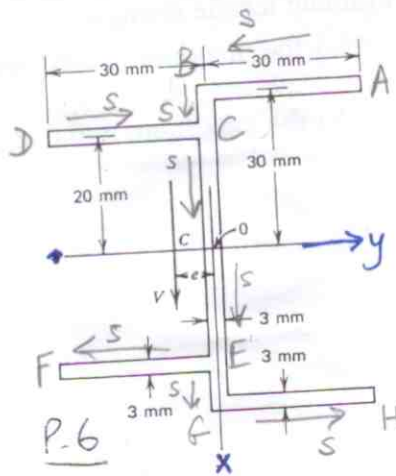


P.6.



O is CG.

$$I_{xx} = \frac{1}{12} \{ 63 \times 3^3 + 2 \times 3 \times 60^3 - 2 \times 3 \times 3^3 \} =$$

$$I_{yy} = \frac{1}{12} \{ 63^3 \times 3 + 4 \times 3^3 \times 30 - 2 \times 3^3 \times 3 \} + 2 \times 28.5 \times 3 \times (30^2 + 20^2) = 285068.25$$

$$I_{xy} = 0 \text{ (symm about y-axis).}$$

$$X_{CF} = 0 \text{ (symm abt y-axis).}$$

$$Q_x^{AB} = \int \int y \, ds \, dn = \int_{-31.5}^{-28.5} \int_{-30}^{30} y \, (dy) \, (dx) = \frac{(30^2 - y^2)}{2} (-3) = -f(y)$$

$$Q_x^{BC} = \int \int y \, (-dy) \, dx + Q_x^{AB} \Big|_{y=0} = -\frac{3}{2} (30)^2 = -c_1$$

$$Q_x^{DC} = \int_{-21.5}^{-18.5} \int_{-30}^{-30} y \, dy \, dx = \frac{(30^2 - y^2)}{2} (3) = +f(y); \quad Q_x^{FE} = \int_{21.5}^{18.5} \int_{-30}^{-30} y \, (-dy) \, (-dx) = \frac{(30^2 - y^2)}{2} (-3) = -f(y)$$

$$Q_x^{GH} = \int_{28.5}^{31.5} \int_{-30}^{30} y \, dy \, dx = \frac{(30^2 - y^2)}{2} (3) = +f(y)$$

$$Q_x^{CE} = \int_{x=1.5}^{-20} \int_{-20}^{-1.5} y \, (-dy) \, dx + Q_x^{BC} \Big|_{x=-20} + Q_x^{DC} \Big|_{y=0} = -\frac{3}{2} (30)^2 + \frac{3}{2} (30)^2 = 0$$

$$Q_x^{EG} = \int_{x=1.5}^{30} \int_{-1.5}^{-1.5} y \, (dy) \, dx + Q_x^{GH} \Big|_{y=0} = \frac{3}{2} \times 30^2 = c_1$$

$$Q_y^{AB} = \int_{-28.5}^{-31.5} \int_{-30}^{30} x \, (-dy) \, (-dx) = \frac{(31.5^2 - 28.5^2)}{2} (30 - y) = g(y)$$

$$Q_y^{BC} = \int_{x=1.5}^{-30} \int_{-1.5}^{-1.5} x \, (-dy) \, dx + Q_y^{AB} \Big|_{y=0} = \frac{(30^2 - x^2)}{2} (3) + \frac{(31.5^2 - 28.5^2)}{2} (30) = p(x)$$

$$Q_y^{DC} = \int_{-21.5}^{-18.5} \int_{-30}^{-30} x \, dy \, dx = \frac{(21.5^2 - 18.5^2)}{2} (30 + y) = h(y)$$

$$Q_y^{FE} = \int_{21.5}^{18.5} \int_{-30}^{-30} x \, (-dy) \, (-dx) = \frac{(21.5^2 - 18.5^2)}{2} (30 + y) = h(y)$$

