CE623 Advanced Mechanics of Solids Midsem 9/9/06

- Marks: **Q1=25, Q2=20, Q3=25, Q4=30.**
- Show all working.
- Attempt all parts of a question in a contiguous manner, i.e., dont scatter parts of the same question all over the answerbook.
- **Only one attempt per question will be graded**. So cancel out any attempt you do not want graded. The first not-cancelled attempt will be graded by default.
- **Formula sheet must be submitted with the answer book. No solved examples allowed on formula sheet.**

1. A pure shear state of stress referred to the *xyz* coordinate system is given as

$$\sigma_{ij} = \begin{pmatrix} a & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & b \end{pmatrix}$$

Thus, there exists a transformation to a coordinate system *XYZ* such that the stress matrix in the *XYZ* system has zero diagonal entries. In addition, it is also given that there exist plane(s) on which the total stress is zero. Determine :

(i) *a, b*, and direction cosines (referred to *xyz* coordinate system) of the plane(s) on which the total stress is zero.

(ii) The transformation from *xyz* system to *XYZ* system.

(Hint: for part (ii) consider a transformation such that $\sigma_{YY} = \sigma_{yy} = 0$ and use the invariant property(s) of the stress tensor)

2. Given the displacement field

$$u_x = \sqrt{2x}\cos y - x; \qquad u_y = \sqrt{2x}\sin y - y; \qquad u_z = 0$$

determine the change in volume per unit original volume.

3 The infinitesimal strains in a solid are obtained as $\varepsilon_{xx} = -\varepsilon_{yy} = -\varepsilon_{zz} = f(y,z)$, and all shear strains are zero. The engineering extensional strains are measured at three points, i.e., at point $P \equiv (2,1,1)$ along the direction parallel to the *x*-axis, at point $Q \equiv (-2,-1,-1)$ along the direction parallel to the line $y - z = 0$, and at point $R \equiv (0,-2,0)$ along the direction parallel to the line $y + z = 0$. All three measurements have value 0.01. Determine the strain tensor for an arbitrary point.
(Hint: the strains must satisfy compatibility)

4. Consider the shear loaded cantilever as a plane stress problem. The loading is only on the top face, and varies linearly from zero at the left end to k/L N/mm^2 at the right end. Determine the solution for the stresses in terms of *k, L, h.*
(Hint: assume a form for one of the stress components based on the variation of the applied load, and use the semi-inverse method).