# CE469 Advanced Mechanics of Solids Tutorial 2

Problems 2, 3, 5, 6, 7, to be turned in.

1. Using the method of transformation of coordinates, derive all three equilibrium equation in cylindrical coordinates. Do this by first principles (i.e., by transforming stresses and partial derivatives) as done in class and check the result using the formulae given in class for transformation between orthogonal curvilinear coordinate systems. For cylindrical coordinates, $r^2 = x_1^2 + x_2^2$, $\theta = \tan^{-1}\left(\dfrac{x_2}{x_1}\right)$, $z = x_3$.

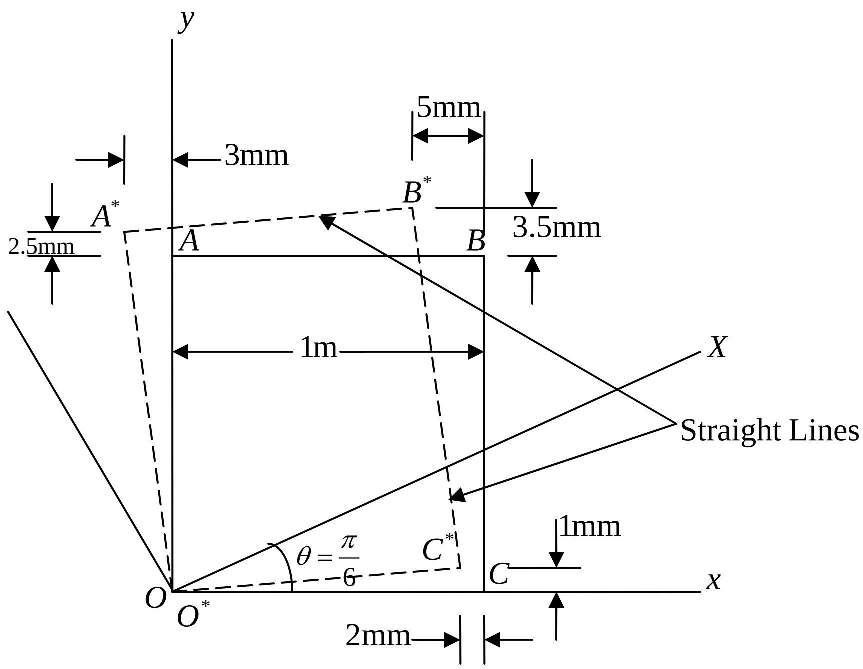
2. A rectangular plate having thickness $t = 1\text{cm}$ lies in the region $0 \le x_1 \le 2b$, $-c \le x_2 \le c$. It is loaded in a manner so as to give the following stress distribution:
$$\sigma_{11} = \frac{q}{2I}\left[x_1^2 x_2 - \frac{2}{3}x_2^3 + \frac{2}{5}c^2 x_2\right], \qquad \sigma_{22} = \frac{q}{2I}\left[\frac{1}{3}x_2^3 - c^2 x_2 + \frac{2}{3}c^3\right], \qquad \sigma_{i3} = 0$$
where $I = \dfrac{2c^3}{3}$, and $q$ is a constant. Assuming that body forces and body moments are absent, what must the shear component $\sigma_{12}$ be to ensure equilibrium. For what boundary conditions is the above solution valid?

3. Consider a light rod having a cross-sectional area of $1\,\text{cm}^2$. It is subjected to a uniformly distributed tensile load $P$ applied to its ends. The rod has the following strength characteristics, beyond which it breaks: maximum permissible shear, tensile, and compressive stresses are $300\times10^3\ \text{N/m}^2$, $10^5\ \text{N/m}^2$, and $10^7\ \text{N/m}^2$, respectively. At what values of $P$ will the rod break? What is the expected angle of inclination of the broken section? Re-work the problem for the case when the maximum permissible shear, tensile, and compressive stresses are $450\times10^3\ \text{N/m}^2$, $10^6\ \text{N/m}^2$, and $10^7\ \text{N/m}^2$, respectively.

