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INCLUSION OF $P-\Delta$ EFFECT IN THE ESTIMATION OF HYSTERETIC ENERGY DEMAND BASED ON MODAL PUSHOVER ANALYSIS

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ABSTRACT

Estimation of hysteretic energy demand is the first significant step in energy-based seismic design of structures. The present paper extends a modal pushover analysis (MPA)-based energy demand estimation method to include the effect of $P-\Delta$ in structures. Efficiency of the extended procedure is tested on three standard steel moment resisting frames by comparing estimates based on this method with results from nonlinear dynamic analyses of MDOF systems for several earthquakes. In addition, three non-standard frames, with artificially increased susceptibility to $P-\Delta$ effects, are also considered. Bias statistics are presented that show the effectiveness of the previously proposed method including the effects of $P-\Delta$ for both the standard and the non-standard designs. The effect of $P-\Delta$ on the actual hysteretic energy demand on a structure is also studied. The MPA-based method including $P-\Delta$ effects remains less demanding on computation and also suitable for adopting in design guidelines.

KEYWORDS: Hysteretic Energy Demand, Modal Pushover Analysis, $P-\Delta$ Effect, Dynamic Stability, Energy-Based Seismic Design

INTRODUCTION

In recent years, many researchers have identified hysteretic energy demand or its equivalent parameters as the demand parameters, which are most closely correlated to seismic damage of structures (Zahrah and Hall, 1984; Fajfar, 1992; Manfredi, 2001). The hysteretic energy demand takes into account the effects of the duration of the earthquake and the cyclic-plastic deformation behavior of the structure. A monotonic demand parameter, such as peak inelastic drift or displacement, cannot represent this cumulative cyclic damage. A design approach based on hysteretic energy demand, thus, has the potential to account for the damage potential explicitly.

The necessity of an energy-based design procedure for future seismic design guidelines has been emphasized by many researchers, including a few attempts at providing a framework for such design procedures. Discussions of these efforts can be found in the works by Ghosh and Collins (2006) and Prasanth et al. (2008). Estimation of hysteretic energy demand on structures is the first significant step in an energy-based design method. With the computing facilities available today, this estimation for a specific structure under a certain earthquake ground motion is not difficult, although it is computation intensive. However, one has to apply this detailed method – nonlinear response history analysis (NL-RHA) of a multi-degree of freedom (MDOF) model – for each individual structure separately. In addition, this method cannot use a single degree oscillator-based design/response spectrum, making this direct method unsuitable for incorporating in a general purpose design methodology based on hysteretic energy demand.

This paper investigates a simpler method for estimating hysteretic energy demand. It is an extension of the work by Prasanth et al. (2008) that used the concepts of modal pushover analysis (MPA) (Chopra and Goel, 2002; Goel and Chopra, 2004) for estimating hysteretic energy demand on MDOF systems. They used multiple modal equivalent single degree of freedom (ESDOF) systems for a structure. This MPA-based approximate method of estimating hysteretic energy demand was found to be very efficient for steel frame structures. However, the multiple modal ESDOF-based method of estimating hysteretic energy demand used by Prasanth et al. (2008) did not consider the effect of gravity load and $P-\Delta$.

The $P-\Delta$ effect ("global/structure P-delta effect") is the second order effect arising due to geometric nonlinearity in the static and dynamic analysis of structures under lateral loads, such as earthquakes. It is the secondary moment effect due to gravity loads combined with large inter-story deformations. This effect may significantly alter the response of an inelastic system susceptible to large deformations during the course of an earthquake. Gravity loads, acting on the large inter-story deformations, may even cause a dynamic instability by reducing the lateral stiffness in a severe ground motion scenario (Bernal 1998). The $P-\Delta$ effect can be very severe on flexible structures, such as steel moment resisting frames, because these flexible structures are subjected to large lateral displacements during a seismic shaking (Gupta and Krawinkler, 1999). A detail review of research works on the various aspects of $P-\Delta$ effects on seismic response of building structures is avoided in this article. Asimakopoulos et al. (2007) compiled a list of literatures available on $P-\Delta$ effect in structures. The present article proposes a modification on the method proposed by Prasanth et al. (2008) to account for the $P-\Delta$ effect, and studies the effectiveness of the modified method for representative low-, mid- and high-rise steel frame building structures, by comparing MPA estimates with the results from NL-RHA of the MDOF system. In addition, the effectiveness of the modified method is checked for non-standard designs that are specifically vulnerable to the $P-\Delta$ effect due to very high gravity loads. The primary focus of this paper is on measuring the effectiveness and checking the robustness of the approximate method through various case studies. The effect of $P-\Delta$ on the computed hysteretic energy demand is also studied. It should be noted that the objective here is not to find out how damage (in terms of hysteretic energy demand) is distributed in the structure. This paper follows the concept of using the overall energy demand in a structure as design criterion, as proposed by previous researchers (Fajfar, 1992; Ghosh and Collins, 2006).

