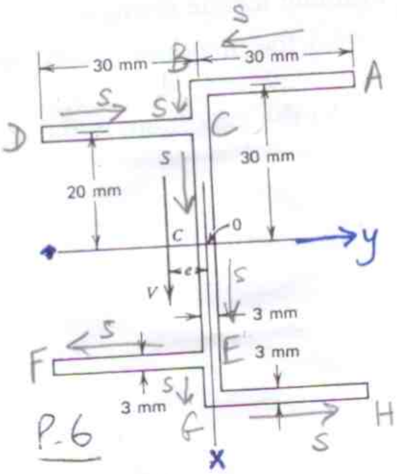
HW-6 solutions (P.3, 4, 6 only) (1)

P. 6.



O is CG.

$$I_{xx} = \frac{1}{12}\{63*3^3 + 2*3*60^3 - 2*3*3^3\} =$$

$$I_{yy} = \frac{1}{12}\{63^3*3 + 4*3^3*30 - 2*3^3*3\} + 2*28.5*3*(30^2+20^2) = 285068.25$$

$I_{xy} = 0$ (symm about y-axis).

$x_{cF} = 0$ (symm abt y-axis).

$$Q_x^{AB} = \iint y\,ds\,dn = \int_{-28.5}^{-31.5}\int_{y}^{30} y(-dy)(-dx) = \frac{(30^2-y^2)}{2}(-3) = -f(y)$$

$$Q_x^{BC} = \int_{x}^{-30}\int_{1.5}^{-1.5} y(-dy)\,dx + Q_x^{AB}\Big|_{y=0} = -\frac{3}{2}(30)^2 = -c_1$$

$$Q_x^{DC} = \int_{-21.5}^{-18.5}\int_{y}^{-30} y\,dy\,dx = \frac{(30^2-y^2)}{2}(3) = +f(y); \quad Q_x^{FE} = \int_{21.5}^{18.5}\int_{y}^{-30} y(-dy)(-dx) = \frac{(30^2-y^2)}{2}(-3) = -f(y)$$

$$Q_x^{GH} = \int_{28.5}^{31.5}\int_{y}^{30} y\,dy\,dx = \frac{(30^2-y^2)}{2}(3) = +f(y)$$

$$Q_x^{CE} = \int_{x}^{-20}\int_{1.5}^{-1.5} y(-dy)\,dx + Q_x^{BC}\Big|_{x=-20} + Q_x^{DC}\Big|_{y=0} = -\frac{3}{2}(30)^2 + \frac{3}{2}(30)^2 = 0$$

$$Q_x^{EG} = \int_{x}^{30}\int_{1.5}^{-1.5} y(-dy)\,dx + Q_x^{GH}\Big|_{y=0} = \frac{3}{2}*30^2 = c_1$$

$$Q_y^{AB} = \int_{-28.5}^{-31.5}\int_{y}^{30} x(-dy)(-dx) = \frac{(31.5^2-28.5^2)}{2}(30-y) = g(y)$$

$$Q_y^{BC} = \int_{x}^{-30}\int_{1.5}^{-1.5} x(-dy)\,dx + Q_x^{AB}\Big|_{y=0} = \frac{(30^2-x^2)}{2}(3) + \frac{(31.5^2-28.5^2)}{2}(30) = p(x)$$

$$Q_y^{DC} = \int_{-21.5}^{-18.5}\int_{y}^{-30} x\,dy\,dx = \frac{(21.5^2-18.5^2)}{2}(30+y) = h(y)$$

$$Q_y^{FE} = \int_{21.5}^{18.5}\int_{y}^{-30} x(-dy)(-dx) = \frac{(21.5^2-18.5^2)}{2}(30+y) = h(y)$$

$$Q_y^{GH} = \int_{28.5}^{31.5}\int_{y}^{30} x\,dy\,dx = \frac{(31.5^2-28.5^2)}{2}(30-y) = g(y)$$

