CE623 Advanced Mechanics of Solids Midsem 21/9/05

Marks: 1 → 10, 2(a) → 10, 2(b) → 10, 3 → 20, 4 → 20, 5 → 30

1. A long cylinder is restrained from longitudinal displacement at its two end faces and it is subject to loading that does not vary along the longitudinal direction. A strain rosette, having geometry as shown, is embedded onto a cross-sectional face at a point P located far away from the end faces. The measured strains are $\varepsilon_{EA} = 200$, $\varepsilon_{EB} = -100$, $\varepsilon_{EC} = 400$ μmm/mm. Using material properties $E = 210 \times 10^9$ N/m², $\nu = 0.25$, and assuming infinitesimal displacement gradients, determine the principal stresses at P.

2. (a) The stresses at a point in a solid are

$$\sigma_{ij} = \begin{pmatrix} -a & 0 & d \\ 0 & b & e \\ d & e & c \end{pmatrix}$$

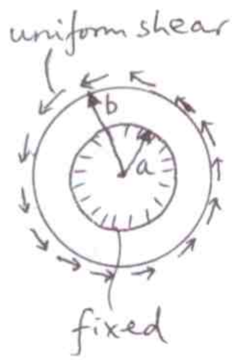
Determine the unit normal (in terms of a, b, c, d, e) of a plane parallel to the x_3 axis on which the resultant stress vector is tangential to the plane.

(b) The strains in a solid are obtained as $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = F[x_1, x_2, x_3]$, $\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0$, and infinitesimal strains can be assumed. Obtain the explicit form of $F[x_1, x_2, x_3]$ in terms of unknown constants to be determined later.

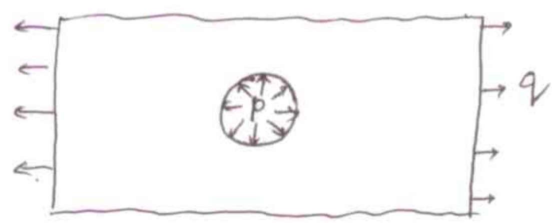
3. Consider the annular ring fixed at $r = a$ as shown. The ring is subjected to a uniform circumferential shear at $r = b$ that forms a resultant couple M. Using the Airy stress function $\phi = C\theta$, where C is a constant, determine the displacements u_r and u_θ in terms of the applied couple.

4. A large plate has a small circular hole of radius a. The plate is loaded by uniform tension q parallel to the x-axis and a uniform pressure p at the hole, as shown. Determine:
   (a) The stresses in the plate
   (b) The least value of p for which σ_θ is tensile everywhere along the hole.

5. The long gravity wall shown has a density ρ and is subject to uniform shear q (constant) along the right face. It is restrained at its two end faces (i.e., where $z =$ constant). Assuming $\sigma_{xy} = f[x]$, determine:
   (a) the stresses in the wall.
   (b) the displacements, assuming zero displacement and rotation at the origin.



P.3



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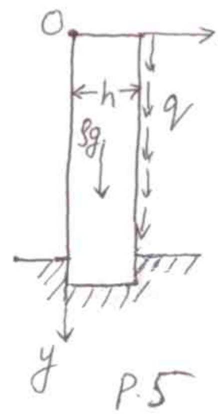

P.5