INCLUDING THE $P-\Delta$ EFFECT IN AN MPA-BASED ANALYSIS

If large inelastic deformations occur during an earthquake, the $P-\Delta$ effect in a structure becomes significant as it further increases the displacement and may reduce the lateral load carrying capacity. It may even result in the dynamic instability or the collapse of a story. Based on these considerations, the inclusion of $P-\Delta$ effect becomes necessary for estimating inelastic force and deformation parameters through nonlinear analysis (Gupta and Krawinkler, 1999).

In MPA (Chopra and Goel, 2002), the elastic mode shapes and frequencies are used to formulate nonlinear ESDOF models for each mode. The nonlinear characteristics of each mode, such as yield point and strain hardening stiffness ratio, are obtained through a nonlinear static pushover analysis using a mode-specific lateral force distribution. The use of multiple modal ESDOF systems in MPA overcomes the limitation of traditional pushover analysis of not being able to account for the higher mode effects. The MPA was used for estimating seismic force and displacement demands on nonlinear (inelastic) as well as linear elastic MDOF systems with sufficient closeness to results obtained from a response history analysis. The advantage of MPA is that it achieves this degree of accuracy without losing the conceptual simplicity and computational attractiveness of the traditional pushover procedure. Goel and Chopra (2004) modified this basic MPA method of estimating inelastic demands by including the $P-\Delta$ effect in the nonlinear static pushover analyses. The modified method was tested on the 9- and 20-story SAC steel frames from Boston, Los Angeles and Seattle, USA. It was observed that $P-\Delta$ effects increased the bias in MPA-based estimation of story drift ratios (over 40% for the Los Angeles 20-story building), where the bias was defined as the ratio of the NL-RHA-based estimation of a parameter to the MPA-based estimation of that parameter.

A similar modification is adopted herein for the MPA-based estimation of hysteretic energy demand. The $P-\Delta$ effect is included in the nonlinear pushover analysis corresponding to each mode. The lateral force distribution $\mathbf{f}_n$ for the nth mode pushover analysis is obtained based on the nth mode shape, after normalizing $\mathbf{m}\boldsymbol{\phi}_n$ to a unit base shear ($\mathbf{f}_n = \mathbf{m}\boldsymbol{\phi}_n / \boldsymbol{\iota}^T\mathbf{m}\boldsymbol{\phi}_n$); here, $\mathbf{m}$ = mass matrix, $\boldsymbol{\phi}_n$ is the nth mode shape normalized to roof displacement component $\phi_{rn} = 1$, $\boldsymbol{\iota}$ = influence vector. For each mode, the pushover analysis is carried out to a maximum interstory drift of 2.5%. As mentioned in similar works earlier (Ghosh and Collins, 2006; Prasanth et al., 2008), the results do not change significantly if a higher maximum drift is considered in pushover analyses including $P-\Delta$ effects. The base shear V_n vs. roof displacement D_n "pushover" curve is approximated by a bilinear function by equating the areas underneath the curves. This bilinear curve gives the elastic stiffness K_{pon}, the yield displacement D_{yn} (= V_{yn}/K_{pon}), and the strain hardening stiffness ratio α_{kn}, from which the critical parameters for the nth mode equivalent system are obtained as described by Prasanth et al. (2008). The inclusion of the $P-\Delta$ effect in the pushover analysis changes the parameters for the corresponding modal ESDOF system. It primarily changes its strain-hardening stiffness ratio (α_{kn}), and yield force (V_{ny}); however, it can also affect elastic parameters such as stiffness (K_n), and period (T_n) or frequency (ω_n). The governing equation of motion for the nth modal ESDOF system subjected to a ground acceleration $\ddot{u}_g$ is written as

$$\ddot{q}_n + 2\xi_n \omega_n \dot{q}_n + \omega_n{}^2 G_n\left(q_n, \operatorname{sign} \dot{q}_n\right) = -\Gamma_n \ddot{u}_g \tag{1}$$

where, ξ_n = modal damping ratio, G_n expresses the force-deformation relation based on the bilinearized pushover plot, and Γ_n is the participation factor for the nth mode, given as

$$\Gamma_n = \frac{\phi_n^{\mathrm{T}} \mathbf{m}\,\iota}{\phi_n^T \mathbf{m} \phi_n} \tag{2}$$

The hysteretic energy demand in each mode (E_{nh}) is obtained by solving the nonlinear dynamic relation of Equation (1). Since E_{nh} is a cumulative (non-decreasing) function in time, the peak hysteretic energy will always occur at the end of the analysis. A simple way to combine the individual E_{nh} values is to add them together. However, this is still an approximation because it ignores any coupling in the inelastic domain that may occur (Prasanth et al., 2008).

The geometric nonlinearity due to flexural deformations within a member, or the $P-\delta$ effect (the "member P-delta" effect) is not considered here because the focus is on the overall building response. Adam and Krawinkler (2004) found this effect to be mostly insignificant for overall seismic response of building structures.