$$Q_y^{CE} = \int_{x}^{-20}\int_{1.5}^{-1.5} x(-dy)\,dx + Q_y^{BC}\Big|_{x=-20} + Q_y^{DC}\Big|_{y=0}$$
$$= \frac{(20^2-x^2)}{2}(3) + \frac{(30^2-20^2)}{2}(3) + \frac{(31.5^2-28.5^2)}{2}(30) + \frac{(21.5^2-18.5^2)}{2}(30) = q(x)$$

$$Q_y^{EG} = \int_{x}^{30}\int_{1.5}^{-1.5} x(-dy)\,dx + Q_y^{GH}\Big|_{y=0} = \frac{(30^2-x^2)}{2}(3) + \frac{(31.5^2-28.5^2)}{2}(30) = p(x)$$

$$y_{cF} = -\frac{I_x}{\Delta}\int Q_y(x\,dy - y\,dx) + \frac{\cancel{I_{xy}}^{\,0}}{\Delta}\int Q_x(x\,dy - y\,dx)$$ ②

So all Q_x are not required. Here $\Delta = I_x I_y - \cancel{I_{xy}^2}^{\,0}$

$$y_{cF} = -\frac{1}{I_y}\int Q_y(x\,dy - y\,dx)$$

$$\int Q_y(x\,dy - y\,dx) = \int_{A\to B} Q_y\,x\,dy + \int_{B\to C} Q_y(-y\,dx) + \int_{D\to C} Q_y\,x\,dy + \int_{C\to E} Q_y(-y\,dx)$$

$$+ \int_{E\to G} Q_y(-y\,dx) + \int_{E\to F} Q_y\,x\,dy + \int_{G\to H} Q_y\,x\,dy$$

$$= \int_{30}^{0} g(y)*(-30)\,dy + \int_{-30}^{-20} p(x)*(0)\,dx + \boxed{\int_{-30}^{0} h(y)*(-20)\,dy} + \int_{-20}^{20} q(x)*(-0)\,dx$$

$$+ \int_{20}^{30} p(x)*(0)\,dx + \int_{0}^{-30} h(y)*(20)\,dy + \int_{0}^{30} g(y)*(30)\,dy$$

$$= 2\left[30\int_0^{30} g(y)\,dy + 20\int_0^{-30} h(y)\,dy\right] = 2\left[30*\left(\frac{31.5^2 - 28.5^2}{2}\right)\left(30*30 - \frac{30^2}{2}\right) + 20*\left(\frac{21.5^2 - 18.5^2}{2}\right)\left(30*(-30) + \frac{30^2}{2}\right)\right]$$

$$= 1350000$$

$$\Rightarrow y_{cF} = -\frac{1350000}{285068.25} = -4.7357 \text{ mm}$$ ◄ (from O, ie CG).

Q: Lets say we reverse direction of 's' in leg DC. Will it affect y_{cF}?

A: $$Q_y^{DC} = \int_{-18.5}^{-21.5}\int_y^{30} x(-dy)(-dx) = -h(y)$$

$$\int_{C\to D} Q_y\,x\,dy = \int_0^{-30} -h(y)*(-20)\,dy = \int_{-30}^{0} h(y)*(-20)\,dy$$ = same as dotted circled term above.

↑ note this change. (C→D)

So y_{cF} wont change, as expected. (See NOTE on next pg) →

This is the advantage of this seemingly lengthy method, ie, no matter what direction of s you assume, as long as you are consistent in the limits of integration for the Q's (ie from cut to free surface) and for the $\int Q_y (x\,dy - y\,dx) + \int Q_x (x\,dy - y\,dx)$ (ie. along increasing assumed s, you simply can't go wrong.

The advantage of this method is that it is programmable & good for non-symmetric sections, as opposed to other short-cut methods.

