Advanced Materials Research Vols. 133-134 (2010) pp 503-508
© (2010) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/AMR.133-134.503

Thrust Line Using Linear Elastic Finite Element Analysis for Masonry Structures

VARMA Mahesh[1, a], JANGID R S[2,b] and GHOSH Siddhartha[3,c]

[1]Nandadeep Building Center, Aurangabad, India

[2,3] Department of Civil Engineering, Indian Institute of Technology Bombay, Mumbai, India

[a]mnvarma@nandadeep.org, [b]rsjangid@civil.iitb.ac.in, [c]sghosh@civil.iitb.ac.in

Abstract Failure of masonry structures are generally studied in terms of the formation of unstable mechanisms and the *thrust line approach* is considered to be the most useful tool for this. Thrust line analysis is a simple technique for studying the stability of masonry structures, although its applicability is limited to specific types of structures because of various implicit assumptions. *Finite element analysis*, on the other hand, is versatile but computationally more intensive. This paper presents a linear elastic finite element analysis based method of obtaining the thrust line of a masonry structure. The proposed method allows the application of the thrust line analysis to structures with any complicated geometry while retaining the simplicity of this approach for studying the stability of a masonry structure. The proposed method is applied to various case study structures and the sensitivity of the results to the adopted material property data in the finite element analysis is studied. The proposed method also allows a structural engineer, who is usually familiar with the finite element analysis, to easily migrate to the stability analysis of masonry systems.

Keywords: Arch, dome, finite element, masonry, thrust line

Introduction

Analysis of masonry structures are performed in various ways which can broadly be categorized into two basic groups: a) the *thrust line* approach and b) the *finite element* approach. Each approach has its pros and cons and these approaches are adopted based on the suitability of analysis in terms of

- if sufficient data are available for the analysis
- complexity in the geometry of the masonry structures and complexity in the material behaviour
- types of results required from the analysis, etc.

The thrust line approach is a relatively simpler approach and cannot be applied for complicated structures. This paper proposes a simple method of combining the thrust line and the finite element approach so that this shortcoming of the thrust line approach is eliminated. It is important to understand these approaches clearly to understand the need to combine these methods to achieve versatility and simplicity with one single approach. This paper then discusses the proposed method and provides two demonstration examples, one with a “thin” arch and the other with a “thick” arch. The last section lists out the significant conclusions of this study.

Thrust Line Method and Finite Element Method

Thrust line method is used from ancient times and is still perused by researchers (Clemente et al. 1995, O’Dwyer 1999, Block et al. 2006). These works are mostly based on the simplified assumptions proposed by Heyman (1969): 1) Stone has no tensile strength, 2) Stone has infinite compressive strength and 3) Sliding failure cannot occur.

This method has two advantages. The equilibrium equations based on these assumptions eliminate the need to obtain the mechanical/material properties of the masonry and rely only on the structure’s geometry and load distributions. The graphical thrust line methods based on Heyman's assumptions are simple to understand and implement. However, these methods are far from being versatile when it comes to handle a structure with complex geometry, boundary conditions and redundancy. Clemente et al. (1995)'s comment provides the essence of the thrust line philosophy: "collapse must be viewed as a geometrical problem rather than a problem in strength of material; failure of an arch is not related to crushing of the material but only to its shape." Clemente et al. have confirmed from analysis of existing bridges that the stresses are usually too low in arch structures to cause any failure due to crushing of material. Failure in such structures is primarily a stability problem, which a thrust line method can very well simulate.

On the other hand, the finite element method focuses on a stress-based analysis. Such an analysis requires a proper estimation of the material properties and the complexity of the analysis increases to a great degree when problems of instability are handled using nonlinear finite element analysis. However, the finite element approach is the only viable option for analyzing arches and domes of complex geometry, boundary conditions, etc., since the thrust line approach can only be applied to structures with simple geometry, boundary conditions and redundancy. Besides, the finite element method can directly account for non-homogeneity of the material.

Masonry finite element modelling can be divided in three categories as: (Zucchini and Lourenco 2004)

1. *Detailed micro-modeling:* Units and mortar in the joints are represented by continuum elements whereas the unit-mortar interface is represented by discontinuum elements. The major difficulty in using this approach is intricacy in predicting mortar joint position or its thickness for new construction as well for old construction. Buhan and Felice (1997), Milani et al. (2006) have pointed out difficulties with this approach in practicing due to numerical difficulties with increased size of the problem.
2. *Simplified micro-modelling:* Expanded units are represented by continuum elements whereas the unit-mortar interface is lumped in discontinuum elements. The limitation of micro modeling applies to this approach as well.
3. *Macro-modeling:* Units, mortar and unit-mortar interface are smeared out in a homogeneous continuum. Mostly homogenization approach is used in predicting the stress value and *the stability is judged by strength criterion*.