CASE STUDY 1: SAC STEEL FRAMES IN LOS ANGELES

In order to test the effectiveness of the modified MPA-based hysteretic energy demand estimation method, it is used to predict energy demands for the 3-, 9- and 20-story "pre-Northridge" SAC Steel moment frame buildings in Los Angeles, CA, USA (Gupta and Krawinkler, 1999) subjected to 21 ground motion records. These buildings are selected for this case study because they represent standard earthquake-resistant designs and have been used in numerous research studies. Based on linear elastic static considerations, these buildings are not expected to show significant susceptibility to $P-\Delta$ effects. A story-specific stability coefficient is used, similar to Equation (4.1) in the report by Gupta and Krawinkler (1999), to measure this susceptibility based on a linear static pushover analysis with the IBC 2006-based lateral force distribution (ICC, 2006):

$$\theta_i = \frac{P_i \Delta_i}{V_i h_i} \tag{3}$$

where, P_i = total vertical load above floor level i, Δ_i = relative deformation at the ith story, V_i = ith story shear, and h_i = ith story height. The maximum stability coefficients obtained for these 3-, 9- and 20-story buildings are 0.030, 0.069 and 0.112, respectively. A value of 0.1 is often cited as a limit above which $P-\Delta$ effects should be considered in design. The effectiveness of the method proposed by Prasanth et al. (2008) is also tested for another set of buildings where significant $P-\Delta$ effects are expected. This will be discussed in the next section.

It should be mentioned that the use of a stability coefficient similar to Equation (3) to predict the vulnerability of a structure to $P-\Delta$ effects is highly questionable. FEMA-355C (FEMA, 2000) states that it "provides inadequate information on the occurrence of a negative post-mechanism stiffness and against excessive drifting of the seismic response." This coefficient cannot properly account for the inelastic and dynamic effects. Gupta and Krawinkler (1999) pointed out that $P-\Delta$ effects are very sensitive to ground motion characteristics once negative post-yield stiffness is attained. However, despite these shortcomings, it is used in this paper because of its familiarity to engineers and researchers.

The Los Angeles SAC building details, including gravity loads, can be obtained from previous publications (Gupta and Krawinkler, 1999; Ghosh, 2003), and are avoided here. The details of the ground motion records, which include the 18 records used by Prasanth et al., along with three additional records (from San Fernando, USA (1971), Tabas, Iran (1978) and Loma Prieta, USA (1989)), are available in a detailed report (Roy Chowdhury, 2008). Only the North-South moment frames of these symmetric buildings are considered for static pushover and nonlinear response history analyses. These analyses are performed using the software DRAIN-2DX (Prakash et al., 1993). The analysis methods and considerations are the same as those adopted by Prasanth et al. except for including the $P-\Delta$ effect in pushover analyses as discussed in the previous section. The nonlinear response-history analyses are carried out by applying the earthquake acceleration at the base of the 2-D frame. The frame members are modeled using the plastic hinge beam column element (Type 02), with rigid-plastic hinges at the ends of a member. P-M interaction is considered in for the hinge capacity. The material behaviour is assumed to be bilinear with 5% strain-hardening stiffness ratio. Strength and stiffness degradations are neglected in the hysteretic behaviour. The flexibility of the joint panel zones and the lateral stiffness of the gravity frames are not considered. A 5% Rayleigh damping is used (for the first two modes) for the NL-RHA. Although Prasanth et al., recommended using only the first three modes even for the 9- and 20-story buildings, here the first five modes are considered for energy estimations while including $P-\Delta$ effects. The various results from this set of case studies are discussed next.

The $P-\Delta$ effects are significant on some ESDOF parameters, primarily on the strain-hardening stiffness ratio (α_{kn}). It also affects other parameters, such as the yield force V_{ny} and stiffness K_n. Table 1 presents, for example, the change in some of these parameters for the inclusion of the $P-\Delta$ effect in pushover analyses for the first five modes of the 20-story frame. The effect of $P-\Delta$ is most significant for the first mode. These observations are similar to those reported by Goel and Chopra (2004).

Table 1: ESDOF Parameters for the First Five Modes of the 20-Story SAC Steel Frame

Mode	**without $P-\Delta$**			**with $P-\Delta$**		
	α_{kn} **(%)**	V_{ny} **(kN)**	T_n **(s)**	α_{kn} **(%)**	V_{ny} **(kN)**	T_n **(s)**
1	13.4	2821	3.81	–9.20	2439	3.99
2	3.00	8200	1.32	3.74	6562	1.32
3	4.76	10220	0.766	3.24	9889	0.771
4	5.85	14370	0.543	4.87	14010	0.552
5	9.10	19490	0.414	8.18	19030	0.414