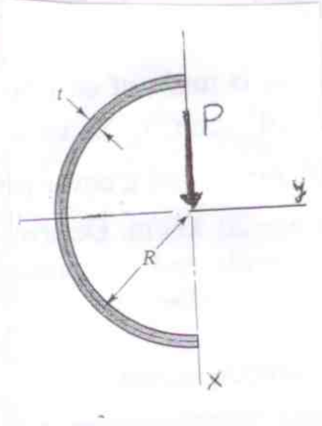
→ NOTE: In this case we would have

$$Q_x^{CE} = \int_x^{-20} \int_{1.5}^{-1.5} y(-dy)\,dx + Q_x^{BC}\Big|_{x=-20} \ominus Q_x^{DC}\Big|_{y=0} = 0 + (-C_1) - (-f(y))\Big|_{y=0} = 0$$

since directions of s is opposite now

ie the same as before.

P. 3



$$I_y = \frac{\pi R t}{2} R^2 = \frac{\pi R^3 t}{2}$$

$$x = -R\cos\theta, \quad y = -R\sin\theta + \frac{2R}{\pi}$$

$$Q_y = \int_\theta^0 x\,ds\,dn = \int_\theta^0 (-R\cos\theta)(R\,d\theta)t = R^2 t \sin\theta$$

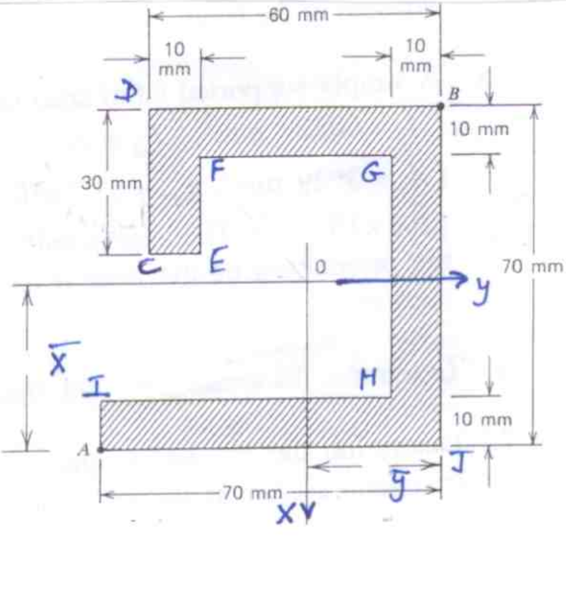
$$y_{CF} = -\frac{1}{I_y}\int_0^\pi Q_y\left[(-R\cos\theta)(-R\cos\theta\,d\theta) - \left(-R\sin\theta + \frac{2R}{\pi}\right)(R\sin\theta\,d\theta)\right]$$

$$= -\frac{1}{I_y}\int_0^\pi R^2 t\sin\theta\left[R^2 - \frac{2R^2}{\pi}\sin\theta\right]d\theta = -\frac{2}{\pi R^3 t}\left\{R^4 t(2) - R^4 t\left(\frac{2}{\pi}\right)\left(\frac{\pi}{2}\right)\right\}$$

$$= -\frac{4R}{\pi} + \frac{2R}{\pi} = -\frac{2R}{\pi} \text{ (from CG).}$$

$$\alpha = \frac{3M}{G\sum a_i b_i^3} = \frac{3 * P * \frac{4R}{\pi}}{G\,\pi R t^3} = \frac{12P}{G\pi^2 t^3}$$

P. 4



CG

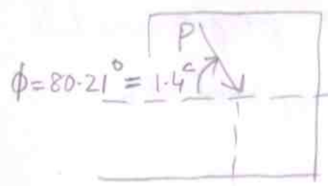
$$\bar{X} = \frac{(70)(10)(5) + (60)(10)(40) + (50)(10)(65) + (20)(10)(50)}{700 + 600 + 500 + 200}$$

$$= 35\text{ mm}$$

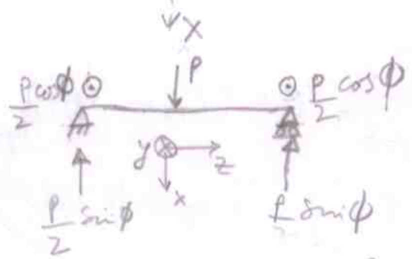
$$\bar{y} = \frac{(700)(35) + (600)(5) + (500)(35) + (200)(55)}{700 + 600 + 500 + 200}$$

$$= 28\text{ mm}$$

Note: x, y axis in problem statement and figure are interchanged.



