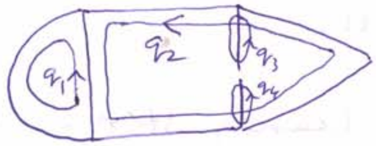
ME623 Quiz II Solution Fall 2005

P.1



$$M = M_1 + M_2 + M_3 + M_4$$

5 $\quad M_3 + M_4 = \frac{G\alpha}{3} \sum a_i \cdot b_i^3 = \frac{G\alpha}{3}\, 2 * \left(3a * (2t)^3\right) = G\alpha\left(16at^3\right)$

5 $$\left\{\begin{aligned} 2G\alpha &= \frac{1}{A_1}\left(a_1 q_1 - a_{12} q_2\right) \\ 2G\alpha &= \frac{1}{A_2}\left(a_2 q_2 - a_{12} q_1\right) \end{aligned}\right\} \Rightarrow \begin{aligned} q_1 &= 2G\alpha \frac{\left(A_1 a_2 + A_2 a_{12}\right)}{a_1 a_2 - a_{12}^2} \\ q_2 &= 2G\alpha \frac{\left(A_1 a_{12} + A_2 a_1\right)}{a_1 a_2 - a_{12}^2} \end{aligned}$$

$$M = M_1 + M_2 + M_3 + M_4$$

5 $$= 2q_1 A_1 + 2q_2 A_2 + M_3 + M_4$$

$$= \underbrace{G\alpha\left[\, 16at^3 + 4\frac{\left(A_1^2 a_2 + A_1 A_2 a_{12} + A_1 A_2 a_{12} + A_2^2 a_1\right)}{a_1 a_2 - a_{12}^2}\right]}_{C/G}$$

5 $$\left\{ A_1 = \frac{9\pi}{2}a^2, \quad A_2 = 48a^2, \quad a_1 = \frac{\pi 3a}{2t} + \frac{6a}{t}, \quad a_2 = \frac{18a}{t} + \frac{10a}{5t} = \frac{20a}{t}, \quad a_{12} = \frac{6a}{t} \right.$$

$$\Rightarrow \frac{C}{G} = 16at^3 + \frac{4\left(\frac{81\pi^2}{4} * 20 + 216\pi * 6 * 2 + 48^2\left[\frac{3\pi}{2} + 6\right]\right) a^5/t}{\left[\left(6 + \frac{3\pi}{2}\right)(20) - 36\right] a^2/t^2}$$

5 $$\Rightarrow C = \left(16at^3 + 826.3\, a^3 t\right)G \simeq \left(826.3 a^3 t\right)G \quad (\because t \ll a)$$

$$\left\{\begin{aligned} (\tau_3)_{max} &= (\tau_4)_{max} = G\alpha\, b_i = G\alpha(2t) \\ q_1 &= 2G\alpha \frac{\left(\frac{9\pi}{2} * 20 + 48 * 6\right)}{\left[(6 + 1.5\pi)20 - 36\right]}\, at = (2G\alpha t) * 3.2019a \\ q_2 &= 2G\alpha \frac{\left(\frac{9\pi}{2} * 6 + 48 \times \left[1.5\pi + 6\right]\right)}{\left[(6 + 1.5\pi)20 - 36\right]}\, at = (2G\alpha t) * 3.3605a \end{aligned}\right.$$

$$\left\{\begin{array}{l}(\tau_1)_{max} = \frac{q_1}{2t} = (2G\alpha a) * \frac{3.2019}{2} \\ (\tau_2)_{max} = \frac{q_2}{t} = (2G\alpha a) * 3.3605\end{array}\right. \quad (5)$$

$\because a \gg t$, $(\tau_2)_{max}$ is max shearing stress — it occurs in horizontal legs of size 6a.

Note: $\because |(q_1 - q_2)| < q_1$ and q_2, the vertical leg of size 6a cannot possess max shear stress.

