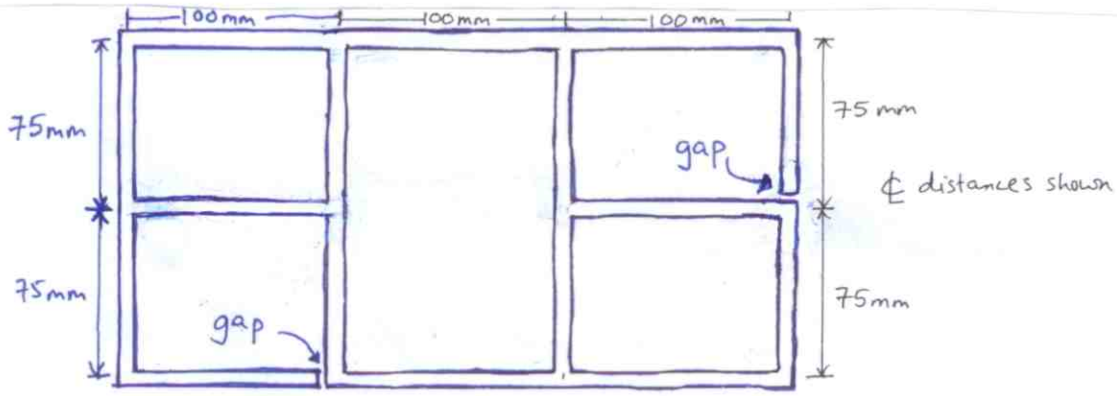
CE623 Advanced Mechanics of Solids Quiz-2 19/10/07

P1=25 marks, P2=25 marks

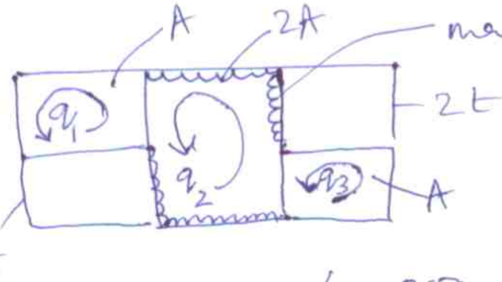
1. A torque M is applied to the thin walled section shown, having shear modulus G and dimensions as indicated in the figure. The open legs have thickness 6 mm while the closed loop legs have thickness 3 mm. Determine:
   (a) The torsional rigidity
   (b) The maximum shear stress and the leg(s) where it occurs.



2. A hollow cylinder with inner radius R and outer radii $2R$ is fitted tightly into a hole of radius $2R$ inside a rigid body. The cylinder is subjected to uniform internal pressure P. Thus, a state of plane strain can be assumed. Determine:
   (a) The stress distribution in the cylinder in terms of ν, R, P.
   (b) The maximum shear stress in the cylinder and its location.
   (c) For a Poisson ratio of 0.3, find the locations where the individual stress components reach their maximum tensile and compressive values.
   (d) The effective external pressure coming on the cylinder.

CE623 Advanced Solid Mechanics Quiz 2 Fall 2007 (1)

P.1



(i)

Loops:
$$2G\alpha A = \frac{1}{t}\left(q_1 350 - q_2 75\right) \longrightarrow (1)$$
$$2G\alpha 2A = \frac{1}{t}\left(q_2 500 - q_1 75 - q_3 75\right) \longrightarrow (2)$$
$$2G\alpha A = \frac{1}{t}\left(q_3 350 - q_2 75\right) \longrightarrow (3) \qquad A = 7500\,mm^2$$

$$(1),(3) \Rightarrow q_1 = q_3.$$
$$(1),(2) \Rightarrow q_1 350 - q_2 75 = q_2 250 - q_1 75$$
$$\Rightarrow q_2 = \frac{425}{325} q_1$$
$$(1) \Rightarrow 2G\alpha A t = q_1\left(350 - 75 * \frac{425}{325}\right)$$
$$\Rightarrow q_1 = 2G\alpha A t\left(\frac{325}{81875}\right)$$

$$M_1 = M_{Loops} = 2 * 2q_1 A + 2q_2(2A) = 4A(q_1 + q_2)$$
$$M_1 = 4 * 7500 * 2G\alpha * 7500 * 3 * \left(\frac{325 + 425}{81875}\right)$$
$$= 12.3664 * 10^6\, G\alpha.$$

Open legs: $M_2 = M_{open\ legs}$.
$$\alpha = \frac{3M_2}{G\sum a_i b_i^3} = \frac{3M_2}{G(2 * 175 * 6^3)}$$

$$C = \frac{M}{\alpha} = \frac{M_1 + M_2}{\alpha} = \frac{G\alpha\left[\frac{350 * 6^3}{3} + 12.3664 * 10^6\right]}{\alpha}$$

$\boxed{C = 12.392 * 10^6\, G}$ $N.mm^2$ if G is in N/mm^2

(ii) Max τ_{sz} in loops correspond to q_2 shear flow

$$(\tau_{sz})_{max,\,loops} = \frac{q_2}{t} = G\alpha\left[2A \times \frac{425}{81875}\right] = 77.86\, G\alpha \ N/mm^2$$

$$(\tau_{sz})_{max,\,legs} = G\alpha (b_i)_{max} = 6G\alpha \ N/mm^2$$

So max occurs in loops, as indicated in fig and value is $77.86\, G\alpha \ N/mm^2 = \frac{77.86\, M}{12.392} \times 10^{-6} \ N/mm^2$

$$\boxed{(\tau_{sz})_{max} = 6.283 \times 10^{-6}\, M} \ N/mm^2$$

P.2. (a) BC's: $-P_i = -P = \sigma_{rr}\big|_{r=r_i=R}$

$$u_r\big|_{r=r_o=2R} = 0$$

Axisymm $\Rightarrow u_\theta = 0 \Rightarrow B = G = H = K = 0$ (as before)
From general axisymm results: (Plane stress).