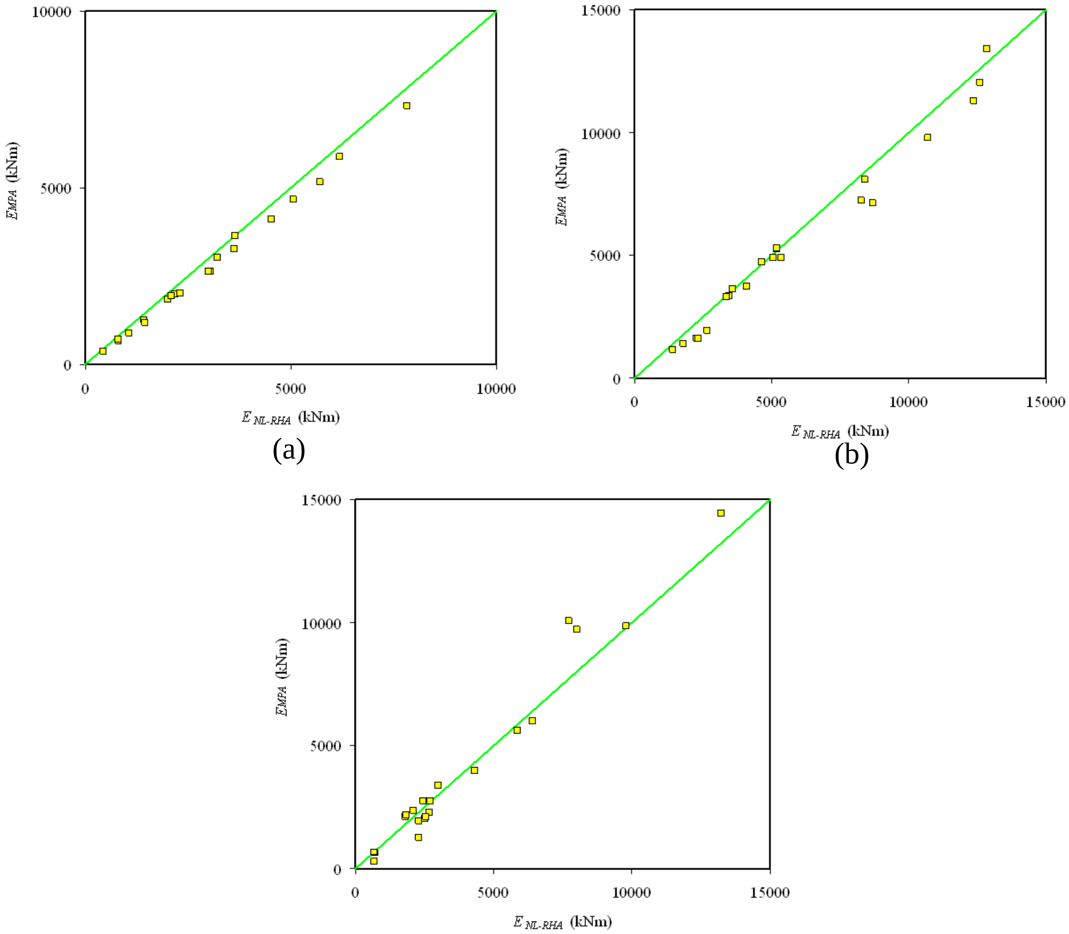
Table 2: Bias Statistics in the Estimation of Hysteretic Energy Demand for the Original SAC Steel Frames

	3-story		**9-story**		**20-story**	
	without $P-\Delta$	**with** $P-\Delta$	**without** $P-\Delta$	**with** $P-\Delta$	**without** $P-\Delta$	**with** $P-\Delta$
Mean	1.08	1.13	1.11	1.12	1.38	1.05
SD	0.057	0.066	0.150	0.147	0.412	0.228
CoV	0.053	0.059	0.135	0.131	0.299	0.218
Maximum	1.18	1.25	1.44	1.44	2.56	1.81
Minimum	0.954	1.00	0.939	0.958	0.906	0.767

The accuracy of the MPA-based estimation is measured using the statistics of a bias factor defined as

$$N_{MPA} = \frac{E_{NL-RHA}}{E_{MPA}} \tag{4}$$

where, $E_{NL\text{-}RHA}$ is the hysteretic energy demand based on the nonlinear response history analysis of the MDOF model of the actual structure, and E_{MPA} is the hysteretic energy demand based on the MPA-based method. This bias factor is calculated for each earthquake, and its statistics (mean, standard deviation, coefficient of variation, maximum and minimum values) are provided in Table 2. Table 2 also presents the results for the same buildings when no $P-\Delta$ effect was considered (neither in NL-RHA nor in MPA). In addition, Figure 1 provides simple scatterplots that compare the values of $E_{NL\text{-}RHA}$ and E_{MPA} for each earthquake, when $P-\Delta$ effects are included. The diagonal line across a scatterplot represents perfect agreement between the NL-RHA and the approximate analysis technique. These scatterplots provide an easy estimation of how effective the MPA method is when the $P-\Delta$ effect is considered.



(a) (b)

(c)

Fig. 1 Scatterplots comparing E_{MPA} with $E_{NL\text{-}RHA}$, with $P-\Delta$ effects included, for the original SAC Steel (a) 3-story, (b) 9-story, and (c) 20-story buildings

The scatterplots and Table 2 show very clearly that the MPA-based method of hysteretic energy demand, including the $P-\Delta$ effect, provides estimates that are comparable to results from “exact” NL-RHA for most of the cases considered. The mean bias is close to its ideal value of 1.0 and the scatter is also low for all the three frames. The largest discrepancies occur for the 20-story building, a phenomenon which was also observed by Goel and Chopra (2004) for MPA-based displacement estimation.

