CE 469 Advanced Mechanics of Solids Tutorial 1

1. At a given point in a solid, the principal stresses are:

$$\lambda(1) = 1, \quad \lambda(2) = 4, \quad \lambda(3) = -2,$$

along with the following direction cosines:

$$n_1(1) = \frac{1}{2}, \quad n_2(1) = \frac{1}{2}, \quad n_3(1) = \sqrt{\frac{1}{2}}, \quad n_1(2) = 0$$

Find all the components of the stress tensor at this point.

2. In a solid circular shaft subject to pure twist, the components of the stress tensor are:

$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & -ax_2 \\ 0 & 0 & ax_1 \\ -ax_2 & ax_1 & 0 \end{bmatrix}$$

where a is a constant.
(a) Find the stress vector and the normal and shear components of the stress vector at the point $(1,2,2)$ for the following surfaces:

i. Plane $2x_1 + x_2 + x_3 = 6$
ii. Sphere $x_i x_i = 9$

(b) At the point $(1,2,2)$ find the principal stresses and corresponding planes, and the maximum shear stress and corresponding plane.
The positive direction of n_i (i.e., normal vector $\underline{\mathbf{n}}$) in each case is the side remote from the origin.

3. At a point in a solid, the components of the stress tensor are

$$\sigma_{ij} = \begin{bmatrix} 1 & -3 & \sqrt{2} \\ -3 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{bmatrix}$$

Find:

a. The principal deviator stresses and direction cosines of the corresponding planes.
b. The normal and shearing components of the stress vector on the octahaedral plane
c. The maximum shearing stresses and direction cosines of the corresponding plane.

4. Show that the octahaedral shear stress S_{OCT} (i.e., the magnitude of the shearing component of the stress vector on the octahedral plane) is given by $S_{\text{OCT}} = \left(-\frac{2}{3}\hat{I}_2\right)^{\frac{1}{2}}$, where $\hat{I}_2$ is the second invariant of the deviatoric stress tensor.

5. Determine the unknown stresses for the volume element, of unit thickness, shown below.