$$u_r\big|_{r=r_o} = 0 = \frac{1}{E}\left[-\frac{(1+\nu)}{r_o}A + 2C(1-\nu)r_o\right] \rightarrow ①$$

$$\sigma_r\big|_{r=r_i} = \frac{A}{r_i^2} + 2C = -P. \rightarrow ②$$

$$② \text{ in } ① \rightarrow -\frac{(1+\nu)}{r_o}A - \left(P + \frac{A}{r_i^2}\right)(1-\nu)r_o = 0$$

Plane stress $\rightarrow$ $$A = \frac{P(\nu-1)r_o}{\left(\frac{1+\nu}{r_o}\right) + \frac{(1-\nu)r_o}{r_i^2}}, \quad 2C = -P\left[1 + \frac{(\nu-1)r_o}{\left(\frac{1+\nu}{r_o}\right) + \frac{(1-\nu)r_o}{r_i^2}} \cdot \frac{1}{r_i^2}\right]$$

Plane strain conversion $\rightarrow \nu \rightarrow \frac{\nu}{1-\nu}$

$$\Rightarrow A = \frac{P(2\nu-1)/(1-\nu) \times r_o}{\left\{\frac{1}{(1-\nu)r_o} + \frac{(1-2\nu)}{(1-\nu)}\frac{r_o}{r_i^2}\right\}} = \frac{P(2\nu-1)}{\frac{1}{r_o^2} + \frac{(1-2\nu)}{r_i^2}}$$

③

$$2C = -P\left[1 + \frac{(2\nu-1)}{\frac{1}{r_0^2} + \frac{(1-2\nu)}{r_i^2}} \cdot \frac{1}{r_i^2}\right] = -P\left[\frac{1/r_0^2}{\frac{1}{r_0^2} + \frac{(1-2\nu)}{r_i^2}}\right]$$

$$\sigma_{rr} = \frac{A}{r^2} + 2C = \left[\frac{\frac{(2\nu-1)}{r^2} - \frac{1}{(2R)^2}}{\frac{1}{(2R)^2} + \frac{(1-2\nu)}{R^2}}\right] P$$

$= \Delta$, denominator is positive.

let $\frac{P}{\Delta} = K$ (+ve).

$$\sigma_{\theta\theta} = \left[\frac{-\frac{(2\nu-1)}{r^2} - \frac{1}{(2R)^2}}{\frac{1}{(2R)^2} + \frac{(1-2\nu)}{R^2}}\right] P$$

$$\sigma_{zz} = \nu(\sigma_{rr} + \sigma_{\theta\theta}) = -\nu\left[\frac{2/(2R)^2}{\frac{1}{(2R)^2} + \frac{(1-2\nu)}{R^2}}\right] P$$

(b)

$$\frac{\sigma_{rr} - \sigma_{\theta\theta}}{2} = \frac{2(2\nu-1)}{r^2}\left\{\frac{P}{\frac{1}{(2R)^2} + \frac{(1-2\nu)}{R^2}}\right\}\frac{1}{2}$$

$\longrightarrow$ by observation it gives S_{max} for $r = r_i = R$.

$$\frac{\sigma_{rr} - \sigma_{zz}}{2} = (2\nu-1)\left(\frac{1}{r^2} + \frac{1}{(2R)^2}\right)\left\{\quad\right\}\frac{1}{2}$$

$$\frac{\sigma_{\theta\theta} - \sigma_{zz}}{2} = (2\nu-1)\left(-\frac{1}{r^2} + \frac{1}{(2R)^2}\right)\left\{\quad\right\}\frac{1}{2}$$

max shear stress $S_{max} = \left|\frac{2(2\nu-1)}{R^2}\left\{\frac{P}{\frac{1}{(2R)^2} + \frac{(1-2\nu)}{R^2}}\right\}\frac{1}{2}\right|$ ◀

occurs at $r = r_i = R$.

(c) For $\nu = 0.3$,

$$\sigma_{rr} = \left(-\frac{0.4}{r^2} - \frac{1}{(2R)^2}\right)K, \quad \sigma_{\theta\theta} = \left(\frac{0.4}{r^2} - \frac{1}{(2R)^2}\right)K$$

$\sigma_{zz} = -0.6/(2R)^2\, K.$ = constant and compressive throughout. ◀

σ_{rr} = compressive throughout, $(\sigma_{rr})_{max}$ at $r = r_i = R$

$\sigma_{\theta\theta} = 0$ at $r = 1.2649R$ (ie $\sqrt{0.4} \times 2R$)

$\Rightarrow (\sigma_{\theta\theta})_{max\ tensile}$ at $r = r_i = R$, $(\sigma_{\theta\theta})_{max\ compressive}$ at $r = r_o = 2R$.

(d) effective external pressure $= \sigma_{rr}\big|_{r=2R}$

$$= \left[\frac{2(\nu-1)}{4} \Big/ \frac{(5-8\nu)}{4}\right] p$$

$$= \frac{2(\nu-1)}{(5-8\nu)}\, p.$$