Table 3 provides a summary of the modal contributions of each ESDOF system to the E_{MPA} estimates presented in Figure 1c for the 20-story building. These results show that considering only the first three modes is sufficient even for the 20-story building. For many of the records, only the first mode contributes significantly. However, for a small number of records the 2nd and 3rd mode contributions are significant. In fact, the 2nd mode contribution is more than the 1st mode contribution in some cases. Prasanth et al. (2008) also observed similar results for these specific records while estimating the hysteretic energy demand for these frames without the $P-\Delta$ effect. As discussed in their paper, these interesting results are attributed to the unique characteristics and frequency content of the input ground motion records.

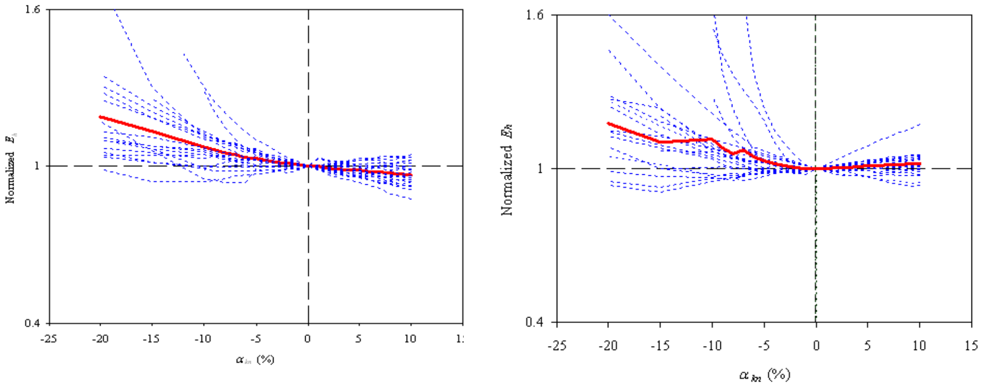
The level of accuracy, measured in terms of the mean bias presented in Table 2, reduces for including the $P-\Delta$ effects by 4.8% and 0.89% for the 3- and 9-story frames, respectively. This may be due to the fact that the effect of $P-\Delta$, based on their stability coefficients, is expected to be insignificant for these buildings. For the 20-story frame, on the other hand, the level of accuracy improves by 24%. It is difficult to ascertain any specific reason why the accuracy improves for the 20-story building. Prasanth et al.

(2008) observed that when no $P-\Delta$ effect was considered, the MPA-based method underestimated the energy demand (on average), and the error (that is, the degree of underestimation) increased for higher buildings. It was suspected that the inability of the MPA-based method to account for inelastic coupling of modes was the reason for this underestimation, which became more significant for the 20-story frame. As shown later in Figure 4a, the actual hysteretic energy demand (that is, $E_{NL\text{-}RHA}$) on these frames does not change significantly for the inclusion or exclusion of the $P-\Delta$ effect. Considering the improvement in the MPA-based estimation while including $P-\Delta$ for the 20-story frame, it can be conjectured that the effect of inclusion of $P-\Delta$ for the MPA-based method is additive in terms of hysteretic energy demand on this structure (that is, the hysteretic energy demand increases if the $P-\Delta$ effect is included for the MPA-based method).

Table 3: Mode-wise Distribution of Hysteretic Energy Demands for the SAC Steel 20-Story Building

Ground motion record	**E_{nh}/E_{MPA} (where, $E_{MPA}=\sum_{n=1}^{5}E_{nh}$)**				
	Mode 1 (%)	**Mode 2 (%)**	**Mode 3 (%)**	**Mode 4 (%)**	**Mode 5 (%)**
s640r005	100	0	0	0	0
s503r005	100	0	0	0	0
s065r005	97.9	2.06	0	0	0
s621r004	100	0	0	0	0
s050r005	89.9	10.1	0	0	0
s212r008	100	0	0	0	0
s305r008	72.1	27.9	0	0	0
s549r009	94.3	5.70	0	0	0
nr	41.4	53.2	5.38	0	0
ns	30.4	67.1	2.49	0	0
chy08036	16.2	81.5	2.27	0	0
chy0809	0	74.4	25.6	0	0
tcu0659	92.7	7.27	0	0	0
syl90	100	0	0	0	0
newh360	62.9	37.1	0	0	0
nh	67.1	32.9	0	0	0
syl360	67.6	32.4	0	0	0
tcu06536	90.9	9.07	0	0	0
lgp00	82.5	14.0	3.45	0	0
pcd164	54.3	45.7	0	0	0
tabln	84.6	15.4	0	0	0

