CE623 Advanced Mechanics of Solids 3-0-0 HW 1 Fall 2005

(5)

1. In a square plate in the region $-1 \leq x_1,\ x_2 \leq 1$ the following stresses hold:

   $$\sigma_{11} = cx_1 + dx_2, \quad \sigma_{22} = ax_1 + bx_2, \quad \sigma_{33} = \sigma_{31} = \sigma_{32} = 0$$

   where a, b, c, d are constants. Assuming that body forces and body moments are absent, what must the shear component σ_{12} be in order to ensure equilibrium? State the boundary conditions (on the edges of the plate) when $a = b = 0$

(5)

2. In plane stress problems, the stress components $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$, and the remaining ones are functions of x_1 and x_2 only. Assuming that body forces and body moments are absent, find a relation between the non-zero stress components and a single stress function $\phi[x_1, x_2]$, so that all equilibrium equations are satisfied.

(5)

3. At a given point in a solid, the principal stresses are:

   $$\sigma(1) = 1, \quad \sigma(2) = 4, \quad \sigma(3) = -2,$$

   along with the following direction cosines:

   $$n_1(1) = \frac{1}{2}, \quad n_2(1) = \frac{1}{2}, \quad n_3(1) = \sqrt{\frac{1}{2}}, \quad n_1(2) = 0$$

   Find all the components of the stress tensor at this point.

(10)

4. In a solid circular shaft subject to pure twist, the components of the stress tensor are:

   $$\sigma_{ij} \Rightarrow \begin{pmatrix} 0 & 0 & -ax_2 \\ 0 & 0 & ax_1 \\ -ax_2 & ax_1 & 0 \end{pmatrix}$$

   where a is a constant.

   (a) Find the stress vector and the normal and shear components of the stress vector at the point $(1, 2, 2)$ for the following surfaces:

   i. Plane $2x_1 + x_2 + x_3 = 6$

   ii. Sphere $x_i x_i = 9$

   The positive direction of n_i (i.e., the normal vector $\vec{n}$) in each case is the side remote from the origin.

   (b) At the point $(1, 2, 2)$ find the principal stresses and axes, and the maximum shearing stress.

(10) 5. At a point in a solid, the components of the stress tensor are

$$\sigma_{ij} \Rightarrow \begin{pmatrix} 1 & -3 & \sqrt{2} \\ -3 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{pmatrix}$$

Find:

(a) The principal deviator stresses

(b) The normal and shearing components of the stress vector on the octahaedral plane

(c) The maximum shearing stresses

(5) 6. A plate is stretched in the x_1 direction, compressed in the x_2 direction, and free in the x_3 direction. The loading is uniform along the edges, i.e., the tensile and compressive loads (per unit area) are spatially constant and given as σ_{11}^* and σ_{22}^*, respectively. There is a flaw in a plane that is parallel to the x_3 axis and inclined at 45° to the x_1 axis. If the shear stress acting on the flaw exceeds a critical value σ_{cr}, the plate will fail. Determine the critical combinations of σ_{11}^* and σ_{22}^* at which the plate fails.

(15) 7. Using the method of transformation of coordinates, derive all three equilibrium equation in cylindrical coordinates. Do this by first principles (i.e., by transforming stresses and partial derivatives) as done in class and check the result using the formulae given in class for transformation between orthogonal curvilinear coordinate systems. For cylindrical coordinates, $r^2 = x_1^2 + x_2^2, \ \theta = \tan^{-1}(\frac{x_2}{x_1}), \ z = x_3$

(5) 8. A rectangular plate having thickness t = 1cm lies in the region $0 \le x_1 \le 2b, \ -c \le x_2 \le c$. It is loaded in a manner so as to give the following stress distribution:

$$\sigma_{11} = \frac{q}{2I}\left[x_1^2 x_2 - \frac{2}{3}x_2^3 + \frac{2}{5}c^2 x_2\right], \quad \sigma_{22} = \frac{q}{2I}\left[\frac{1}{3}x_2^3 - c^2 x_2 + \frac{2}{3}c^3\right], \quad \sigma_{i3} = 0$$

where $I = \dfrac{2c^3}{3}$, and q is a constant. Assuming that body forces and body moments are absent, what must the shear component σ_{12} be to ensure equilibrium. For what boundary conditions is the above solution valid?

(15) 9. (a) Find an expression in terms of the principal stresses for the magnitude σ_{OCT} of the stress vector on an octahaedral plane.

(b) Show that the normal component N_{OCT} of the stress vector on the octahaedral plane of part (a) (i.e., the octahaedral normal stress) equals $\frac{1}{3}I_1$ where I_1 is the first invariant of the stress tensor.

(c) Show that the octahaedral shear stress S_{OCT} (i.e., the magnitude of the shearing component of the stress vector on the octahaedral plane) is given by $S_{OCT} = \left(-\frac{2}{3}\hat{I}_2\right)^{1/2}$, where $\hat{I}_2$ is the second invariant of the deviatoric stress tensor.

10. Consider a light rod having a cross-sectional area of $1\,cm^2$. It is subjected to a uniformly distributed tensile load P applied to its ends. The rod has the following strength characteristics, beyond which it breaks: maximum permissible shear, tensile, and compressive stresses are $300 \times 10^3\,N/m^2$, $10^5\,N/m^2$, and $10^7\,N/m^2$, respectively. At what value of P will the rod break? What is the expected angle of inclination of the broken section? Re-work the problem for the case when the maximum permissible shear, tensile, and compressive stresses are $450 \times 10^3\,N/m^2$, $10^6\,N/m^2$, and $10^7\,N/m^2$, respectively.

11. Determine the principal stresses, maximum shearing stress, and octahaedral shearing stress at a point at which $\sigma_{xx} = 80$ MPa, $\sigma_{yy} = -35$ MPa, $\sigma_{zz} = -50$ MPa, and $\sigma_{xy} = 45$ MPa.

12. Determine the principal stresses and maximum shearing stress at a point at which $\sigma_{xx} = -150$ MPa, $\sigma_{yy} = 0$ MPa, $\sigma_{zz} = 80$ MPa, $\sigma_{xy} = -40$ MPa, $\sigma_{yz} = 0$ MPa, $\sigma_{zx} = 50$ MPa.

13. Determine the unknown stresses for the volume element shown.