← applied point load at midspan shown here. So bending moments at midspan are largest. They are,



$$M_y = -\frac{P}{2}\sin\phi\left(\frac{L}{2}\right) = -\frac{P}{2}\sin\phi = -2.5\sin(80.21)\ \text{kN.m}$$

$$M_x = \frac{P}{2}\cos\phi\left(\frac{L}{2}\right) = \frac{P}{2}\cos\phi = 2.5\cos(80.21)\ \text{kN.m}$$

$$I_{xx} = \frac{1}{12}\left[10*70^3 + 60*10^3 + 10*50^3 + 20*10^3\right] + 700*(28-35)^2 + 600*(28-5)^2 + 500*(28-35)^2 + 200*(28-55)^2$$

$$= 918666.6667\ \text{mm}^4$$

$$I_{yy} = \frac{1}{12}\left[10^3*70 + 60^3*10 + 10^3*50 + 20^3*10\right] + 700*(35-5)^2 + 600*(35-40)^2 + 500*(35-65)^2 + 200*(35-50)^2$$

$$= 1336666.667\ \text{mm}^4$$

$$I_{xy} = 700(35-5)(28-35) + 600(35-40)(28-5) + 500(35-65)(28-35) + 200(35-50)(28-55)$$

$$= -30000\ \text{mm}^4.$$

$$\sigma_{zz} = -\left(\frac{M_x I_{xy} + M_y I_x}{I_{xx}I_{yy} - I_{xy}^2}\right)x + \left(\frac{M_x I_{yy} + M_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)y$$

→ (x, y) w.r.t CG, 'O'!

$$= Ax + By, \quad A = 0.5391\ \text{N.mm}^{-3}, B = 0.2821\ \text{N.mm}^{-3}$$

Method-1 (to find max compressive / tensile stresses)

Evaluate stresses at all corners & choose max tensile & compressive values. This will work only if boundaries are vertical or horizontal.

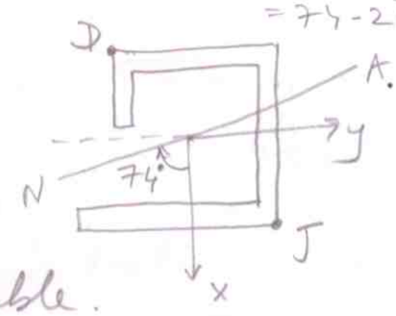
Point	A (35,-42)	B (-35,28)	C (-5,-32)	D (-35,-32)	E (-5,-22)	F (-25,-22)	G (-25,18)	H (25,18)	I (25,-42)	J (35,28)
σ_{zz}	7.02	-10.97	-11.7	-27.89 ✓	-8.9	-19.68	-8.39	18.55	1.629	26.76 ✓

Method -2 Shorter, will work any boundaries.

Find N.A inclination, β. Then find stresses of points farthest from N.A. These are the extreme values sought.

$$\tan\beta = \frac{M_x I_{xy} + M_y I_{xx}}{M_x I_{yy} + M_y I_{xy}} = -3.5445, \quad \beta = -74.24^\circ$$

CCW from x-axis
= 74.24° CW from x-axis.



So D, J are farthest points, so evaluate σ_{zz} at D, J for extreme values — Result matches with Method – I Table.

$(\sigma_{zz})_D = -27.89\ N/mm^2$ (max compressive)

$(\sigma_{zz})_J = 26.76\ N/mm^2$ (max tensile)

Can guess it (intuitively) from the physical application of load & deformation expected, that probably 'D' will be max compression fiber & J will be max tensile fiber.