The mean ratio of E_{MPA} with and without the P-Δ effect is found to be 0.979, 0.867 and 1.16, respectively for the 3-, 9-, and 20-story frames. The primary reason for this increase in EMPA for the 20-story frame is the level of reduction in the ESDOF parameter α_{kn}. For example, for the 1st mode ESDOF system, α_{kn} changes from 13.4% to –9.20% for the 20-story frame, 11.8% to 2.88% for the 9-story frame, and 11.4% to 8.91% for the 3-story frame. A simple study is performed to monitor the effect of change in α_{kn} on the hysteretic energy demand on an inelastic SDOF system. For this, two SDOF systems (one having the properties corresponding to the 1st modal ESDOF system of the 20-story frame, and the other corresponding to the 1st modal ESDOF system of the 9-story frame), are analyzed subjected to the selected set of 21 earthquake records, and the hysteretic energy demand (E_h) is monitored with varying α_{kn} values. Figure 2 illustrates the variation in E_h (normalized to E_h at α_{kn} = 0) with α_{kn} (the thick unbroken lines represent the mean value for all the records). The illustration shows that this variation changes with the earthquake record and with the SDOF system selected. As mentioned earlier, the P-Δ effects are very sensitive to ground motion characteristics once the negative post-yield stiffness is attained. The mean curves show that, on average, E_h increases when there is a decrease in α_{kn} in the range of negative post-yield stiffness. For the 1st modal ESDOF of the 20-story frame, this increase occurs for any decrease in α_{kn} from 10% down to –20%. With the change in the other ESDOF parameters being almost insignificant, a definite increase in hysteretic energy demand occurs where the reduction in the post-yield stiffness is more and to the range of 0% and below, that is, for the high-rise frames having high stability coefficients.



(a) (b)

Fig. 2 Variation of hysteretic energy demand (normalized to E_h at $\alpha_{kn} = 0$) with α_{kn} under 21 records (and the mean variation), on the 1st modal ESDOF systems for the (a) 20-story and (b) 9-story frames. (The dashed lines represent results for individual earthquakes and the solid line represents their mean.)

CASE STUDY 2: BUILDINGS WITH INCREASED $P-\Delta$ EFFECTS

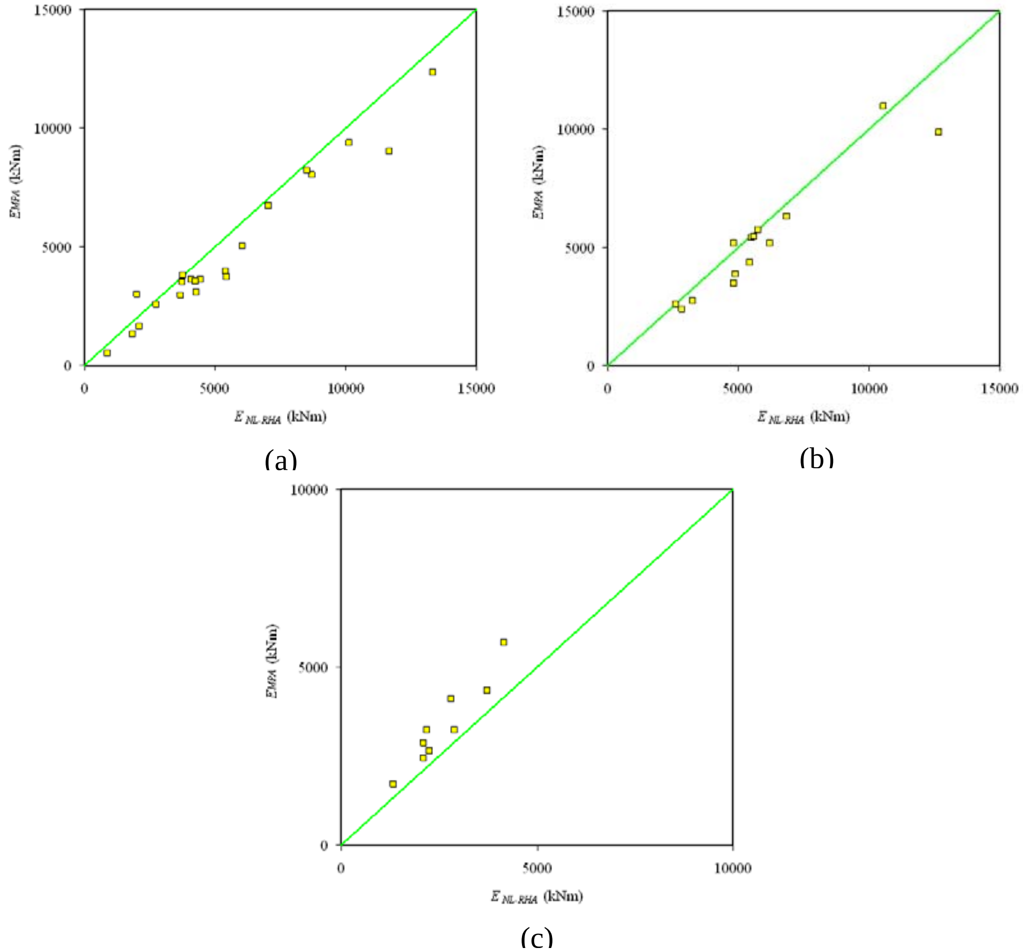
The previously proposed MPA-based method (Prasanth et al., 2008) of estimating hysteretic energy demand is also tested for building frames susceptible to increased $P-\Delta$ effects during large earthquakes. For this case study of non-standard designs, the same 3-, 9- and 20-story “Pre-Northridge” SAC Steel frames from Los Angeles are considered, but with artificially increased gravity loads (along with suitable increase in the inertial masses for response-history analyses). The new 3-, 9- and 20-story frames have maximum stability coefficients of 0.060, 0.137 and 0.168, respectively, as per Equation (3). It should be noted that the new 3-story frame with the increased mass has a stability coefficient less than what is conventionally accepted as the minimum value (= 0.1) for having significant $P-\Delta$ effects. The 9- and 20-story frames with artificially increased stability coefficients become unstable (the DRAIN-2DX solver fails to converge) for several of the large magnitude earthquakes considered in the previous section. Also, the MPA-based method fails to deliver any result for some earthquakes where a very high negative strain-hardening stiffness of the modal ESDOF system causes its collapse. For example, the 1st mode ESDOF system for the new 20-story frame has $\alpha_{kn} = -30\%$, leading to a zero force carrying capacity under several earthquake records. These unstable cases are excluded from the results to follow. Similar cases of instability were also reported by previous researchers (Goel and Chopra, 2004) for MPA-based displacement estimation.

