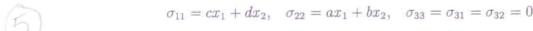
1. In a square plate in the region  $-1 \le x_1, x_2 \le 1$  the following stresses hold:



where a, b, c, d are constants. Assuming that body forces and body moments are absent, what must the shear component  $\sigma_{12}$  be in order to ensure equilibrium? State the boundary conditions (on the edges of the plate) when a=b=0

- 2. In plane stress problems, the stress components  $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$ , and the remaining ones are functions of  $x_1$  and  $x_2$  only. Assuming that body forces and body moments are absent, find a relation between the non-zero stress components and a single stress function  $\phi[x_1, x_2]$ , so that all equilibrium equations are satisfied.
- 3. At a given point in a solid, the principal stresses are:

$$\sigma(1) = 1$$
,  $\sigma(2) = 4$ ,  $\sigma(3) = -2$ ,

along with the following direction cosines:

$$n_1(1) = \frac{1}{2}, \quad n_2(1) = \frac{1}{2}, \quad n_3(1) = \sqrt{\frac{1}{2}}, \quad n_1(2) = 0$$

Find all the components of the stress tensor at this point.

4. In a solid circular shaft subject to pure twist, the components of the stress tensor are:

$$\sigma_{ij} \Rightarrow \begin{pmatrix} 0 & 0 & -ax_2 \\ 0 & 0 & ax_1 \\ -ax_2 & ax_1 & 0 \end{pmatrix}$$

where a is a constant.

- (a) Find the stress vector and the normal and shear components of the stress vector at the point (1, 2, 2) for the following surfaces:
  - i. Plane  $2x_1 + x_2 + x_3 = 6$
  - ii. Sphere  $x_i x_i = 9$

The positive direction of  $n_i$  (i.e., the normal vector  $\vec{n}$ ) in each case is the side remote from the origin.

(b) At the point (1, 2, 2) find the principal stresses and axes, and the maximum shearing stress.

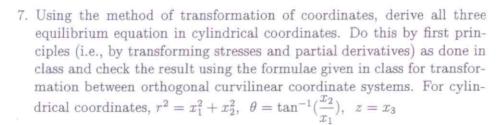
5. At a point in a solid, the components of the stress tensor are

$$\sigma_{ij} \Rightarrow \begin{pmatrix} 1 & -3 & \sqrt{2} \\ -3 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{pmatrix} .$$

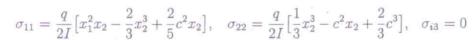
(10)

Find:

- (a) The principal deviator stresses
- (b) The normal and shearing components of the stress vector on the octahaedral plane
- (c) The maximum shearing stresses
- 6. A plate is stretched in the x<sub>1</sub> direction, compressed in the x<sub>2</sub> direction, and free in the x<sub>3</sub> direction. The loading is uniform along the edges, i.e., the tensile and compressive loads (per unit area) are spatially constant and given as σ<sub>11</sub>\* and σ<sub>22</sub>\*, respectively. There is a flaw in a plane that is parallel to the x<sub>3</sub> axis and inclined at 45° to the x<sub>1</sub> axis. If the shear stress acting on the flaw exceeds a critical value σ<sub>cr</sub>, the plate will fail. Determine the critical combinations of σ<sub>11</sub>\* and σ<sub>22</sub>\* at which the plate fails.



8. A rectangular plate having thickness t=1cm lies in the region  $0 \le x_1 \le 2b, -c \le x_2 \le c$ . It is loaded in a manner so as to give the following stress distribution:



where  $I = \frac{2c^3}{3}$ , and q is a constant. Assuming that body forces and body moments are absent, what must the shear component  $\sigma_{12}$  be to ensure equilibrium. For what boundary conditions is the above solution valid?

- (a) Find an expression in terms of the principal stresses for the magnitude σ<sub>OCT</sub> of the stress vector on an octahaedral plane.
  - (b) Show that the normal component  $N_{OCT}$  of the stress vector on the octahaedral plane of part (a) (i.e., the octahaedral normal stress) equals  $\frac{1}{3}I_1$  where  $I_1$  is the first invariant of the stress tensor.

- (c) Show that the octahaedral shear stress  $S_{OCT}$  (i.e., the magnitude of the shearing component of the stress vector on the octahaedral plane) is given by  $S_{OCT} = \left(-\frac{2}{3}\hat{I}_2\right)^{1/2}$ , where  $\hat{I}_2$  is the second invariant of the deviatoric stress tensor.
- 10. Consider a light rod having a cross-sectional area of  $1\,cm^2$ . It is subjected to a uniformly distributed tensile load P applied to its ends. The rod has the following strength characteristics, beyond which it breaks: maximum permissible shear, tensile, and compressive stresses are  $300 \times 10^3\,N/m^2$ ,  $10^5\,N/m^2$ , and  $10^7\,N/m^2$ , respectively. At what value of P will the rod break? What is the expected angle of inclination of the broken section? Re-work the problem for the case when the maximum permissible shear, tensile, and compressive stresses are  $450 \times 10^3\,N/m^2$ ,  $10^6\,N/m^2$ , and  $10^7\,N/m^2$ , respectively.
- 11. Determine the principal stresses, maximum shearing stress, and octahaedral shearing stress at a point at which  $\sigma_{xx}=80$  MPa,  $\sigma_{yy}=-35$  MPa,  $\sigma_{zz}=-50$  MPa, and  $\sigma_{xy}=45$  MPa.
- 12. Determine the principal stresses and maximum shearing stress at a point at which  $\sigma_{xx} = -150$  MPa,  $\sigma_{yy} = 0$  MPa,  $\sigma_{zz} = 80$  MPa,  $\sigma_{xy} = -40$  MPa,  $\sigma_{yz} = 0$  MPa,  $\sigma_{zx} = 50$  MPa.
- 13. Determine the unknown stresses for the volume element shown.