The advantages in using finite element with homogenization approach are:

1. Homogenization makes it possible to employ the rough discretization necessary for actual large scale structures (Milani et al. 2006);
2. It gives the possibility to use standard material models and software codes for isotropic materials (Zucchini and Lourenco 2004).

Limitations of using finite element method with homogenization approach are:

1. The type of texture, that is the way in which the blocks and mortar is arranged, deeply influence the mechanical response of masonry (Cecchi and Marco 2002). This texture is usually non predictable in existing structure. This lead to difficulties in predicting homogenized mechanical properties for masonry.
2. The mechanical properties required by model are derived from experimental data and the results are limited to the loading conditions under which the data are obtained. This means that introduction of new material (or even texture) or application of well known material in different loading condition will require separate testing program on masonry specimen for derivation of mechanical properties (Milani et al. 2006, Zucchini and Lourenco 2004).
3. Homogenization is used mostly to predict the failure, based on the stress based criterions. Stress results are sensitive to mechanical properties (Pegon et al 2001), and with slight variation at site conditions, the assumed mechanical properties could lead to incorrect prediction of failure.
4. Stability is not represented in simple terms as it is indicated in the thrust line method (Block et al. 2006).

The work presented in this paper eliminates all the above four limitations of the finite element homogenization approach while retaining the advantages of the thrust line method for masonry structure analysis mentioned earlier. Moreover as the stability is judged using homogenized approach, the need for complex macro modelling is eliminated.

Proposed Methodology

Here the method is proposed to post process the linear elastic FEA stress results for plotting the thrust line. The software tool developed, extracts stress output (in Cartesian coordinate system) generated by the finite element analysis package ANSYS (http://www.ansys.com/) for nodes in ‘Section file’ (generated by user, containing node number for each section). The stresses so extracted and resolved normal to section, ‘S1 and S2’, are used to locate the position of resultant force as shown in Fig. 1.

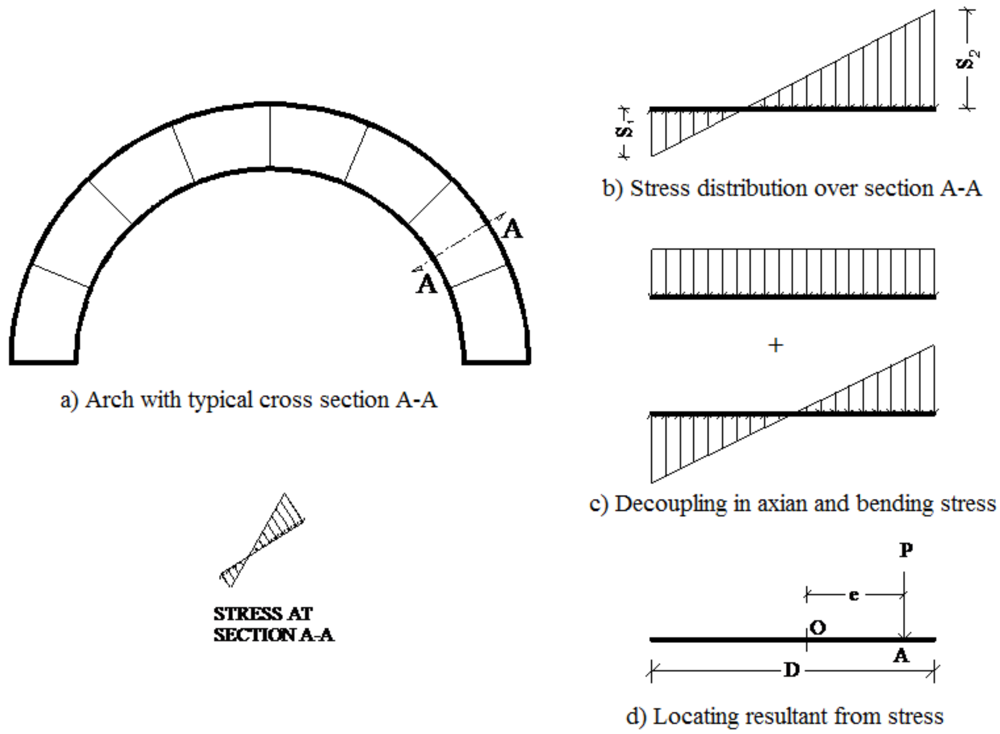

Figure 1: Methodology for plotting thrust line from stress results

Equivalent resultant moment acting over the section is

$$M = (S2 - S1)\, D2 / 12 \tag{1}$$

Equivalent resultant axial force over the section is

$$P = (S1 + S2)\, D / 2 \tag{2}$$

Resultant distance from midpoint of section is

$$e = M / P \tag{3}$$

This eccentricity ‘e’ from middle of section ‘O’ gives point ‘A’, as shown in Figure 1(d). Point ‘A’ represents the position of the resultant of stress over the section. The line of thrust indicates the position of the resultant of the stress acting at a section (O'Dwyer 1999); hence, joining such points along the circumferential length of the arch gives the thrust line. The output from software are ‘Thrust line calculation’ and drawing in ‘.dxf’ format as shown in the next section.