The bias statistics summary for these estimates, based on all 21 records for the 3-story frame, 14 records for the 9-story frame, and 9 records for the 20-story frame, is presented in Table 4. These results (Table 4 and the scatterplots in Figure 3) show that the MPA-based method of estimating hysteretic energy works quite well even for systems where $P-\Delta$ effects are exaggerated. The mean bias for this 9-story frame is at the same level of that for the original SAC 9-story frame discussed in the previous section. For the 3-story frame, the estimates are good, with the level of accuracy slightly deteriorating from that of the original SAC frame. However, for the 20-story frame, the mean bias goes down to 0.789 for the frame with a very high stability coefficient. This may be due to the additive effect of high $P-\Delta$ on SDOF energy demand for the MPA-based method as discussed earlier in the previous section ($\alpha_{kn} = -30\%$ for the 1st modal ESDOF system of the modified 20-story frame). Overall, the results, with mean bias close to 1.0 and low coefficients of variation, show that the MPA-based method is effective even for non-standard designs with very high stability coefficients. The discrepancies introduced by the inclusion of the $P-\Delta$ effect in the MPA-based hysteretic energy demand are lower than those for the MPA-based displacement estimation (Goel and Chopra, 2004). It should be mentioned here that the number of samples for these data is small, specifically for the new 20-story estimates, and conclusions drawn here may need modifications based on a larger sample size.

Table 4: Bias Statistics in the Estimation of Hysteretic Energy Demand for the Modified 3-, 9- and 20-Story Frames

	3-story[a]		**9-story**[b]		**20-story**[c]	
	without $P-\Delta$	**with** $P-\Delta$	**Without** $P-\Delta$	**with** $P-\Delta$	**without** $P-\Delta$	**With** $P-\Delta$
Mean	1.21	1.20	1.18	1.13	1.13	0.789
SD	0.164	0.223	0.159	0.141	0.275	0.085
CoV	0.136	0.186	0.135	0.125	0.244	0.107
Maximum	1.76	1.78	1.47	1.38	1.81	0.895
Minimum	1.05	0.67	1.02	0.935	0.903	0.674

[a] Results based on 21 records.
[b] Results based on 14 records (7 excluded due to instability).
[c] Results based on 9 records (12 excluded due to instability).



(a)

(b)

(c)

Fig. 3 Scatterplots comparing E_{MPA} with E_{NL-RHA} for the (a) 3-story, (b) 9-story, and (c) 20-story buildings with increased stability coefficient

EFFECT OF $P-\Delta$ ON HYSTERETIC ENERGY DEMAND

The effect of the inclusion of $P-\Delta$ on the actual hysteretic energy demand on a structure, as obtained from NL-RHA of the MDOF model, is also studied for the three original SAC Steel frames and the three modified frames for the same sets of acceleration records. Gupta and Krawinkler (1999), based on their study of the SAC Steel frames, concluded that the dynamic effect of $P-\Delta$ can be additive or subtractive in terms of inelastic displacement demand (unlike the static effect which is always additive). However, there are no specific data available in published literature quantifying the effect of $P-\Delta$ in terms of hysteretic energy demand. Based on the results from the three original SAC Steel frames, the $P-\Delta$ effect has almost no impact (mean ratio of E_{NL-RHA} with and without $P-\Delta$ effect is 0.97 for the three frames together, with a standard deviation of 0.057) on the hysteretic energy demand for the whole frame. Figure 4a illustrates this fact for all the three frames using a scatterplot. For the modified 3-, 9- and 20-story frames with higher stability coefficients, the effect of $P-\Delta$ is slightly more; however, there is no specific trend of increase or decrease in the energy demand, with the average demand remaining almost the same (Figure 4b) (mean ratio of E_{NL-RHA} with and without $P-\Delta$ effect is 0.962 for the three frames together, with a standard deviation of 0.117).

