightarrow a = 0.025$

$u_2(1000,0) = 12.5 = 1000f \Rightarrow f = 0.0125$

$u_1(0,1000) = 0 = 1000b \Rightarrow b = 0$

$u_2(0,1000) = 12.5 = 1000g \Rightarrow g = 0.0125$

$u_1(1000,1000) = -12.5 = 1000a + 10^6 c \Rightarrow c = -3.75E-5$

$u_2 = (1000,1000) = 12.5 = 1000f + 1000g + 10^6 h \Rightarrow h = -1.25E-5.$

For $P \Rightarrow (500, 500)$ Not required

$\varepsilon_{11} = a + cx_2 = 6.25E-3$

$\varepsilon_{22} = g + hx_1 = 6.25E-3$

$\varepsilon_{12} = \frac{1}{2}\left[(b + cx_1) + (f + hx_2)\right] = -6.25E-3.$

$(6.25 - \varepsilon)^2 - 6.25^2 = \lambda^2 - 12.5\lambda = 0 \Rightarrow \varepsilon = 0, 12.5$

$\varepsilon(1) = 0 \Rightarrow 6.25n_1 - 6.25n_2 = 0 \Rightarrow \underline{n}(1) = \frac{\underline{e}_1}{\sqrt{2}} + \frac{\underline{e}_2}{\sqrt{2}}$

$\varepsilon(2) = 12.5 \Rightarrow -6.25n_1 - 6.25n_2 = 0 \Rightarrow \underline{n}(2) = \frac{\underline{e}_1}{\sqrt{2}} - \frac{\underline{e}_2}{\sqrt{2}}$

$(\varepsilon_{12})_{max} = \frac{|12.5 - 0|}{2} = 6.25$ in direction $\underline{e}_1$

$\varepsilon_{11} = a + cx_2 = -0.0125$

$\varepsilon_{22} = g + hx_1 = 0$

$\varepsilon_{12} = \frac{1}{2}\left[(b + cx_1) + (f + hx_2)\right] = -0.01875$

$\varepsilon(\varepsilon + 0.0125) - 0.01875^2 = 0$

$\varepsilon(1) = -0.02601, \quad \varepsilon(2) = 0.01351$

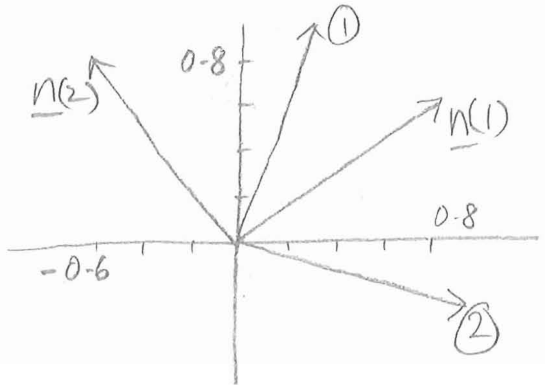
$\varepsilon(1) = -0.02601 : (-1250 + 2601)n_1 - 1875n_2 = 0$

$\Rightarrow \underline{n}(1) = 0.8113\underline{e}_1 + 0.5866\,\underline{e}_2$

$\varepsilon(2) = 0.01351 \; : \; (-1250 - 1351) n_1 - 1875 n_2 = 0$

$$\Rightarrow \underline{n}(2) = -0.5848 \underline{e}_1 + 0.8112 \underline{e}_2$$

$$(\varepsilon_{12})_{max} = \frac{(0.01351 + 0.02601)}{2} = 0.01976$$



lines ① & ② correspond to max shear strain.

P(2) $I_1 = c + d + 2 = 5 \rightarrow c + d = 3 \rightarrow$ ①

$I_3 = -5b^2 - a(ab) - a(ab) = -5b^2 - 2a^2 b = 2cd \rightarrow$ ②

$I_2 = 2c + 2d + cd = -a^2 - a^2 - b^2 \longrightarrow$ ③

①, ③ $\rightarrow cd < 0$, say $c > 0$, $d < 0 \rightarrow$ ④

①, ④ $\rightarrow c > 3 \Rightarrow S_{max} = \dfrac{c-d}{2} = 5.5 \rightarrow$ ⑤

①, ⑤ $\rightarrow c = 7$, $d = -4$

③ $\rightarrow -2a^2 = -22 + b^2$ $\quad a = \pm i\sqrt{\frac{27}{2}}, \boxed{\pm\sqrt{3}, \pm 3}$

②, ⑤ $\rightarrow b^3 - 5b^2 - 22b + 56 = 0 \rightarrow b = 7, \boxed{-4, 2}$ ✓

So a, b not unique.