$$Q_y^{GH} = \int_{28.5}^{31.5} \int_{-30}^{30} x \, dy \, dx = \frac{(31.5^2 - 28.5^2)}{2} (30 - y) = g(y)$$

$$Q_y^{EG} = \int_{x=1.5}^{30} \int_{-1.5}^{-1.5} x \, (-dy) \, dx + Q_y^{GH} \Big|_{y=0} = \frac{(30^2 - x^2)}{2} (3) + \frac{(31.5^2 - 28.5^2)}{2} (30) = p(x)$$

$$Q_y^{CE} = \int_{x=1.5}^{-20} \int_{-20}^{-1.5} x \, (-dy) \, dx + Q_y^{BC} \Big|_{x=-20} + Q_y^{DC} \Big|_{y=0} = \frac{(20^2 - x^2)}{2} (3) + \frac{(30^2 - 20^2)}{2} (3) + \frac{(31.5^2 - 28.5^2)}{2} (30) + \frac{(21.5^2 - 18.5^2)}{2} (30) = p(x)$$

$$y_{CF} = -\frac{I_x}{\Delta} \int Q_y (x dy - y dx) + \frac{I_{xy}}{\Delta} \int Q_x (x dy - y dx) \quad (2)$$

So all Q_x are not required. Here $\Delta = I_x I_y - I_{xy}^2$

$$y_{CF} = -\frac{1}{I_y} \int Q_y (x dy - y dx)$$

$$\begin{aligned} \int Q_y (x dy - y dx) &= \int_{A \rightarrow B} Q_y x dy + \int_{B \rightarrow C} Q_y (-y dx) + \int_{D \rightarrow C} Q_y x dy + \int_{C \rightarrow E} Q_y (-y dx) \\ &\quad + \int_{E \rightarrow G} Q_y (-y dx) + \int_{E \rightarrow F} Q_y x dy + \int_{G \rightarrow H} Q_y x dy \\ &= \int_0^{30} g(y) \times (-30) dy + \int_{-30}^{-20} p(x) \times (0) dx + \int_{-30}^0 h(y) \times (-20) dy + \int_{-20}^0 q(x) \times (-0) dx \\ &\quad + \int_{-20}^{30} p(x) \times (0) dx + \int_0^{-30} h(y) \times (20) dy + \int_0^{30} g(y) \times (30) dy \\ &= 2 \left[30 \int_0^{30} g(y) dy + 20 \int_0^{-30} h(y) dy \right] = 2 \left[30 \times \left(\frac{31.5^2 - 28.5^2}{2} \right) \left(30 \times 30 - \frac{30^2}{2} \right) \right. \\ &\quad \left. + 20 \times \left(\frac{21.5^2 - 18.5^2}{2} \right) \left(30 \times (-30) + \frac{30^2}{2} \right) \right] \\ &= 1350000 \end{aligned}$$

$$\Rightarrow y_{CF} = -\frac{1350000}{285068.25} = -4.7357 \text{ mm} \quad \leftarrow \text{(from O, i.e. CG)}$$

Q: Lets say we reverse direction of 's' in leg DC. Will it affect y_{CF} ?

$$A: Q_y^{DC} = \int_{-18.5}^{30} \int_y^x (-dy)(-dx) = -h(y)$$

$$\int Q_y x dy = \int_0^{-30} -h(y) \times (-20) dy = \int_{-30}^0 h(y) \times (-20) dy = \text{same as dotted circled term above.}$$

(C→D) note this change.

So y_{CF} won't change, as expected.

(See NOTE on next pg) →

This is the advantage of this seemingly lengthy method, i.e., no matter what direction of 's' you assume, as long as you are consistent in the limits of integration for the Q 's (i.e. from cut to free surface) and for

the $\int Q_y(x dy - y dx) + \int Q_x(x dy - y dx)$ (ie along increasing s) assumed s , you simply can't go wrong. ③

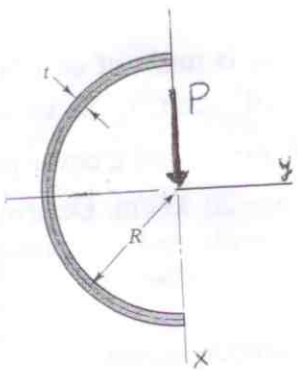
The advantage of this method is that it is programmable & good for non-symmetric sections, as opposed to other short-cut methods.