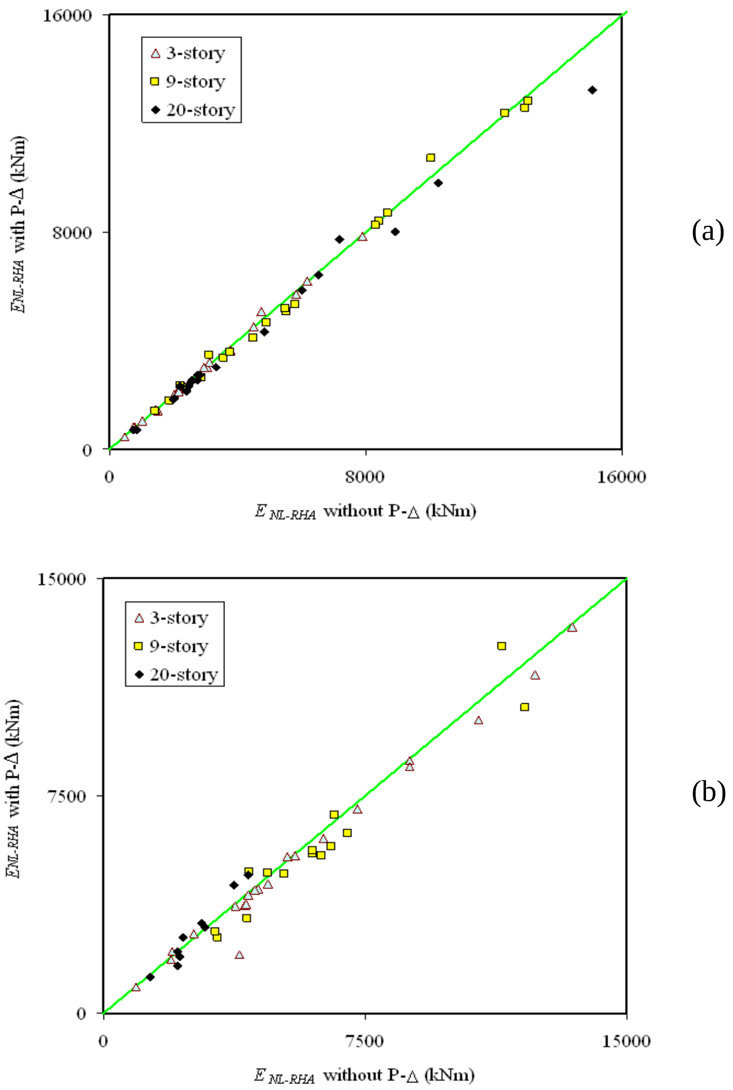


Fig. 4 Comparison of $E_{NL\text{-}RHA}$ with and without the $P-\Delta$ effect for the (a) original SAC Steel frames, and the (b) modified frames with increased stability coefficient

SUMMARY AND CONCLUSIONS

This paper extends a MPA-based hysteretic energy demand estimation technique to include the $P-\Delta$ effects in structures. The modified method is tested on three low-to-high-rise steel frames conforming to design standards and on three non-standard low-to-high-rise steel frames with relatively high susceptibility to $P-\Delta$ effects. The effect of $P-\Delta$ on the hysteretic energy demand (obtained from NL-RHA of the MDOF system) is also studied.

The following general conclusions are drawn based on the study of the effect of inclusion of the $P-\Delta$ effect in the MPA-based method of hysteretic energy demand:

- The proposed procedure (Prasanth et al., 2008) remains a simple and effective method of estimating hysteretic energy demand on a structure even with the inclusion of the $P-\Delta$ effect.
- Based on the analyses of the 3-, 9- and 20-story SAC Steel buildings, this procedure is found to provide consistently good estimates of hysteretic energy demand, with slightly increasing level of accuracy for taller frames.

- The MPA-based method also works well for non-standard designs where very high $P-\Delta$ effects are expected. The level of accuracy goes down when the stability coefficient is increased to a very high value.
- The $P-\Delta$ effect may increase or decrease the value of energy demand estimated using MPA (or E_{MPA}), depending on the amount of negative post-yield stiffness caused by it.
- Based on NL-RHA, the $P-\Delta$ effect does not significantly affect the hysteretic energy demand of a frame. The effect of $P-\Delta$ on hysteretic energy demand increases for buildings with large stability coefficients; however, there is no specific trend of increase or decrease in the demand.

These conclusions are based on the two sets of case studies conducted herein. Future extensions of this work should primarily focus on: a) estimating member/story-level hysteretic energy demands, and b) estimating energy demands for other structures having a different hysteretic behavior (such as reinforced concrete buildings).

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