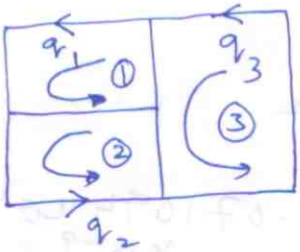
P.1 $\tau_i = \dfrac{3M b_i}{\sum a_i b_i^3} = 3M b_i \dfrac{G\alpha}{3M} = G\alpha b_i$ for open legs.

Case-I Max stress in open legs. Then it must occur in leg CA for which $b = 5$ mm as compared to leg EF for which $b = 3$ mm. In that case.

$$\alpha = \frac{\tau_{max}}{G * 5} \longrightarrow (A)$$

Case-II Max stress in closed loops. There are 3 loops, i.e. $i = 1, 2, 3$.



We have $2G A_i \alpha = \oint_{i^{th}\,loop} \dfrac{q}{t}\, ds$.

$$\Rightarrow_{i=1:}\ 2G\,(100*75)\alpha = \frac{1}{5}\left[350q_1 - 100q_2 - 75q_3\right] \to (1)$$

$$i=2:\ 2G\,(100*75)\alpha = \frac{1}{5}\left[350q_2 - 100q_1 - 75q_3\right] \to (2)$$

$$i=3:\ 2G\,(100*150)\alpha = \frac{1}{5}\left[500q_3 - 75q_1 - 75q_3\right] \to (3)$$

Soln of (1), (2), (3) is $q_1 = q_2 = q_3 = q$.

Put $q = \tau_{max} * 5$ in say (1), get,

$$75000\,G\alpha = 175q = 175\,\tau_{max} * 5$$

$$\Rightarrow \alpha = \frac{\tau_{max}}{G}\left(\frac{875}{75000}\right) \longrightarrow (B)$$

Compare (A) & (B) $\Rightarrow$ critical case is (B) ie Case II.

$$\Rightarrow M = \underbrace{\sum_{i=1}^{3} 2q_i A_i}_{M_{loops}} + \underbrace{\frac{\alpha G}{3}\sum_{i=1}^{2} a_i b_i^3}_{M_{legs}}$$

$(\alpha G)_{critical} = (70)\left(\dfrac{875}{75000}\right)$; $\displaystyle\sum_{i=1}^{3} 2q_i A_i = 2*70*5\begin{pmatrix}100*75+\\100*75+\\100*150\end{pmatrix}$

$\displaystyle\sum_{i=1}^{2} a_i b_i^3 = 100*5^3 + 75*3^3$

$M = 21011$ N-m ◄

$\because q_1 = q_2 = q_3$, $\tau_{BC} = \tau_{CD} = \tau_{CF} = 0$ ◄

P.2. BC's are:

$u_r = 0$ at $r = 100$ mm.

$\sigma_{rr} = 0$ at $r = 400$ mm.

$$0 = (.100)C_1 + \frac{C_2}{(.100)} + \frac{1-0.32^2}{8*12*10^9} * 2000\,\omega^2(0.100)^3$$

$$0 = \frac{12*10^9}{1-0.32^2}\left[(1+0.32)C_1 - (1-0.32)\frac{C_2}{(0.400)^2} - (3+0.32)\frac{(1-0.32^2)}{8*12*10^9} * 2000\,\omega^2(0.400)^2\right]$$

Solve C_1, C_2 in terms of ω^2.

$$0.1C_1 + 10C_2 = 1.87*10^{-11}\omega^2$$

$$1.32C_1 - 4.25C_2 = 9.93344*10^{-9}\omega^2$$

$$\Rightarrow C_2 = \frac{-9.6866*10^{-9}}{136.25}\omega^2 = -0.071094\,\omega^2 * 10^{-9}$$

$$C_1 = 7.2964*10^{-9}\,\omega^2$$

$$\sigma_{rr}^* = \frac{\sigma_{rr}}{E/1-\nu^2} = 9.631248*10^{-9}\omega^2 + \frac{4.834392*10^{-11}}{r^2}\omega^2 - 62.084*10^{-9}\,\omega^2 r^2$$

$$\sigma_{\theta\theta}^* = \frac{\sigma_{\theta\theta}}{E/1-\nu^2} = 9.631248*10^{-9}\omega^2 - \frac{4.834392*10^{-11}}{r^2}\omega^2 - 36.652*10^{-9}\,\omega^2 r^2$$

$$\frac{d\sigma_{rr}^*}{dr} = 0 = \left(-\frac{9.668784*10^{-11}}{r^3} - 124.168*10^{-9}\,r\right)\omega^2$$

gives complex r or r = 0 which is out of domain.

So σ_{rr}^* is max at either r = 100 mm or 400 mm

$$\Rightarrow \sigma_{rr}^*\Big|_{r=100} = 1.38448*10^{-8}\,\omega^2, \quad \sigma_{rr}^*\Big|_{400} = 0$$ (comes $-4.25*10^{-14}$ from computation)

$$\frac{d\sigma_{\theta\theta}}{dr} = 0 = \left[\frac{4.834392*10^{-11}*2}{r^3} - 36.652*10^{-9}*2*r\right] = 0$$

$\Rightarrow r = 0$ or $(1.318998 \times 10^{-3})^{1/4}$

$= 0.190573\ m = 190.573\ mm$

So, $(\sigma_{rr})_{max} = \frac{E}{1-\nu^2} \sigma_{rr}^{*}\Big|_{r=100mm} = 185.0909\omega^2\ N/m^2$

$(\sigma_{\theta\theta})_{max} = \frac{E}{1-\nu^2} \sigma_{\theta\theta}^{*}\Big|_{r=190.573mm} = 93.1683\,\omega^2\ N/m^2$

$\Rightarrow (\sigma_{rr})_{max}$ is the critical one.

$(\sigma_{rr})_{max} = \frac{\sigma_u}{SF} = \frac{20 \times 10^6}{2} \Rightarrow \omega_{max} = 232.438$ rad/sec