CE469 Advanced Mechanics of Solids Tutorial 3

1. Consider the infinitesimal strain distribution $\varepsilon_{11} = \varepsilon_{22} = 2x_1$, $\varepsilon_{12} = x_1 + 2x_2$, $\varepsilon_{33} = 2x_3$, $\varepsilon_{13} = \varepsilon_{23} = 0$. Show that this is a possible strain distribution. Then determine the displacements $u_i[x_1, x_2, x_3]$ if the origin has zero displacement and any infinitesimal line element at the origin has zero rotation. (Hint: integrate the strain displacement relations to obtain the displacements, and then determine the constants of integration by using the conditions given at the origin).

2. The solution for the displacement field of a problem done in cylindrical coordinates is as follows:

$$u_r = \frac{A}{r} + Br(\ln r - 1) + C\sin\theta$$
$$u_\theta = Br\theta + Dr + E\cos\theta, \qquad u_z = 0$$

where A, B, C, D, E are constants. Is displacement compatibility assured when considering arbitrary values of the constants? Which of these constants can be determined on the basis of physical arguments, and what are their values?

3. The displacement components of an incompressible solid are:

$$u_1 = (1 - x_2^2)(a + bx_1 + cx_1^2), \quad u_2\Big|_{x_2 = \pm\sqrt{3}} = 0, \; u_3 = 0$$

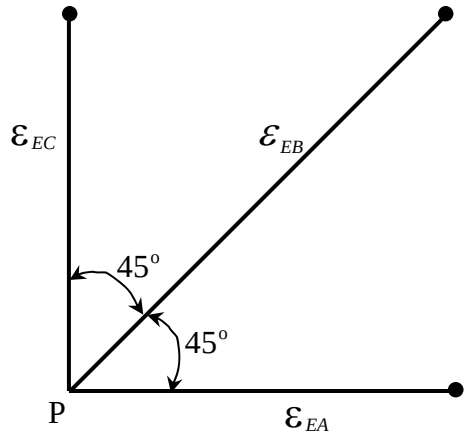
where a, b, c are infinitesimal constants. Determine u_2.

4. A 50mm cube of an isotropic solid ($E = 210\times10^9\ \mathrm{Nm}^{-2}$, $v = 0.25$) is loaded in a manner that yields a uniform state of stress given as,

$$\sigma_{ij} \Rightarrow 10^5 \times \begin{pmatrix} 700 & 300 & 40 \\ 300 & -15 & 0 \\ 40 & 0 & 100 \end{pmatrix} \mathrm{Nm}^{-2}$$

Assuming that the loading (and resulting stresses) are within limits for which the assumptions of linear elastic behavior is valid, determine the total change in volume induced by this stress field.

5. A thin plate is made of a linear, isotropic, homogeneous, elastic material. It is loaded along its edges with inplane loads, and body forces are negligible. The top and bottom faces are not loaded. A rectangular rosette (shown in the fig.) is attached to a point P on the upper face. The strain measurements recorded by the rosette are $\varepsilon_{EA} = -100$ $\varepsilon_{EB} = -200$ and $\varepsilon_{EC} = -400$ microinches per inch. Find the principal stresses at the point P, if $E = 30\times10^6$ psi and $v = 0.3$.



STRAIN ROSETTE

6. The displacement field in an isotropic , homogeneous, linear, elastic medium in equilibrium is given as:

$$u_1 = cx_1 \ , \ \ u_2 = -cvx_2 \ , \ \ u_3 = -cvx_3$$

where v is the Poisson's ratio and c is an infinitesimal constant. Find the body forces in terms of v and E.