P-2 Eqn of boundary is,

$$B(x,y) = (y-a)\left(x - [\tan\alpha]y - \underset{2a\tan\alpha}{q_1}\right)\left(x + [\tan\alpha]y - \underset{-2a\tan\alpha}{q_2}\right) = 0$$

(4) $\Rightarrow \underbrace{(y-a)}_{S_1} \cup \underbrace{\left(x - \tan\alpha(y+2a)\right)\left(x + \tan\alpha(y+2a)\right)}_{S_2} = 0$

(4) $I_{xy} = 0$, $W_y = 0 \Rightarrow K_y = 0 \Rightarrow g(x) = 0$. Also $x=0$ $\because$ load applied thru CF.

$$K_x = \frac{W_x}{EI_y} = \frac{P}{EI_y}$$

(4) $f(y) = \frac{1}{2}EK_x x^2$ on S ($= \frac{1}{2}EK_x x^2$ on that part of S for which $\frac{dy}{ds} \neq 0$, ie excluding $y=a$ boundary), i.e., S_2 part of bndry)

$$= \frac{P}{2I_y}(2a+y)^2\tan^2\alpha$$

$$\Rightarrow \nabla^2\phi = 2\overset{E/2(1+\nu)}{G}\nu\frac{P}{EI_y}y - \frac{P}{I_y}(2a+y)\tan^2\alpha$$

$$= \overset{1/3}{\cancel{\frac{\nu}{1+\nu}}}\frac{P}{I_y}y - \frac{P}{I_y}(2a+y)\overset{1/3}{\cancel{\tan^2\alpha}} = \frac{1}{3}\frac{P}{I_y}(-2a) \longrightarrow (1)$$

→ Try $\phi = B(x,y)$ so that $\phi = 0$ on S

(4) Now $\nabla^2\left[(y-a)\left[x^2 - \tan^2\alpha(y+2a)^2\right]\right] = \nabla^2\left[yx^2 - y\tan^2\alpha(y+2a)^2 - ax^2 + a\tan^2\alpha(y+2a)^2\right]$

$$= 2y - 2a + \underset{1/3}{\cancel{\tan^2\alpha}}(-6y - 8a + 2a)$$

$$= -4a.$$

So $\phi = \frac{P}{6I_y}(y-a)\left[x^2 - \tan^2\alpha\,(y+2a)^2\right]$ solves ①.

$$\tau_{xz} = \phi_{,y} + f(y) - \tfrac{1}{2} E K_x x^2 \qquad ④$$

$$\tau_{yz} = -\phi_{,x} - \cancel{g}(x) - \tfrac{1}{2} E \cancel{K_y} y^2$$

$$\tau_{xz} = \frac{P}{6I_y}\left[x^2 - \tfrac{1}{3}(y+2a)^2\right] + \frac{P}{6I_y}\left[-(y-a)\tfrac{2}{3}(y+2a)\right] + \frac{P}{2I_y}\,\tfrac{1}{3}(2a+y)^2 - \frac{P}{2I_y}x^2$$

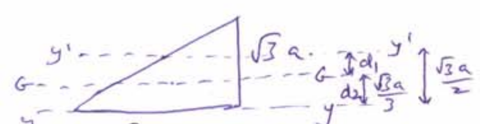
$$= \frac{P}{6I_y}\left[-2x^2 + \tfrac{2}{3}\left(y^2+4a^2+4ay-y^2+2a^2-ay\right)\right]$$

see below for I_y

$$= \frac{P}{6I_y}\left[-2x^2 + \tfrac{2}{3}(6a^2+3ay)\right] = \frac{2\sqrt{3}}{54a^4}P\left[-2x^2+4a^2+2ay\right]$$

$$\tau_{yz} = -\frac{Px}{3I_y}(y-a) = -\frac{2\sqrt{3}}{27}\frac{Px}{a^4}(y-a)$$

Calculation of I_y.



$$I_{y'y'} = \frac{1}{2}\,\frac{3a(\sqrt{3}a)^3}{12}$$

$$I_{GG} = I_{y'y'} - Ad^2$$

$$= \frac{9\sqrt{3}}{24}a^4 - \frac{1}{2}3a\sqrt{3}a\left(\frac{1}{6}\sqrt{3}a\right)^2$$

$$= \left(\frac{9\sqrt{3}}{24} - \frac{3\sqrt{3}}{2}\cdot\frac{3}{36}\right)a^4 = \frac{\sqrt{3}}{4}a^4$$

$$I^*_{yy} = I_{GG} + Ad_2^2$$

$$= \frac{\sqrt{3}}{4}a^4 + \frac{1}{2}3a\sqrt{3}a\left(\frac{1}{3}\sqrt{3}a\right)^2$$

$$= \frac{3\sqrt{3}}{4}a^4$$

$$I_y = I_{yy} = 2I_{yy} = \frac{3\sqrt{3}}{2}a^4$$