

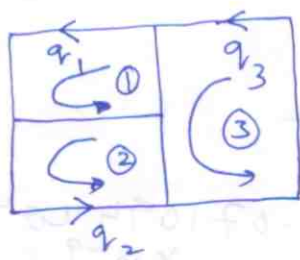
P.1 
$$\tau_i = \frac{3M b_i}{\sum a_i b_i^3} = 3M b_i \frac{6\alpha}{3M} = 6\alpha b_i \text{ for open legs}$$

Case-I Max stress in open legs. Then it must occur in leg CA for which  $b=5\text{mm}$  as compared to leg EF for which  $b=3\text{mm}$ . In that case.

$$\alpha = \frac{\tau_{\max}}{6 \times 5} \rightarrow \textcircled{A}$$

Case-II Max stress in closed loops. There are 3 loops,

ie  $i=1, 2, 3$ .



We have  $2GA_i\alpha = \oint_{i^{\text{th}} \text{ loop}} \frac{q}{t} ds$ .

$$\Rightarrow_{i=1} 2G(100 \times 75)\alpha = \frac{1}{5} [350q_1 - 100q_2 - 75q_3] \rightarrow \textcircled{1}$$

$$i=2: 2G(100 \times 75)\alpha = \frac{1}{5} [350q_2 - 100q_1 - 75q_3] \rightarrow \textcircled{2}$$

$$i=3: 2G(100 \times 150)\alpha = \frac{1}{5} [500q_3 - 75q_1 - 75q_2] \rightarrow \textcircled{3}$$

Soln of  $\textcircled{1}, \textcircled{2}, \textcircled{3}$  is  $q_1 = q_2 = q_3 = q$ .

Put  $q = \tau_{\max} \times 5$  in say  $\textcircled{1}$ , get,

$$75000G\alpha = 175q = 175\tau_{\max} \times 5$$

$$\Rightarrow \alpha = \frac{\tau_{\max}}{G} \left( \frac{875}{75000} \right) \rightarrow \textcircled{B}$$

Compare  $\textcircled{A}$  &  $\textcircled{B} \Rightarrow$  critical case is  $\textcircled{B}$  ie Case II.

$$\Rightarrow M = \underbrace{\sum_{i=1}^3 2q_i A_i}_{M_{\text{loops}}} + \underbrace{\frac{\alpha G}{3} \sum_{i=1}^3 a_i b_i^3}_{M_{\text{legs}}}$$

$$(\alpha G)_{\text{critical}} = (70) \left( \frac{875}{75000} \right); \sum_{i=1}^3 2q_i A_i = 2 \times 70 \times 5 \left( \frac{100 \times 75 + 100 \times 75 + 100 \times 150}{3} \right)$$

$$\sum_{i=1}^3 a_i b_i^3 = 100 \times 5^3 + 75 \times 3^3$$

$$M = 21011 \text{ N-m} \leftarrow$$

$$\therefore q_1 = q_2 = q_3, \tau_{BC} = \tau_{CD} = \tau_{CF} = 0 \leftarrow$$

P.2. BC's are:

$$u_r = 0 \text{ at } r = 100 \text{ mm.}$$

$$\sigma_{rr} = 0 \text{ at } r = 400 \text{ mm.}$$

$$0 = (100)C_1 + \frac{C_2}{(100)} + \frac{1-0.32^2}{8 \times 12 \times 10^9} \times 2000 \omega^2 (0.100)^3$$

$$0 = \frac{12 \times 10^9}{1-0.32^2} \left[ (1+0.32)C_1 - \frac{(1-0.32)C_2}{(0.400)^2} - \frac{(3+0.32)(1-0.32)}{8 \times 12 \times 10^9} \times 2000 \omega^2 (0.400)^2 \right]$$

Solve  $C_1, C_2$  in terms of  $\omega^2$ .

$$0.1C_1 + 10C_2 = 1.87 \times 10^{-11} \omega^2$$

$$1.32C_1 - 4.25C_2 = 9.93344 \times 10^{-9} \omega^2$$

$$\Rightarrow C_2 = \frac{-9.6866 \times 10^{-9} \omega^2}{136.25} = -0.071094 \times 10^{-9} \omega^2$$

$$C_1 = 7.2964 \times 10^{-9} \omega^2$$

$$\sigma_{rr}^* = \frac{\sigma_{rr}}{E/(1-\nu^2)} = 9.631248 \times 10^{-9} \omega^2 + \frac{4.834392 \times 10^{-11} \omega^2}{r^2} - 62.084 \times 10^{-9} \omega^2$$

$$\sigma_{\theta\theta}^* = \frac{\sigma_{\theta\theta}}{E/(1-\nu^2)} = 9.631248 \times 10^{-9} \omega^2 - \frac{4.834392 \times 10^{-11} \omega^2}{r^2} - 36.652 \times 10^{-9} \omega^2$$

$$\frac{d\sigma_{rr}^*}{dr} = 0 = \left( -\frac{9.668784 \times 10^{-11}}{r^3} - 124.168 \times 10^{-9} r \right) \omega^2$$

gives complex  $r$  or  $r=0$  which is out of domain.

So  $\sigma_{rr}^*$  is max at either  $r=100$  mm or  $400$  mm

$$\Rightarrow \left. \sigma_{rr}^* \right|_{r=100} = 1.38448 \times 10^{-8} \omega^2, \quad \left. \sigma_{rr}^* \right|_{400} = 0 \text{ (comes } -4.25 \times 10^{-14} \text{ from computation)}$$

$$\frac{d\sigma_{\theta\theta}^*}{dr} = 0 = \left[ \frac{4.834392 \times 10^{-11} \times 2}{r^3} - 36.652 \times 10^{-9} \times 2 \right] \omega^2 = 0$$

$$\Rightarrow r = 0 \text{ or } (1 - 3.18998 \times 10^{-3})^{1/4}$$

$$= 0.190573 \text{ m} = 190.573 \text{ mm}$$

$$\text{So, } (\sigma_{rr})_{\max} = \frac{E}{1-\nu^2} \sigma_{rr}^* \Big|_{r=100 \text{ mm}} = 185.0909 \omega^2 \text{ N/m}^2$$

$$(\sigma_{\theta\theta})_{\max} = \frac{E}{1-\nu^2} \sigma_{\theta\theta}^* \Big|_{r=190.573 \text{ mm}} = 93.1683 \omega^2 \text{ N/m}^2$$

$\Rightarrow (\sigma_{rr})_{\max}$  is the critical one.

$$(\sigma_{rr})_{\max} = \frac{\sigma_u}{SF} = \frac{20 \times 10^6}{2} \Rightarrow \omega_{\max} = 232.438 \text{ rad/sec}$$