→ NOTE: In this case we would have

$$Q_x^{CE} = \int_{x=1.5}^{-20} \int_{y=1.5}^{-20} y(-dy) dx + Q_x^{BC} \Big|_{x=-20} - Q_x^{DC} \Big|_{y=0} = 0 + (-C_1) - (-fy) \Big|_{y=0} = 0$$

Side directions of s is opposite now
ie the same as before.

P.3



$$I_y = \frac{\pi R t}{2} R^2 = \frac{\pi R^3 t}{2}$$

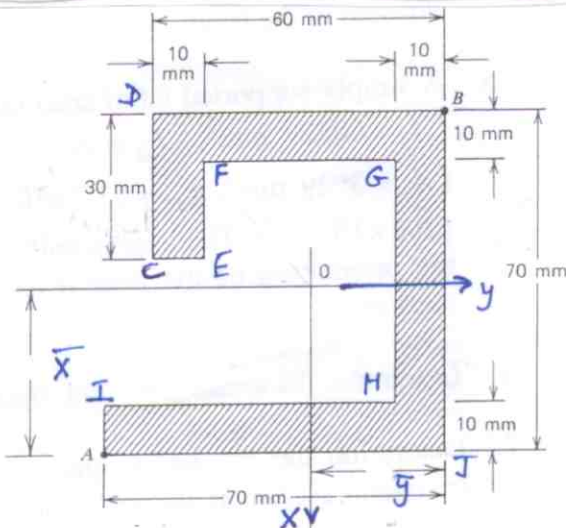
$$x = -R \cos \theta, \quad y = -R \sin \theta + \frac{2R}{\pi}$$

$$Q_y = \int_{\theta} x ds dn = \int_{\theta} (-R \cos \theta) (R d\theta) t = R^2 t \sin \theta$$

$$\begin{aligned} y_{CF} &= -\frac{1}{I_y} \int_0^{\pi} Q_y [(-R \cos \theta)(-R \cos \theta d\theta) - (-R \sin \theta + \frac{2R}{\pi})(R \sin \theta d\theta)] \\ &= -\frac{1}{I_y} \int_0^{\pi} R^2 t \sin \theta [R^2 - \frac{2R^2 \sin \theta}{\pi}] d\theta = -\frac{2}{\pi R^3 t} \left\{ R^4 t \left(\frac{\pi}{2} \right) - R^4 t \left(\frac{2}{\pi} \right) \left(\frac{\pi}{2} \right) \right\} \\ &= -\frac{4R}{\pi} + \frac{2R}{\pi} = -\frac{2R}{\pi} \text{ (from CG).} \end{aligned}$$

$$\alpha = \frac{3M}{G \sum a_i b_i^3} = \frac{3 \times P \times \frac{4R}{\pi}}{G \pi R t^3} = \frac{12P}{G \pi^2 t^3}$$

P.4

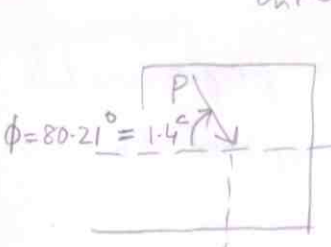


CG

$$\begin{aligned} \bar{X} &= \frac{(70)(10)(5) + (60)(10)(40) + (50)(10)(65) + (20)(10)(50)}{700 + 600 + 500 + 200} \\ &= 35 \text{ mm} \end{aligned}$$

$$\begin{aligned} \bar{Y} &= \frac{(700)(35) + (600)(5) + (500)(35) + (200)(55)}{700 + 600 + 500 + 200} \\ &= 28 \text{ mm} \end{aligned}$$