4. The stress components in the half-space $x_3 > 0$ are:
$$\sigma_{ij} = \frac{a x_i x_j}{r^5} x_3, \quad r \ne 0, \quad a > 0$$
Find the total force on the surface of the hemisphere $r = a,\ x_3 > 0$

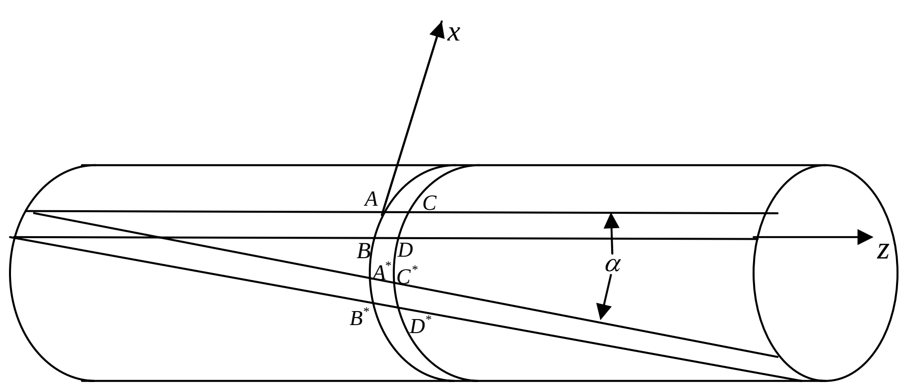
5. The square plate shown is loaded so that it is in a state of plane strain (i.e., $\varepsilon_{zx} = \varepsilon_{zy} = \varepsilon_{zz} = 0$).
   a) Determine the displacements $u_x \equiv u$, $u_y = v$ for the plate for the deformations shown in Fig.P5.
   b) Determine the strain components in the $(x, y)$ system.
   c) Determine the strain components in the $(X, Y)$ system.
   d) Determine the principal axes and principal strains.

**Fig.P5**




6. When solid circular torsion members are used to obtain material properties for finite strain applications, an expression for the engineering strain $\gamma_{xz}$ is needed, where $(x, z)$ is the tangent plane and the *z*-axes is along the axis of the member. Consider the element *ABCD* for the undeformed member as shown in Fig.P6. As an approximation, assume that the members deforms such that the volume and diameter remain constant. Thus for the deformed element we have $A^*B^* = AB$, $C^*D^* = CD$, and the distance (along the *z*-axis) between the parallel curved lines $A^*B^*$ and $C^*D^*$ remains unchanged. Show that the engineering shear strain $\gamma_{xz} = \tan\alpha$ where $\alpha$ is the angle between $AC$ and $A^*C^*$.

7. Consider the deformation field $x_1^* = x_1 + kx_2$, $x_2^* = x_2$, $x_3^* = x_3$. A differential square element $ABCD$ lies in the $x_1 - x_2$ plane and has sides of length *dL* with sides *AB* and *AD* parallel to the $x_1$ and $x_2$ axis, respectively. Calculate the unit extension for sides *AB*, *AD*, and diagonals *AC*, *DB* considering both the linear and the nonlinear theory. Under what restriction (if any) is the linear theory valid.



8. The displacement field in a solid is given as:

$$u_1 = \frac{a}{4} x_1 (x_2 + x_3)^2, \quad u_2 = \frac{a}{4} x_2 (x_1 + x_3)^2, \quad u_3 = \frac{a}{4} x_3 (x_1 + x_2)^2$$

where $a$ is an infinitesimal constant. Find the maximum and minimum values of the elongation per unit length at the point $P \Rightarrow (1,1,1)$

9. (a) Consider the displacement field $u_i \Rightarrow [cx_2x_3,\ cx_3x_1,\ cx_1x_2]$, where $c$ is an infinitesimal constant. Does this displacement field represent a state of (a) pure straining, (b) pure rigid body rotation, or (c) general motion.

   (b) Consider the displacement field $u_i \Rightarrow [cx_1x_2,\ cx_1x_2,\ 2c(x_1 + x_2)x_3]$, where $c$ is an infinitesimal constant. What is the rotational component of relative displacement between two neighbouring points originally lying on the line (in the $x_1x_2$ plane) that makes equal acute angles with positive $x_1$ and $x_2$ axes.