Demonstration Examples

The two arches for these demonstration problems is taken from Block et al.(2006) with thickness to Radius (t/R) ratio of 0.08 (referred as thin arch) and 0.16 (referred as thick arch) is considered for demonstration.

Block et al., through this example, revealed: "The FEA outputs of the two arches are very similar and it is difficult for the elastic FE analyst to note any significant difference between the two arches. A simple thrust line analysis immediately reveals the major difference between the two arches."

Problem Description Geometry and material properties considered for modelling are summarized in Table 1 and Table 2. 'Plane42' element is used in ANSYS for finite element modelling with plane stress option. Nodes at springing level of arch are restrained in both x and y directions.

Table 1: Geometry

Problem	*Radius (mid surface)*	*Thickness*
a) t/R = 0.08	5 meter	0.4 meter
b) t/R = 0.16	5 meter	0.8 meter

Table 2: Material Properties (Assumed for analysis)

Property	*Value*
Modulus of elasticity	2×10^9 N/m^2
Poisson's ratio	0.1
Density	2000 Kg/ m^3

Thin Arch Analysis The analysis result of thin arch is summarized in Table 3. Thrust line departs at 'Section 1' at bottom and from 'Section 9' at top as shown in Figure2. This thin arch is not safe as the thrust line is not contained within arch geometry (Heyman, J. 1967, Heyman, J. 1969)

Table 3: Thrust Line calculation for thin arch

Section	*S1*	*S2*	*P*	*M*	*e*	*e/D*
1	624060.00	-909010.00	-56990.00	20440.93	-0.36	**-0.90**
2	-11770.77	-272975.08	-56949.17	3482.72	-0.06	-0.15
3	-364139.63	84722.05	-55883.52	-5984.82	0.11	0.27
4	-479375.12	221144.84	-51646.05	-9340.27	0.18	0.45
5	-427204.51	199893.71	-45462.16	-8361.31	0.18	0.46
6	-275420.44	82817.26	-38520.64	-4776.50	0.12	0.31
7	-85757.64	-73677.28	-31886.98	-161.07	0.01	0.01
8	89655.37	-221975.14	-26463.96	4155.07	-0.16	-0.39
9	211343.36	-325978.13	-22926.95	7164.29	-0.31	**-0.78**
10	254690.00	-363200.00	-21702.00	8238.53	-0.38	**-0.95**

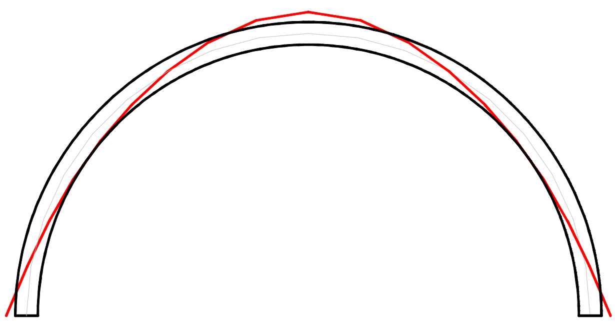

Figure 2: Thrust Line for thin Arch

Thick Arch Analysis The arch with t/R = 0.16,thick arch, is approaching safety limit by forming three hinges (Heyman 1969) as shown in Figure 3 and Table 4.

Table 4: Thrust Line calculation for thick arch

Section	*S1*	*S2*	*P*	*M*	*e*	*e/D*
1	245200.00	-545450.00	-120100.00	42168.00	-0.35	-0.44
2	-111336.50	-176051.35	-114955.14	3451.46	-0.03	-0.04
3	-277320.73	-4516.21	-112734.78	-14549.57	0.13	0.16
4	-320458.31	61096.74	-103744.63	-20349.60	0.20	0.25
5	-277980.60	50990.84	-90795.90	-17545.14	0.19	0.24
6	-185659.94	-5131.33	-76316.51	-9628.19	0.13	0.16
7	-76098.25	-80195.42	-62517.47	218.52	0.00	0.00
8	23221.39	-151338.47	-51246.83	9309.86	-0.18	-0.23
9	91479.78	-201239.45	-43903.87	15611.69	-0.36	-0.44
10	115700.00	-219100.00	-41360.00	17856.00	-0.43	**-0.54**