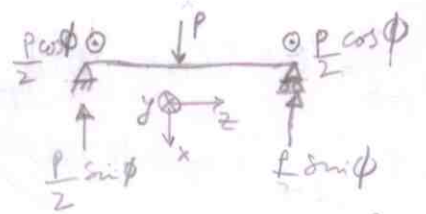
Note: x, y axis in problem statement and figure are interchanged. (4)



← applied load at midspan shown here.
 So bending moments at midspan are largest. They are,

$$M_y = -\frac{P}{2} \sin \phi \left(\frac{L}{2}\right) = -\frac{P}{2} \sin \phi = -2.5 \sin(80-21) \text{ kNm}$$

$$M_x = \frac{P}{2} \cos \phi \left(\frac{L}{2}\right) = \frac{P}{2} \cos \phi = 2.5 \cos(80-21) \text{ kNm}$$



$$I_{xx} = \frac{1}{12} [10 \times 70^3 + 60 \times 10^3 + 10 \times 50^3 + 20 \times 10^3] + 700 \times (28-35)^2$$

$$+ 600 \times (28-5)^2 + 500 \times (28-35)^2 + 200 \times (28-55)^2$$

$$= 918666.6667 \text{ mm}^4$$

$$I_{yy} = \frac{1}{12} [10^3 \times 70 + 60^3 \times 10 + 10^3 \times 50 + 20^3 \times 10] + 700 \times (35-5)^2$$

$$+ 600 \times (35-40)^2 + 500 \times (35-65)^2 + 200 \times (35-50)^2$$

$$= 1336666.667 \text{ mm}^4$$

$$I_{xy} = 700(35-5)(28-35) + 600(35-40)(28-5)$$

$$+ 500(35-65)(28-35) + 200(35-50)(28-55)$$

$$= -30000 \text{ mm}^4$$

$$\sigma_{zz} = -\left(\frac{M_x I_{xy} + M_y I_x}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} + M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y$$

= Ax + By, A = 0.5391 N/mm³, B = 0.2821 N/mm³ CG, '0'

Method-1 (to find max compressive/tensile stress).
 Evaluate stress at all corners & choose max tensile & compressive values. This will work only if boundaries are vertical or horizontal.

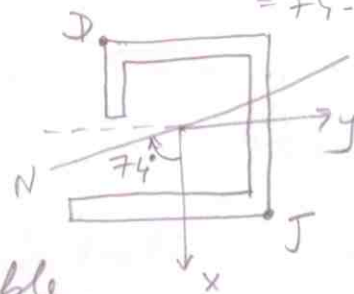
| Point | A | B | C | D | E | F | G | H | I | J |
|---------------|----------|----------|-----------|------------|-----------|------------|-----------|----------|-----------|----------|
| | (35, 43) | (35, 28) | (-5, -32) | (-35, -32) | (-5, -22) | (-25, -22) | (-25, 18) | (25, 18) | (25, -42) | (35, 28) |
| σ_{zz} | 7.02 | -10.97 | -11.7 | -27.89 | -8.9 | -19.68 | -8.39 | 18.55 | 1.629 | 26.76 |

Method-2 Shorter, will work any boundaries. (5)
 Find N.A inclination, β . Then find stresses of points farthest from N.A. These are the extreme values sought.

$$\tan \beta = \frac{M_x I_{xy} + M_y I_{xx}}{M_x I_{yy} + M_y I_{xy}} = -3.5445, \quad \beta = -74.24^\circ$$

ccw from x-axis
 $= 74.24^\circ$ CW from x-axis.

So D, J are farthest points, so evaluate σ_{zz} at D, J for extreme values. Result matches with Method-I Table.



$$(\sigma_{zz})_D = -27.89 \text{ N/mm}^2 \text{ (max compressive)}$$

$$(\sigma_{zz})_J = 26.76 \text{ N/mm}^2 \text{ (max tensile)}$$

Can guess it (intuitively) from the physical application of load & deformation expected, that probably 'D' will be max compression fiber & J will be max tensile fiber.