

1. The state of stress at a point in two Cartesian systems is

**(20 marks)**

$$\sigma_{ij} = \begin{pmatrix} 5 & a & -a \\ a & 0 & b \\ -a & b & 0 \end{pmatrix}, \quad \sigma_{ij} = \begin{pmatrix} c & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & d \end{pmatrix}$$

where  $a, b, c, d$ , are constants

If the magnitude of the maximum shearing stress at this point is 5.5 MPa

- Determine the principal stresses at this point
- Also find  $a$  and  $b$  (you need not solve for them, just give the equation that yields their solution)

2. A solid is subject to the deformation field

**(10 marks)**

$$x_1^* = x_1 + \phi x_2; \quad x_2^* = x_2 + \psi x_1; \quad x_3^* = x_3$$

where  $\phi, \psi$  are functions of time. Determine the limit(s) on  $\phi, \psi$  for the deformation field to represent the deformation of a real material.

3. Consider a bar of uniform cross-section suspended vertically by **fixing** the **centroid** of the upper base, as shown. Thus, an arbitrary line element at the centroid of the upper base has **zero rotation**. Obtain the stresses, strains, and displacements. (Hint: Assume a suitable distribution of stresses  $\sigma_{ij}$  such that all but one stress components are zero, and the non-zero stress varies only with the  $x_3$  coordinate, the variation being linear. Check that your stress distribution satisfies boundary condition, equilibrium, compatibility. Then, use this distribution to obtain strains and hence displacements).

**(20 marks)**