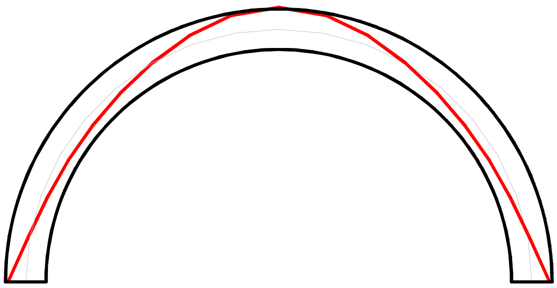

Figure 3: Thrust line for thick arch

Above, the thrust line is plotted using finite element output to compare with the thrust line shown by Block et al and it can be seen that the outcome from two works are identical. This demonstrates the capability of linear elastic finite element analysis to interpret stability in simple terms using thrust line.

Effect of Modulus of Elasticity on Thrust Line

Table 5: Thrust Line calculation for thin arch with higher modulus of elasticity

Section	*S1*	*S2*	*P*	*M*	*e*	*e/D*
1	624060.00	-909010.00	-56990.00	20440.93	-0.36	**-0.90**
2	-11770.77	-272975.08	-56949.17	3482.72	-0.06	-0.15
3	-364139.63	84722.05	-55883.52	-5984.82	0.11	0.27
4	-479375.12	221144.84	-51646.05	-9340.27	0.18	0.45
5	-427204.51	199893.71	-45462.16	-8361.31	0.18	0.46
6	-275420.44	82817.26	-38520.64	-4776.50	0.12	0.31
7	-85757.64	-73677.28	-31886.98	-161.07	0.01	0.01
8	89655.37	-221975.14	-26463.96	4155.07	-0.16	-0.39
9	211343.36	-325978.13	-22926.95	7164.29	-0.31	**-0.78**
10	254690.00	-363200.00	-21702.00	8238.53	-0.38	**-0.95**

Heyman considered masonry analysis based on geometrical parameters only with no relevance to mechanical properties of material, the same is found used by Block and O'Dwyer. Here, the assumption is validated by plotting thrust line for thin arch with modulus of elasticity, E, as 2×10^{10} N/m^2.

The thrust line is not found shifted, as per Table 5 with ten times change in modulus of elasticity of material and hence for simple cases the hypothesis made by Heyman can be justified.

Conclusion

This paper demonstrates the capability of linear elastic finite element analysis to incorporate thrust line and open new possibility to study complex masonry structures for stability. Following are the primary advantages of using this method:

- This method is an extension of finite element method, and hence can be readily used by large group of engineers and researchers, acquainted with finite element, to study historical masonry structures without learning new methodology.
- This method offers versatility of finite element for incorporating complex geometry and boundary conditions.
- The other methods based on only the geometry of structure and loading distribution for plotting thrust line fails to consider effect of different masonry type in three leaf masonry. Present method is amenable to include effect of different modulus of elasticity on thrust line of three leaf masonry structure.

References:

[1] Block, P, Ciblac, T, and Ochsendorf, J (2006). “Real time limit analysis of vaulted masonry buildings.” *Computers and Structures*, 84, 1841-1852.
[2] Buhan, P, and Felice, G (1997). “A homogenization approach to the ultimate strength of brick masonry.” *J. Mech. Phys. Solids*, 45, 1085-1104.
[3] Cecchi, A, and Marco, R (2002). “Homogenized strategy towards constitutive identification of Masonry.” *Journal of Engineering Mechanics*, 128, 688-697.
[4] Clemente, P, Occhiuzzi, A, and Rathel, A (1995). “Limit behavior of stone arch bridges.” *Journal of Structural Engineering*, 121, 1045-1050.
[5] Heyman, J (1967). “On shell solutions of masonry domes.” *International Journal of Solids and Structures*, 3, 227-241.
[6] Heyman, J (1969). “The safety of masonry arches.” *International Journal Mech. Sci.*, 11, 368-385.
[7] Milani, G, Lourenco, P, and Tralli, A (2006). “Homogenised limit analysis of masonry walls, Part I: Failure Surfaces.” *Computers and Structures*, 84, 166-180.
[8] O'Dwyer, D (1999). “Funicular analysis of masonry vaults.” *Computers and Structures*, 73, 187-197.
[9] Pegon, P, Pinto, A, and Geradin, M (2001). “Numerical modelling of stone-block monumental structures.” *Computers and Structures*, 79, 2165-2181.
[10] The ANSYS software website. [Online]. http://www.ansys.com/
[11] Zucchini, A, and Lourenco, P (2004). “A coupled homogenisation - damage model for masonry cracking.” *Computers and Structures*, 82, 917-929.