# CE623 Quiz I Solution.

**P.1**

$I_1 = c + d + 2 = 5 \rightarrow c + d = 3 \rightarrow (1)$

$I_3 = -5b^2 - a(ab) - a(ab) = -5b^2 - 2a^2 b = 2cd \rightarrow (2)$

(2*3) $I_2 = 2c + 2d + cd = -a^2 - a^2 - b^2 \rightarrow (3).$

(3) $(1) \& (3) \rightarrow cd < 0 \rightarrow (4).$

Let $c > d \rightarrow c > 0, d < 0 \rightarrow (4a)$

(2) $(1) \& (4a) \rightarrow c > 3$ and also that $\frac{c-d}{2}$ = max shear stress $= 5.5$

(4) $\Rightarrow c - d = 11 \rightarrow \boxed{c = 7,\ d = -4.}$

$(3) \rightarrow -2a^2 = 2c + 2d + cd + b^2 = -22 + b^2 \rightarrow (5).$

(4) $(5), (2) \rightarrow \boxed{b^3 - 5b^2 - 22b + 56 = 0} \rightarrow$ solve for 'b', then use (2) or (3) to get 'a'.

**P-2** $dV(t) = J(x,t)\, dV(0) \Rightarrow J(x,t) > 0$

(4) (6) $J = \det \left| \frac{\partial x_i^*}{\partial x_j} \right| = \begin{vmatrix} 1 & \phi & 0 \\ \psi & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 - \phi\psi > 0 \Rightarrow \boxed{1 > \phi\psi}$

**P-3.** Given that there is only one non-zero stress, obviously it is $\sigma_{33}$. Further it is given that it is a function of $x_3$ only. Thus,

Static Equil: $\sigma_{33}\big|_{x_3=0} A = \rho A L g \rightarrow \sigma_{33}\big|_{x_3=0} = \rho L g.$

BC at $x_3 = L$ : $\sigma_{33}\big|_{x_3=L} = 0$

Also given that the non-zero component varies linearly with $x_3$.

(3x3) $\Rightarrow \boxed{\sigma_{33} = \rho g (L - x_3)}$ $\boxed{\text{all other } \sigma_{ij} = 0}$

$$\boxed{\begin{aligned}&\varepsilon_{11} = -\frac{\nu}{E}\sigma_{33} = \varepsilon_{22} = -\frac{\nu\rho g}{E}(L-x_3)\\ &\varepsilon_{33} = \frac{\sigma_{33}}{E} = \frac{\rho g}{E}(L-x_3)\\ &\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0.\end{aligned}}$$

4

$$\left.\begin{aligned}&u_{1,1} = u_{2,2} = -\frac{\nu\rho g}{E}(L-x_3)\\ &u_{3,3} = \frac{\rho g}{E}(L-x_3)\\ &u_{1,2}+u_{2,1}=0, \quad u_{2,3}+u_{3,2}=0, \quad u_{3,1}+u_{1,3}=0\end{aligned}\right\} \begin{aligned}&5(a,b)\\ &5(c)\\ &5(d-f)\end{aligned}$$

$$(5c) \rightarrow u_3 = \frac{\rho g}{E}\left(Lx_3 - \frac{x_3^2}{2}\right) + f_3(x_1,x_2) \longrightarrow (6)$$

$$(6),(5(e,f)) \rightarrow \left.\begin{aligned}u_1 &= -x_3 f_{3,1} + f_1(x_1,x_2)\\ u_2 &= -x_3 f_{3,2} + f_2(x_1,x_2)\end{aligned}\right\} \longrightarrow (7).$$

$$(7),(5(a,b)) \rightarrow -x_3 f_{3,11} + f_{1,1} = -\frac{\nu\rho g}{E}(L-x_3)$$

$$\Rightarrow \left.\begin{aligned}f_{3,11} &= -\frac{\nu\rho g}{E}, \quad f_{1,1} = -\frac{\nu\rho g L}{E}\\ f_{3,22} &= -\frac{\nu\rho g}{E}, \quad f_{2,2} = -\frac{\nu\rho g L}{E}\end{aligned}\right\} \rightarrow (8)$$

$$(7),(5(d)) \rightarrow -x_3(f_{3,12}+f_{3,21}) + f_{1,2} + f_{2,1} = 0$$

$$\Rightarrow f_{3,12}=0, \quad f_{1,2} = -f_{2,1} \longrightarrow (9).$$

$$(8,9) \rightarrow \begin{aligned}f_1 &= -\frac{\nu\rho g L}{E}x_1 + a_1x_2 + c_1\\ f_2 &= -\frac{\nu\rho g L}{E}x_2 - a_1x_1 + c_2\\ f_3 &= -\frac{\nu\rho g}{2E}(x_1^2+x_2^2) + a_3x_1 + b_3x_2 + c_3\end{aligned}$$

3

$$u_1 = -\frac{\nu \rho g}{E}(L-x_3)x_1 - a_3x_3 + a_1x_2 + c_1$$

$$u_2 = -\frac{\nu \rho g}{E}(L-x_3)\,x_2 - a_3x_3 - a_1x_1 + c_2$$

$$u_3 = -\frac{\rho g}{2E}\left[(x_3-2L)x_3 + \nu\left(x_1^2+x_2^2\right)\right] + a_3x_1 + b_3x_2 + c_3 \quad \longrightarrow (10)$$

BC's

(i) $u_1 = u_2 = u_3$ at $(x_1, x_2, x_3) = (0,0,0) \longrightarrow c_1 = c_2 = c_3 = 0$ (by observation).

ii) $\bar{\omega}_{ij} = u_{i,j} - u_{j,i} = 0$ at $(x_1, x_2, x_3) = (0,0,0)$ (ie zero rot at origin).

$$\bar{\omega}_{12} = a_1 + a_1 = 0 \Rightarrow a_1 = 0.$$

$$\bar{\omega}_{23} = \cancel{\frac{\nu\rho g}{E}x_2} - a_3 - \cancel{\frac{\nu\rho g}{E}x_2} + b_3 = 0 \qquad \Rightarrow b_3 = 0.$$

$$\bar{\omega}_{13} = \cancel{\frac{\nu\rho g}{E}x_1} - a_3 - \cancel{\frac{\nu\rho g}{E}x_1} + a_3 = 0. \Rightarrow a_3 = 0$$

Observe that rotation tensor actually vanishes everywhere, ie not only at the origin ~~was~~ is required.

only BC's part (4)

Thus $u_1, u_2, u_3$ are as in (10) with only first term retained in each eqn (ie all constants are zero).